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| Abstract | This paper investigates the performance of Reed-Solomon and Convolutional codes applicable to Mode A and Reed-Solomon and Parity check codes applicable to Mode B. |
| Purpose | This paper is submitted to assist the IEEE 802.16.1 working group to evaluate the performance of various coding options in selecting the FEC codes suitable for BWA applications. |
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# On the Performance of Reed-Solomon, Convolutional and Parity Check Codes for BWA Applications

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## 1 Introduction

This paper provides the performance of several coding schemes proposed in the current draft physical layer specifications [1]. One is the Reed-Solomon (RS) code with various codeword length and error correction capability. Another is the RS code concatenated with block code. The third is the RS code concatenated with convolutional code of various code rate. All these codes have been used in most of digital communication systems for many years, and the technology for implementation is mature. Therefore, they can provide performance close to the theoretical prediction with low complexity and low cost, which makes them attractive.

## 2 Reed-Solomon Code

RS code is nonbinary code based on symbols from Galois Field $GF(2^m)$, where each symbol consists of $m$ bits. A $(n,k)$ RS code is generated by a polynomial $g(X)$ of degree $n-k$ with coefficients from $GF(2^m)$. In this paper, the RS codes considered are all from $GF(2^8)$. For a $t$-error-correcting RS code, its generator polynomial is

$$g(X) = \prod_{i=1}^{2t}(x+\alpha^i)$$

where $n-k=2t$ and $\alpha$ is a primitive element in $GF(2^8)$. RS code is proposed in the Mode B of the current draft physical layer specifications for both uplink and downlink, as well as in the Mode A for uplink.

According to the standard decoding algorithm of RS code, if the number of symbol errors in the received codeword is not larger than $t$, the decoder can correct all of them. When the number of symbol errors is larger than $t$, the decoder either provides a mis-decoding result or declares decoding failure and passes the codeword unchanged. For a reasonably high value of $t(\geq 8)$ and $n \geq 5t$, the probability of mis-decoding is much smaller than that of decoding failure, and hence, can be ignored. It has been verified by simulation results that this approximation is valid, as all the codeword errors correspond to decoding failure but not mis-decoding. Therefore, the ratio of bit error rate (BER) to symbol error rate (SER) is the same at the input and output of RS decoder. Consequently, the BER at the output of RS decoder can be given by the following expression;

$$\begin{aligned} P_{bo} &= \frac{P_{bi}}{P_{si}} P_{so} \\ &= \frac{P_{bi}}{P_{si}} \frac{1}{n} \sum_{i=t+1}^{n} i C_n^i {P_{si}}^i (1-P_{si})^{n-i} \end{aligned} \quad (1)$$

where $P_{bi}$ and $P_{si}$ are channel BER and SER at the input of RS decoder and $P_{bo}$ and $P_{so}$ are the post-coding BER and SER at the output of RS decoder. Similarly, the word error rate (WER) $P_w$ can be expressed as,

$$P_w = \sum_{i=t+1}^{n} C_n^i {P_{si}}^i (1-P_{si})^{n-i} \quad (2)$$

Due to the excellent error detection capability of RS code, no additional code, such as CRC, is required for error detection.

Table 1: Performance of RS code.

| Code Type | Rs(204,188) | RS(144,128) | RS(138,128) | RS(69,53) |
|---|---|---|---|---|
| Overall code rate | 0.9216 | 0.8889 | 0.9275 | 0.7681 |
| Uplink/Downlink | Both | Both | Both | Both |
| Required $E_b/N_o$(dB) at BER = $10^{-6}$(QPSK) | 6.99 | 6.87 | 7.36 | 6.84 |
| WER at BER = $10^{-6}$ (QPSK) | $1.6 \times 10^{-4}$ | $1.2 \times 10^{-4}$ | $2.0 \times 10^{-4}$ | $6.0 \times 10^{-5}$ |
| Required $E_b/N_o$(dB) at BER = $10^{-9}$(QPSK) | 7.69 | 7.60 | 8.25 | 7.67 |
| WER at BER = $10^{-9}$ (QPSK) | $2.0 \times 10^{-7}$ | $1.2 \times 10^{-7}$ | $1.8 \times 10^{-7}$ | $7.0 \times 10^{-8}$ |
| Required $E_b/N_o$(dB) at BER = $10^{-6}$(QAM64) | 14.90 | 14.75 | 15.33 | 14.65 |
| WER at BER = $10^{-6}$ (QAM64) | $1.8 \times 10^{-4}$ | $1.1 \times 10^{-4}$ | $2.0 \times 10^{-4}$ | $5.0 \times 10^{-5}$ |
| Required $E_b/N_o$(dB) at BER = $10^{-9}$(QAM64) | 15.68 | 15.58 | 16.30 | 15.58 |
| WER at BER = $10^{-9}$ (QAM64) | $1.5 \times 10^{-7}$ | $1.0 \times 10^{-7}$ | $1.5 \times 10^{-7}$ | $6.0 \times 10^{-8}$ |
| Block size, in payload data bits | 1504 | 1024 | 1024 | 424 |

The BER and WER obtained from computer simulations are the same as that of theoretical calculation, which confirm that the theoretical calculation is valid. In order to save time for running simulations, analytical results are presented in this section. 3 types of packet size are studied, which are 188 bytes (MPEG packet size), 128 bytes and 53 bytes(ATM packet size). An error correction capability of 8 is considered for all codes, while it is possible to reduce it to achieve higher code rate. The performance of some typical RS codes in additive white Gaussian noise (AWGN) channel with QPSK and QAM64 modulations are summarized in Table 1. Gray code is used in the mapping for QAM64.

For the same code rate, the longer the codeword, the better the performance, as the error correction capability is increased. Therefore, if the codeword length is not constrained to certain small value, a better result can be obtained, for example, if 2 ATM packets are transmitted in 1 codeword, the required $E_b/N_o$ for certain BER will be smaller.

It is worthwhile to note that for the same codeword length, we cannot improve the performance by merely increasing the error correction capability. When the error correction capability exceeds certain value, the reduced code rate is not compensated for by the increased error correction capability of the RS code. Therefore, $t = 8$ can provide a satisfactory performance for data block, but it is not a good choice for small block used for signalling purpose.

## 2.1 Parity Check Code

In Mode B of the current draft physical layer specifications, a simple concatenated coding scheme is proposed, where RS code is used as the outer code and a parity check code P(9,8) with soft decoding is used as the inner code. Since inner code is very simple, soft decoding is used to achieve better result without adding much load for decoder.

The BER and WER performance is calculated through a semi-analytical approach. The values of channel

Table 2: Performance of RS code + Parity check code.

| Code Type | Rs(204,188) & P(9,8) | RS(144,128) & P(9,8) | RS(138,128) & P(9,8) | RS(69,53) & P(9,8) |
|---|---|---|---|---|
| Overall code rate | 0.8192 | 0.7901 | 0.8245 | 0.6828 |
| Uplink/Downlink | Both | Both | Both | Both |
| Required $E_b/N_o$(dB) at BER = $10^{-6}$(QPSK) | 5.55 | 5.48 | 5.90 | 5.54 |
| WER at BER = $10^{-6}$ (QPSK) | $1.0 \times 10^{-4}$ | $6.0 \times 10^{-5}$ | $1.0 \times 10^{-4}$ | $4.0 \times 10^{-5}$ |
| Required $E_b/N_o$(dB) at BER = $10^{-9}$(QPSK) | 6.16 | 6.11 | 6.62 | 6.23 |
| WER at BER = $10^{-9}$ (QPSK) | $1.0 \times 10^{-7}$ | $7.5 \times 10^{-8}$ | $1.0 \times 10^{-7}$ | $3.2 \times 10^{-8}$ |
| Block size, in payload data bits | 1504 | 1024 | 1024 | 424 |

Table 3: Puncturing pattern of various code rate.

| Code rate | 1/2 | 2/3 | 3/4 | 5/6 | 7/8 |
|---|---|---|---|---|---|
| Puncturing | 1 | 1 1 | 1 1 0 | 1 1 0 1 0 | 1 1 1 1 0 1 0 |
| pattern | 1 | 1 0 | 1 0 1 | 1 0 1 0 1 | 1 0 0 0 1 0 1 |

BER and SER at the input of RS decoder as used in Eq(1) and (2) are accurately obtained through extensive simulations. The post-coding BER and SER at the output of RS decoder are calculated using the equations mentioned before. The performance of the same RS codes in AWGN channel with QPSK modulation are summarized in Table 2. It can be seen that there is significant improvement over RS code only.

# 3 Concatenation of RS code and convolutional code

The concatenation of RS code and convolutional code is proposed in Mode A of current draft physical layer specifications [1]. The code is applicable in continuous transmission system, therefore, it can only be used in downlink in a frequency division duplexed (FDD) system. The outer RS code is fixed to be a (204,188) code in $GF(2^8)$ so as to facilitate the transmission of MPEG packet. The inner convolutional code has various code rates, which is punctured from the same mother code as specified in [1]. The mother code is a rate 1/2 convolutional code with constraint length $K = 7$ and generator vector $g_1 = (133)_{oct}$ and $g_2 = (171)_{oct}$. The puncturing patterns for rate 2/3, 3/4, 5/6 and 7/8 code are listed in Table 3. The interleaver is a convolutional interleaver with depth $I = 12$ as given in [1].

For the proposed coding scheme, the decoder employs soft decoding of convolutional code, which means no quantization is employed at the decoder, and the metric calculation is floating point operation. When convolutional code is transmitted with BPSK/QPSK modulation in AWGN channels, an upper bound on the BER can be obtained using a union bound argument on the transfer function of the convolutional code. The BER bound $P_b$ for a rate $k/n$ convolutional code is given by

$$P_b = \frac{1}{k} \sum_{j=d_{free}}^{\infty} b_j Q\left(\sqrt{2jRE_b/N_o}\right) \tag{3}$$

where $b_j$ is the total number of nonzero information bits on all weight $j$ paths, $R$ is the code rate, and $d_{free}$ is the free distance of the code. The values of $b_j$ and $d_{free}$ can be found in [2]. The BER performance calculated from the above equation is plotted in Figure 1 together with simulation results.

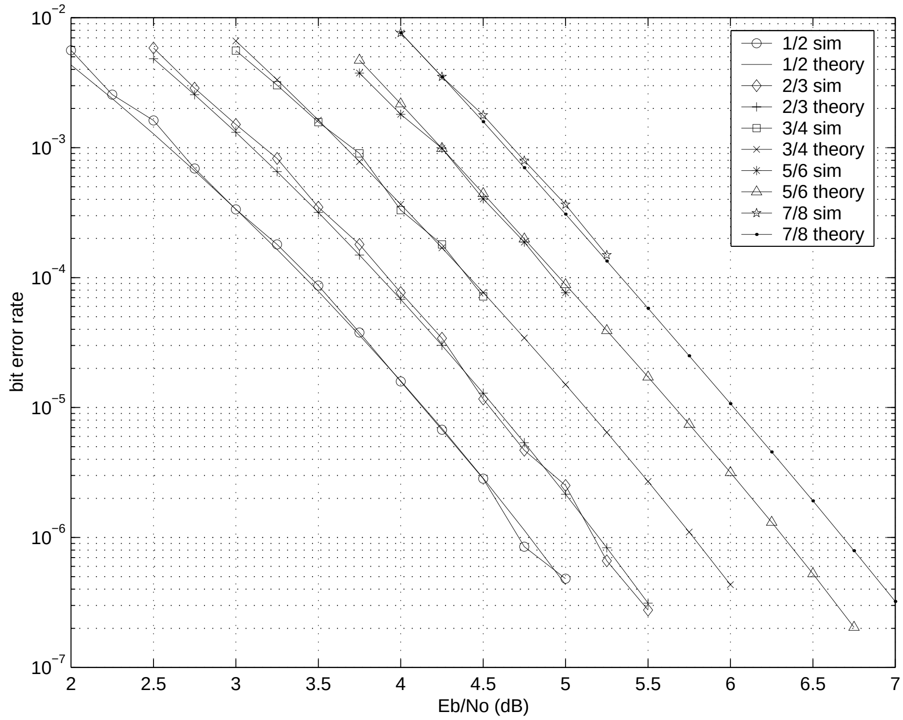


Figure 1: Bit error probability of convolutional code with BPSK/QPSK modulation in AWGN channel.

It is easy to see that the theoretical results match the simulation results for all code rates. When quantization is used at decoder, the performance loss is about 0.2 – 0.3 dB for 8 level quantization.

Table 4: Performance of RS (204,188) concatenated with different convolutional codes.

| Inner code rate | 1/2 | 2/3 | 3/4 | 5/6 | 7/8 |
|---|---|---|---|---|---|
| Overall code rate | 0.4608 | 0.6144 | 0.6912 | 0.7680 | 0.8064 |
| Uplink/Downlink | Downlink | Downlink | Downlink | Downlink | Downlink |
| Required $E_b/N_o$(dB) at BER = $10^{-6}$(QPSK) | 2.56 | 3.11 | 3.58 | 4.15 | 4.55 |
| WER at BER = $10^{-6}$ (QPSK) | $5.5 \times 10^{-5}$ | $8.0 \times 10^{-5}$ | $6.0 \times 10^{-5}$ | $4.5 \times 10^{-5}$ | $4.5 \times 10^{-5}$ |
| Required $E_b/N_o$(dB) at BER = $10^{-9}$(QPSK) | 2.95 | 3.48 | 3.95 | 4.50 | 4.89 |
| WER at BER = $10^{-9}$ (QPSK) | $5.0 \times 10^{-8}$ | $7.0 \times 10^{-8}$ | $6.0 \times 10^{-8}$ | $5.0 \times 10^{-8}$ | $4.5 \times 10^{-8}$ |
| Block size, in payload data bits | 1504 | 1504 | 1504 | 1504 | 1504 |

In [3], the performance of such concatenated coding scheme has been provided analytically. However, there are a few inappropriate assumption and formulation that make the result inaccurate. First, it was assumed that the ratio of $P_{si}$ over $P_{bi}$ at the input of RS decoder is equal to 8 (number of bits in one symbol). However, this is only valid for AWGN channel. It has been known that the errors at the output of Viterbi decoder are bursty, which cannot be approximated as AWGN channel. Due to the bursty nature, the ratio is much smaller than 8. In our simulations, it is found that the ratio varies between 2 – 4 depending on the code rate and the value of $E_b/N_o$. Second, on page 4 of [3], the formula used to calculate the $P_{bo}$ at the output of RS decoder is different from Eq(1) in our paper as well as that used in [4, 5]. By comparing these two formulae, it can be seen that the term $\frac{P_{bi}}{P_{si}}$ is missed in [3], which causes error for the BER.

In this paper, the BER and WER performance for the concatenated coding scheme are obtained through computer simulation. Alternatively, the BER and SER at the input of RS decoder are first obtained through computer simulation. Then, the BER and WER are calculated by using Eq(1) and (2). It is found that these two results match each other quite well. The results for the concatenated coding scheme are summarized in Table 4.

Compared to the coding schemes with the same code rate in Tables 1 and 2, this coding scheme performs better. This is achieved by increasing complexity at the decoder.

# 4 Conclusion

In this paper, the performance of three types of coding schemes proposed in the current draft physical layer specifications [1] has been presented. All of them can provide satisfactory coding gain. Since the technology is mature for such coding schemes, they can be implemented with low complexity.

# References

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