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Re:	Call for comments and contributions for session#44 (IEEE 802.16j-06/006)	
Abstract	This contribution compares different path-loss and shadow fading models and proposes the appropriate models for performance evaluation in IEEE 802.16j Relay TG.	
Purpose	Response the chair's call for comments on Evaluation Methodology (C802.16j_06/040)	
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Comparison on Path-loss and Shadow Fading Models for IEEE 802.16j Relay Task Group

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1 Introduction

This contribution aims to compare several popular path-loss and shadow fading models to assist IEEE 802.16j Relay TG to conclude appropriate channel models for performance evaluation. Three kinds of channel models are considered here: existing IEEE 802.16 channel model [1], ITU channel model [2] and WINNER channel model [3], which will be introduced in section 2. In section 3, two types of correlation models for simulating shadow fading are introduced. In section 4, comparison on path-loss and shadow fading model will be presented.

2 Path-loss and Shadow Fading Models

The unit of following path-loss and shadow fading models are all in dB.

2.1 IEEE 802.16 Model

The IEEE 802.16 path-loss and shadow fading model is given by [1,4]

$$PL = A + 10 \cdot \gamma \cdot \log_{10}(d/d_0) + \Delta PL_f + \Delta PL_h + s \text{ dB} \quad (1)$$

where $d_0=100\text{m}$, $A=20 \cdot \log_{10}(4 \pi d_0 / \lambda)$ and $\gamma=(a - b \cdot h_b + c / h_b)$. λ is the wavelength in meter and h_b is the base station antenna height, which is between 10m and 80m. "s" is the log-normal shadow fading component in dB and d is the distance in meters. Three propagation scenarios are categorized as

Terrain Type A: Hilly terrain with moderate-to-heavy tree densities

Terrain Type B: Intermediate path-loss condition

Terrain Type C: Flat terrain with light tree densities

The corresponding parameters for each propagation scenario are

Model Parameter	Terrain Type A	Terrain Type B	Terrain Type C
a	4.6	4	3.6
b	0.0075	0.0065	0.005
c	12.6	17.1	20
Standard deviation of "s"	10.6 dB	9.6 dB	8.2 dB

Moreover, the correction factors for carrier frequency (ΔPL_f) and receive antenna height (ΔPL_h) are:

$$\Delta PL_f = 6 \cdot \log_{10}(f / 2000) \text{ dB} \quad (2)$$

where f is the carrier frequency in MHz.

$$\Delta PL_h = -10.8 \cdot \log_{10}(h / 2) \text{ dB} ; \text{ for Terrain Type A and B} \quad (3)$$

$$\Delta PL_h = -20 \cdot \log_{10}(h / 2) \text{ dB} ; \text{ for Terrain Type C}$$

where h is the receive antenna height between 2m and 10m.

2.2 ITU Model

The following ITU models are referenced from [2].

2.2.1 Indoor Office Test Environment

The path-loss model for indoor environment is

$$PL = 37 + 30 \cdot \log_{10}(d) + 18.3 \cdot n^{((n+2)(n+1)-0.46)} \text{ dB} \quad (4)$$

where d is the distance in meters and n is the number of floors in the path.

The corresponding standard deviation of log-normal shadow fading is 12dB.

(Note: There is no frequency correction term for this model in [2], therefore we apply Equation 2 for frequency correction in following comparison results.)

2.2.2 Outdoor to Indoor and Pedestrian Test Environment

The following model is used for the outdoor to indoor and pedestrian test environment:

$$PL = 49 + 40 \cdot \log_{10}(d) + 30 \cdot \log_{10}(f) \text{ dB} \quad (5)$$

where d is the distance in kilometers and f is the carrier frequency of 2000 in MHz

The corresponding standard deviation of log-normal shadow fading is 10dB for outdoor users and 12 dB for indoor users and the average building penetration loss is 12dB with 8dB standard deviation.

2.2.2.1 Model for Manhattan-like Environment

A more detailed model is introduced to consider the line-of-sight (LOS) and non line-of-sight (NLOS) situations:

(Note: There is no frequency correction term for this model in [2], therefore we apply Equation 2 for frequency correction in following comparison results.)

$$PL = \min(PL_{\text{Manhattan}}, PL_{\text{macro}})$$

$$PL_{\text{Manhattan}} = 20 \cdot \log_{10} \left(\frac{4\pi d_n}{\lambda} \cdot D \left(\sum_{j=1}^n s_{j-1} \right) \right) \text{ dB} \quad (6)$$

$$\text{where } D(x) = \begin{cases} x/x_{br} & x > x_{br} \\ 1 & x \leq x_{br} \end{cases}$$

$$PL_{\text{macro}} = 24 + 45 \cdot \log(d + 20) \text{ dB}$$

The parameter illusory distance d_n can be obtained by $k_n = k_{n-1} + d_{n-1} \cdot c$ and $d_n = k_n \cdot s_{n-1} + d_{n-1}$ with $d_0 = 0$ and $k_0 = 1$. The break point x_{br} is set to 300m, and c is set to be 0.5 for 90 degree street crossing in Manhattan-like environment. x is the distance from transmitter to receiver and s_{n-1} is the length of the last segment. d is the shortest physical geographical distance from transmitter to receiver in meters, and other detail description of each parameter can refer to [5] and [6].

2.2.3 Vehicular Test Environment

This model is applicable for the test scenarios in urban and suburban areas outside the high rise core:

$$PL = 40 \cdot (1 - 4 \times 10^{-3} \Delta h_b) \cdot \log_{10}(d) - 18 \cdot \log_{10}(\Delta h_b) + 21 \cdot \log_{10}(f) + 80 \text{ dB} \quad (7)$$

Δh_b is the base station antenna height in meters, which is measured from average rooftop level and valid from 0 to 50m. d is the distance in kilometers, and f is the carrier frequency of 2000 in MHz. The standard deviation of log-normal shadow fading is considered as 10dB here.

2.3 WINNER Model

Currently, only part of the WINNER channel models are in public [3]. We select the following ones whose propagation scenarios are close to the target of IEEE 802.16j project.

2.3.1 WINNER B1 Scenario

This scenario is defined for Manhattan-like urban micro-cell environment where both transmit and receive antenna heights are below surrounding buildings. The models for this scenario are classified as LOS and NLOS cases, which are shown below. (Note: Original B1 and C2 models do not provide the frequency correction term, the terms in following models are referenced from B5a model in [3].)

For B1 LOS scenario:

$$PL = 41 + 22.7 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f/5) \text{ dB} \quad (8)$$

where d is the distance in meters and valid from 10m to 650m.

For B1 NLOS scenario:

$$PL = 65 + 0.096 \cdot d_1 + (28 - 0.024 \cdot d_1) \cdot \log_{10}(d_2) + 20 \cdot \log_{10}(f/5) \text{ dB} \quad (9)$$

where d_1 is the distance along the main street in meters and d_2 is for perpendicular street.

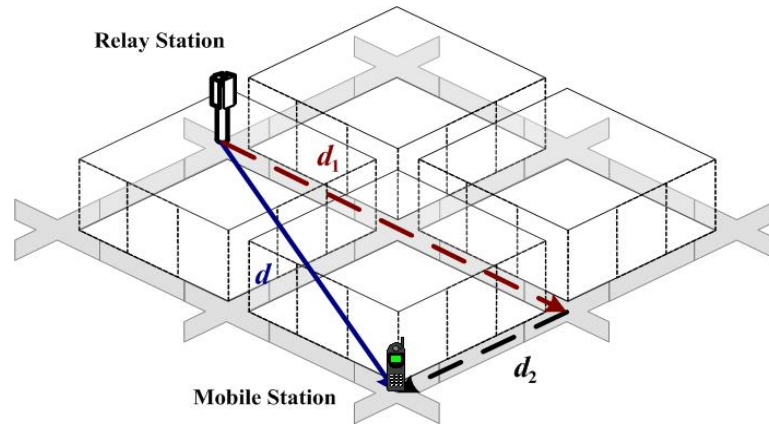


Fig.1 Geography of WINNER B1 NLOS Scenario

The probability to have LOS condition is given as:

$$P_{LOS} = \begin{cases} 1 & d \leq 15m \\ 1 - \left(1 - (1.56 - 0.48 \cdot \log_{10}(d))^3\right)^{1/3} & d > 15m \end{cases} \quad (10)$$

The standard deviation of log-normal shadow fading is 2.3dB for LOS scenario and 3.1 dB for NLOS scenario.

(Note: Currently there is no frequency correction term for this model, the term $20 \cdot \log_{10}(f/5)$ is referenced from WINNER B5a model with the carrier frequency of 5GHz.)

2.3.2 WINNER B5a Scenario

This scenario is defined for LOS stationary transmission, where both transmit and receive antenna are above rooftop.

$$PL = 36.5 + 23.5 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f/2.5) \quad (11)$$

where d is the distance in meters and valid from 30m to 8km, and f is the carrier frequency in GHz.

The standard deviation of log-normal shadow fading is 3.4dB for this scenario.

2.3.3 WINNER C2 Scenario

This scenario is defined for urban macro cell, where base station antenna height is above rooftop and the mobile station is at street level. NLOS transmission is the typical case in this scenario, the model is given as

$$PL = 38.4 + 35 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f/5) - 0.7 \cdot h_m \quad (12)$$

where d is the distance in meters and valid from 50m to 5km, and f is the carrier frequency in GHz.

The standard deviation of log-normal shadow fading is 8dB for this scenario.

(Note: Currently there is no frequency correction term for this model, the term $20 \cdot \log_{10}(f/5)$ is referenced from WINNER B5a model with the carrier frequency of 5GHz.)

3 Correlation Models for Shadow Fading

By given mean (usually equal to 0dB) and standard deviation, the level of shadow fading (in dB) is usually simulated by dropping a Normal distributed random variable. This refers to typical log-normal shadow fading model. However, the correlation of the propagation environment for different observation time or different radio links can not be presented if the simulator drops these variables independently. Two types of correlation models for shadow fading are introduced in this section.

3.1 Auto-correlation Model for Shadow Fading

The auto-correlation of shadow fading indicates the correlation among the shadowing effects among the same radio link in different locations, which is illustrated in Figure 2.

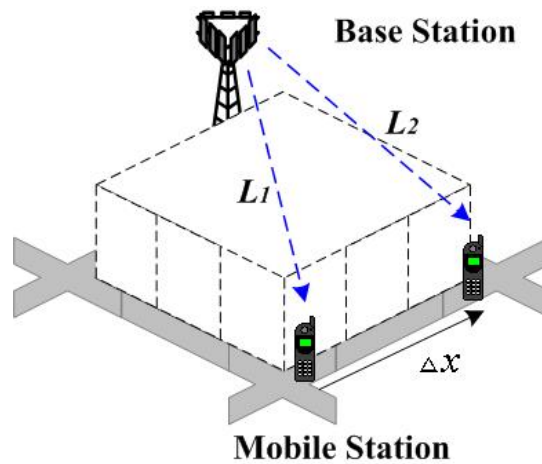


Fig.2 Auto-correlation of shadow fading

A popular model proposed by Gudmundson [7] is well understood in following form [2]:

$$\rho(\Delta x) = e^{-\frac{|\Delta x| \ln 2}{d_{cor}}} \quad (13)$$

where ρ is the correlation coefficient and Δx is the distance between adjacent observation locations. d_{cor} is the de-correlation distance, which was suggested as 20m in vehicular test environment in [2].

The way to apply this model in system level simulation is briefly described as follows:

- ◆ Consider the log-normal shadow fading model with zero mean and variance σ^2 in dB. If L_1 is the log-normal component at position P_1 and L_2 is the one for P_2 , which is Δx away from P_1 . Then L_2 will be normally distributed in dB with mean $\rho(\Delta x) \cdot L_1$ and variance $(1 - [\rho(\Delta x)]^2) \cdot \sigma^2$.

This result can be derived through the analysis in Appendix A [8].

3.2 Cross-correlation Model for Shadow Fading

Instead of the aforementioned auto-correlation model, the cross-correlation of shadow fading indicates the correlation among the shadow effect of different radio links at the same time. In general, longer common propagation path will induce higher correlation. For example, the cross-correlation among the shadow fading of the radio links in Figure 3(a) should be lower than the one in Figure 3(b).

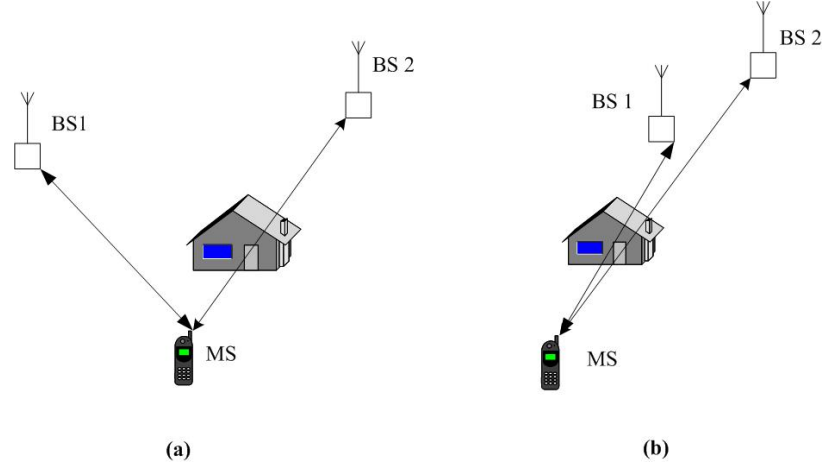


Fig.3 Cross-correlation of shadow fading

A simple cross-correlation model proposed in [9] can present this effect:

$$\rho(\theta) = \begin{cases} 0.8 - \frac{|\theta|}{150} & \text{if } |\theta| \leq 60^\circ \\ 0.4 & \text{if } |\theta| > 60^\circ \end{cases} \quad (14)$$

where ρ is the correlation coefficient and θ is the angle of arrival difference.

More complicated model was also introduced in another contribution [10].

In order to apply this effect in system level simulation, following procedure can be considered:

1. Consider the shadow fading effect of the radio links of one mobile station and N base stations at specific time instance, drop N independent Normal distributed random variables $X = [X_1, X_2, X_3, \dots, X_N]$.
2. Obtain the matrix Γ , which is

$$\Gamma = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{bmatrix} \quad (15)$$

where $\rho_{12} = \rho_{21}$ is the correlation coefficient of the shadow fading component among the radio link to base station #1 and #2.

Since Γ is a symmetric and positive definite matrix, it can be decomposed into a lower and upper triangular matrix by Cholesky decomposition technique. Therefore, $\Gamma = C^T C$, where C is an upper triangular matrix.

3. $Y = XC = [Y_1, Y_2, Y_3, \dots, Y_N]$ will be the cross-correlated log-normal shadow fading components for each radio link.

4 Comparison Results

4.1 Comparison on Path-loss Models

In this section, the aforementioned path-loss models are compared in following figures. The corresponding system parameters are:

- ◆ Carrier frequency: 3.5GHz
- ◆ Base station antenna height: 30m
- ◆ Average rooftop height: 15m
- ◆ Mobile station antenna height: 2m
- ◆ Number of floors in the path for ITU indoor model: 0

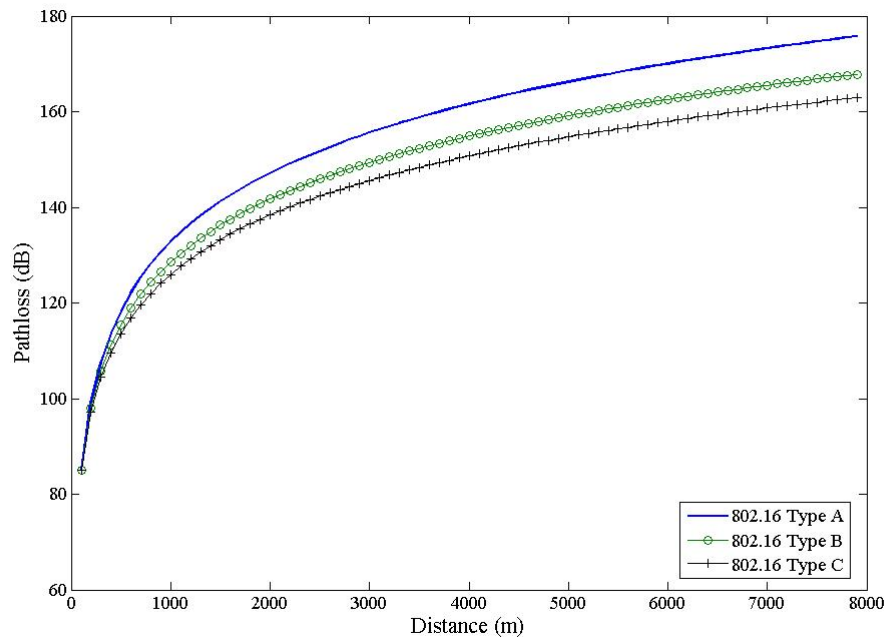


Fig.4 Comparison on IEEE 802.16 path-loss models

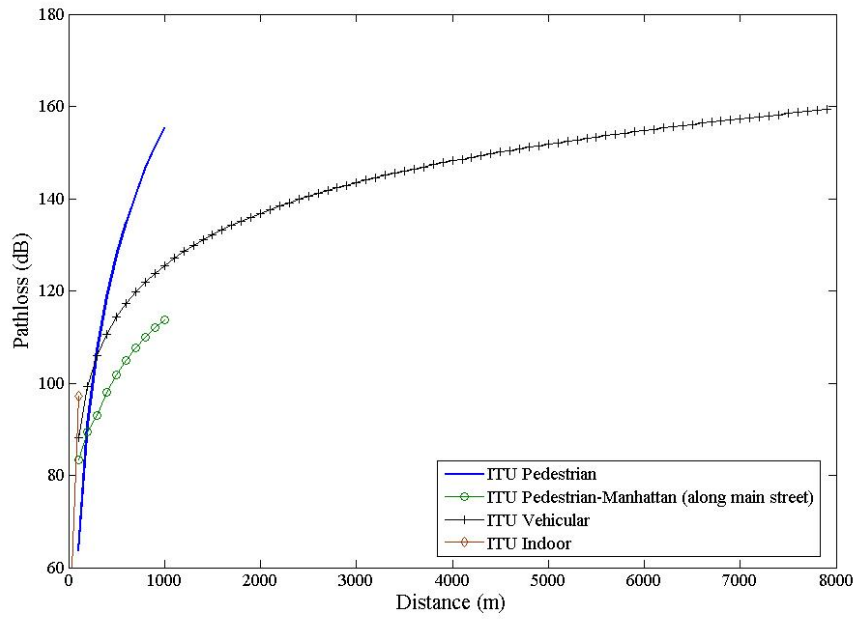


Fig.5 Comparison on ITU path-loss models

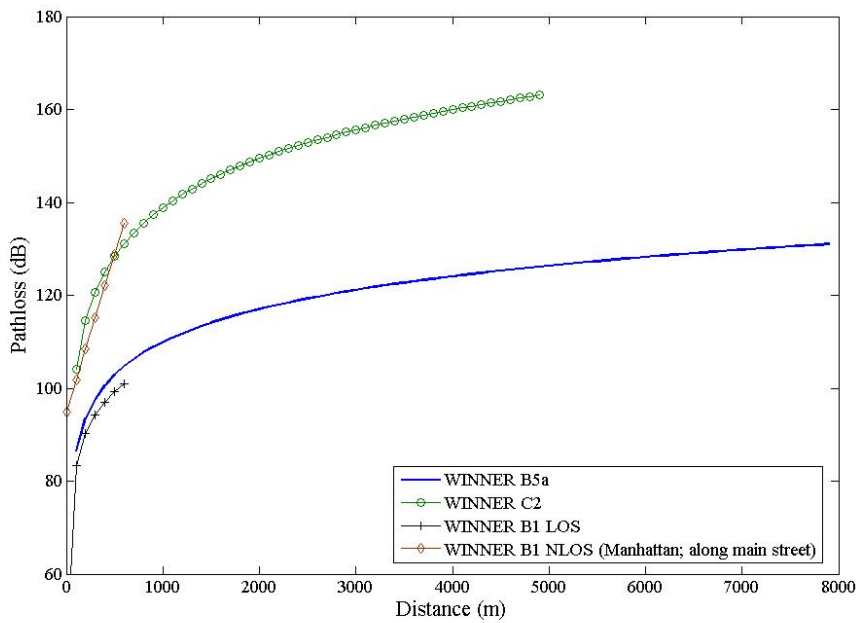


Fig.6 Comparison on WINNER path-loss models

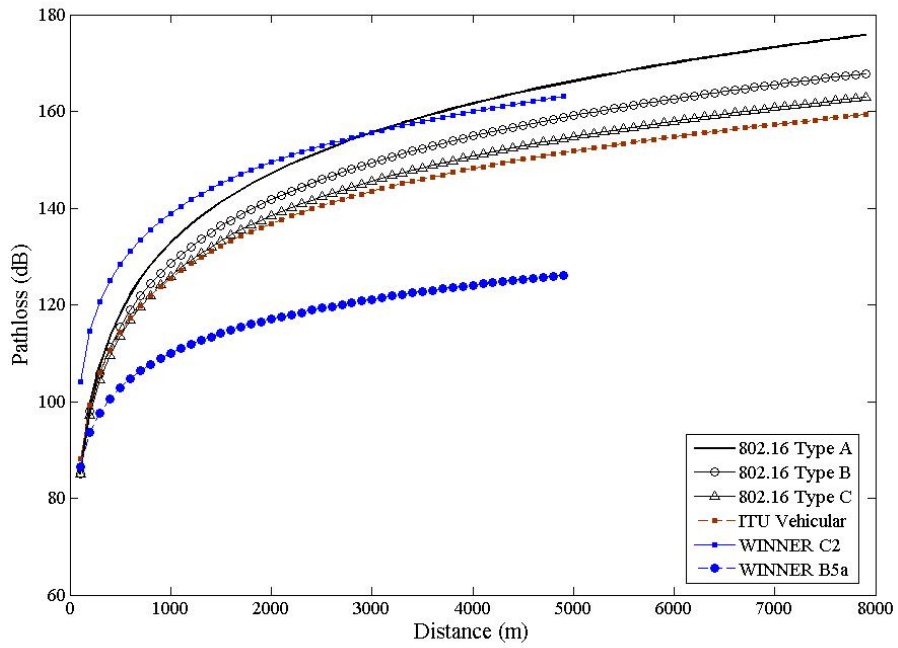


Fig.7 Joint comparison on 802.16, ITU and WINNER path-loss models

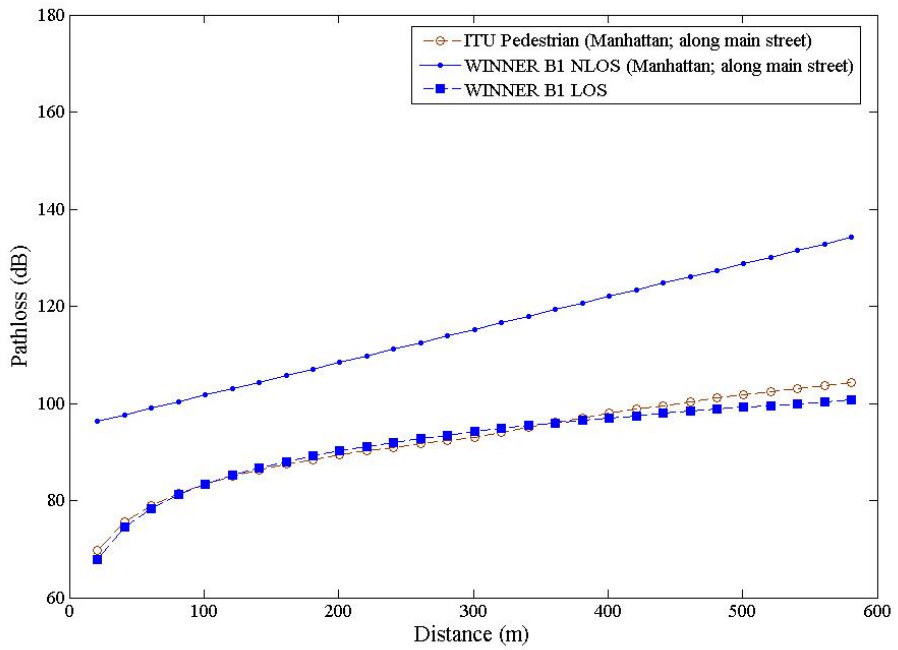


Fig.8 Joint comparison on ITU and WINNER path-loss models

4.2 Comparison on Shadow Fading Models

Scenario	Environment	Standard Deviation of Log-normal Shadow Fading [dB]
IEEE 802.16 Type A	Sub-urban macro cell	10.6
IEEE 802.16 Type B		9.6
IEEE 802.16 Type C		8.2
ITU Indoor	Indoor small office	12
ITU Pedestrian	Urban macro cell	10
ITU Pedestrian - Manhattan	Urban micro cell – Manhattan layout	10
ITU Vehicular	Sub-urban macro cell	10
WINNER B1 LOS	Urban micro cell	2.3
WINNER B1 NLOS	Urban micro cell – Manhattan layout	3.1
WINNER B5a	LOS fixed station (rooftop to rooftop)	3.4
WINNER C2	Urban macro cell	8

Informative Notes:

- ♦ The log-normal fading effect for WINNER B1 NLOS scenario comes from the tunneling effect, and part of the shadowing effect by building has already been incorporated into the path-loss model.
- ♦ The log-normal fading effect for LOS scenario represents the different level of first Fresnel zone clearance. The break distance of the first Fresnel zone is $d_B=4 \cdot h_t \cdot h_r / \lambda$, where h_t is transmit antenna height, h_r is the receive antenna height and λ is the propagation wavelength.

5 Summary

According to aforementioned comparison results, following comments are concluded for C802.16j_06/040 [11]:

1. In order to specify appropriate path-loss and shadow fading models for performance evaluation, we would like to propose following table to be included into C802.16j_06/040:

Category	Propagation Environment	Description	Proposed Model	Applicability Range	Std. of Log-normal Shadow Fading [dB]
Type A	Sub-urban macro cell	Hilly terrain with moderate-to-heavy tree densities	Existing IEEE 802.16 Type A model	$100\text{m} < d < 8\text{km}$	10.6
Type B		Intermediate path-loss condition	Existing IEEE 802.16 Type B model		9.6
Type C		Flat terrain with light tree densities	Existing IEEE 802.16 Type C model		8.2
Type D	LOS fixed stations	Both Tx & Rx antennas are above rooftop	WINNER B5a model	$30\text{m} < d < 8\text{km}$	3.4
Type E	Urban macro cell	Tx antenna is above rooftop but Rx antenna is below rooftop (vice versa)	WINNER C2 model	$50\text{m} < d < 5\text{km}$	8
Type F	Urban micro cell	Both Tx & Rx antennas are below rooftop	WINNER B1 LOS model	$10\text{m} < d < 650\text{m}$	2.3
			WINNER B1 NLOS model	$10\text{m} < d_1 < 550\text{m}$ $15\text{m} < d_2 < 450\text{m}$	3.1
Type G	Indoor	Indoor small office	ITU Indoor model	Indoor range	12

Informative Note:

- I. The carrier frequency from 450MHz to 11.2GHz will be valid by applying the frequency correction term of each model.
- II. The frequency correction term in Equation 2 is proposed to be applied for Type G path-loss model.
- III. For multi-hop relay system, each hop may apply different channel model. For example:
 - i. Type D model may be appropriate for BS↔RS LOS link, RS↔RS LOS link.
 - ii. Type E model may be appropriate for BS↔RS NLOS link, RS↔RS NLOS link, BS↔MS NLOS link and RS↔MS NLOS link.
 - iii. Type F model may be appropriate for RS↔MS LOS link and RS↔MS NLOS link.

The propagation models for each hop should be specified when simulating multi-hop relay systems. More detail description on each propagation scenario can refer to [12].

2. In order to specify the appropriate correlation model for shadow fading, we would like to propose following text to be included into C802.16j_06/040:

“The auto-correlation model for shadow fading is given as:

$$\rho(\Delta x) = e^{-\frac{|\Delta x|}{d_{cor}} \ln 2}$$

where ρ is the correlation coefficient and Δx is the distance between adjacent observation locations. d_{cor} is the de-correlation distance, which was suggested to be 20m. The way to apply this correlation model can refer to C802.16j-06/045.”

6 References

- [1] IEEE 802.16a-03/01, “Channel Models for Fixed Wireless Applications”.
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- [10] IEEE C802.16j-06/009, “Correlated Lognormal Shadowing Model”,
- [11] IEEE C802.16j-06/040, “Multi-hop System Evaluation Methodology (Channel Model and Performance Metric),”
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Appendix A

- Consider X and Y are both Normal distributed random variables. Given $Y = y$ and the correlation coefficient as ρ , the probability distribution function $f_X(x|y)$ can be derived as:

Consider m_1, m_2 and σ_1^2, σ_2^2 are the mean and variance of X and Y

$$\begin{aligned}
 f_X(x|y) &= \frac{f_{X,Y}(x,y)}{f_Y(y)} \\
 &= \frac{\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-m_1}{\sigma_1}\right)\left(\frac{y-m_2}{\sigma_2}\right) + \left(\frac{y-m_2}{\sigma_2}\right)^2\right]\right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{X,Y}^2}} \left[\frac{\exp\left\{-\frac{(y-m_2)^2}{2\sigma_2^2}\right\}}{\sqrt{2\pi}\sigma_2} \right]^{-1} \\
 &= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}} \exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-m_1}{\sigma_1}\right)\left(\frac{y-m_2}{\sigma_2}\right) + \left(\frac{y-m_2}{\sigma_2}\right)^2 - (1-\rho_{X,Y}^2)\left(\frac{y-m_2}{\sigma_2}\right)^2\right]\right\} \\
 &= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}} \exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left(\frac{x-m_1}{\sigma_1} - \rho_{X,Y}\frac{y-m_2}{\sigma_2}\right)^2\right\} \\
 &= \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2(1-\rho_{X,Y}^2)}} \exp\left\{-\frac{\left[x-m_1 - \rho_{X,Y}\frac{\sigma_1}{\sigma_2}(y-m_2)\right]^2}{2\sigma_1^2(1-\rho_{X,Y}^2)}\right\}
 \end{aligned}$$

\therefore the mean of $f_X(x|y)$ is $m_1 + \rho_{X,Y}\frac{\sigma_1}{\sigma_2}(y-m_2)$ and variance is $\sigma_1^2(1-\rho_{X,Y}^2)$

If X and Y are both with zero mean and variance σ^2 , and the correlation coefficient between X and Y are equal to ρ . Then, the mean of $f_X(x|y)$ will be $\rho \cdot y$ and its variance will be $\sigma^2(1-\rho^2)$.