Space-Time Codes and Signal Processing for Slow Fading Channels

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Base Document: The presentation provides an overview of recent advances in space-time codes and signal processing. Since space-time techniques have potentially large benefits for MMDS systems, it is recommended that 802.16.3 investigate their applications in the new standard under development.

Purpose: Discussion

Space-Time Codes and Signal Processing for Slow Fading Channels

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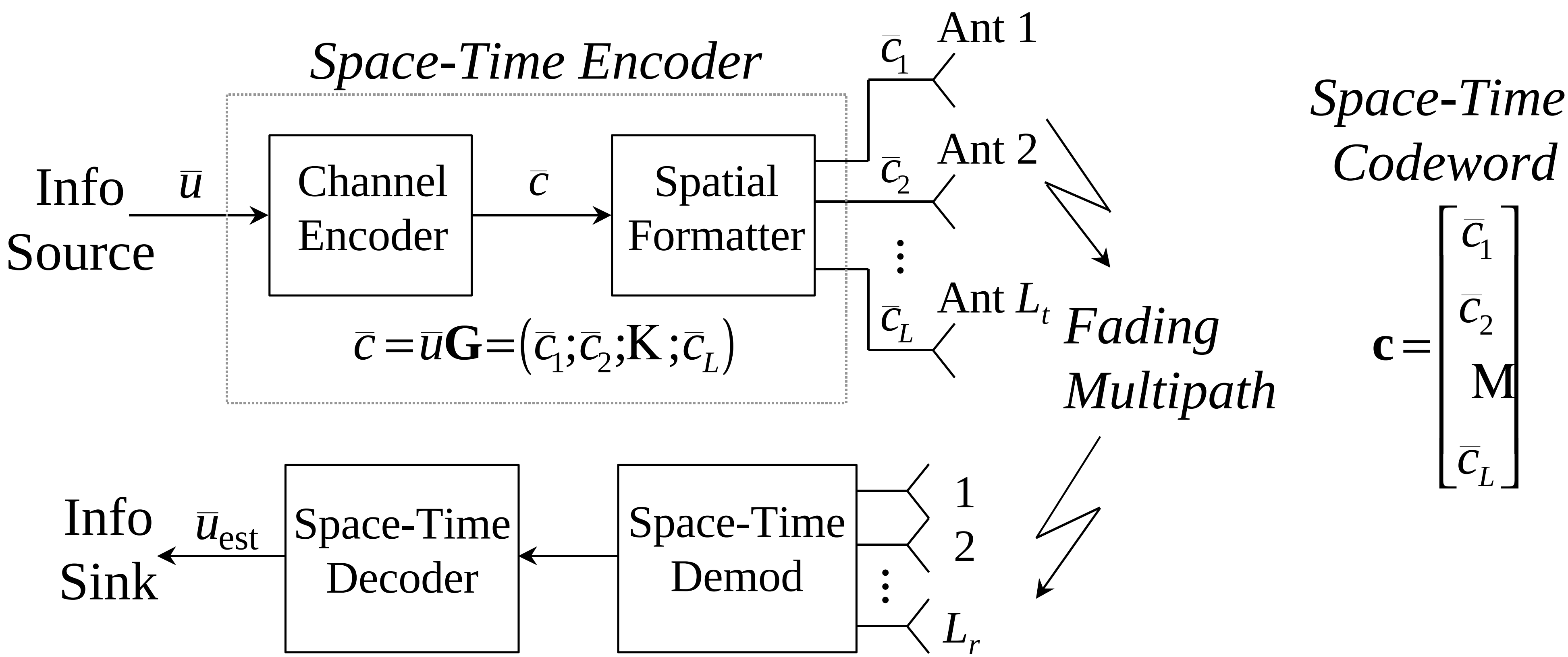
Space-Time Modems Exploit "Hidden" Capacity of the Multipath Channel



- **Space-time codes**: $L_t > L_r$. Seek diversity through code design.
- **BLAST**: $L_r = L_t$. Seek high throughput through signal processing.
- **Hybrid schemes**: Also possible.

Space-Time Technology: Code Design and Signal Processing

AT&T Research has popularized "Space-Time Channel Codes" (*Tarokh*, *Seshadri*, *Calderbank*)

- **Primary objective**: Increased diversity
- **Method**: Channel coding performed across antennas as well as time

Lucent has popularized "Layered Space-Time Architecture" or "BLAST" (*Foschini*, *Gans*)

- **Primary objective**: Increased throughput
- **Method**: Independent spatial channels via interference avoidance and cancellation

We investigated synergistic approaches that advance the state of the art in both space-time codes and space-time modems.

Design Criteria for Space-Time Codes

Pairwise error probability for Rayleigh fading channel:

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left(\frac{hE_s}{4N_0} \right)^{-r}$$

where $r = \text{rank}\big(f(\mathbf{c}) - f(\mathbf{e}) \big)$ $\leftarrow$ rank of baseband difference

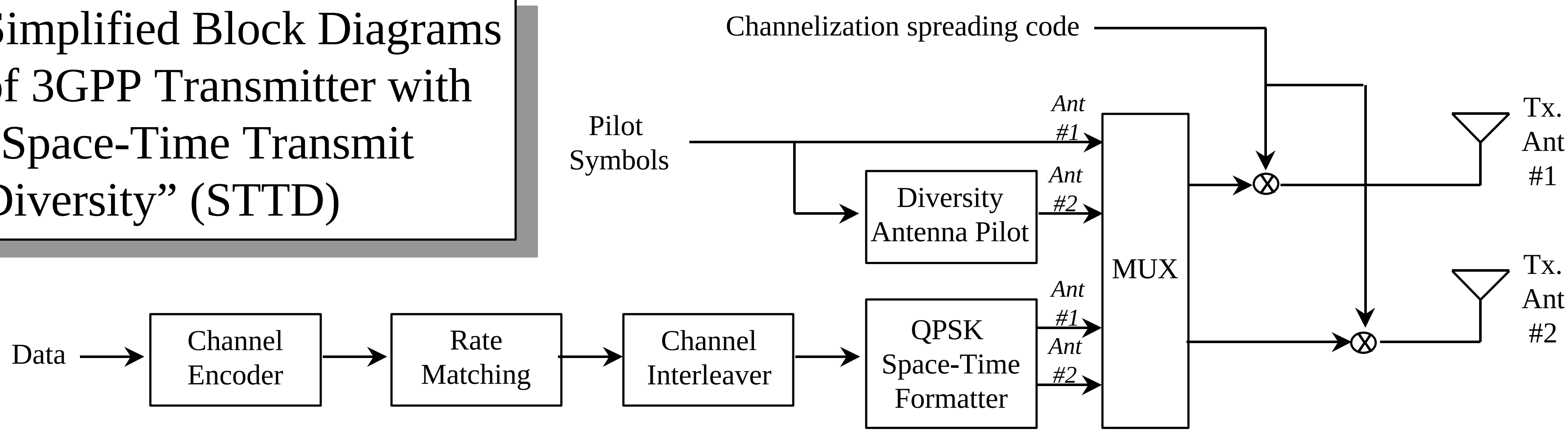
$h = \left(l_1 l_2 \Lambda\ l_r \right)^{1/r}$ $\leftarrow$ geometric mean of eigenvalues

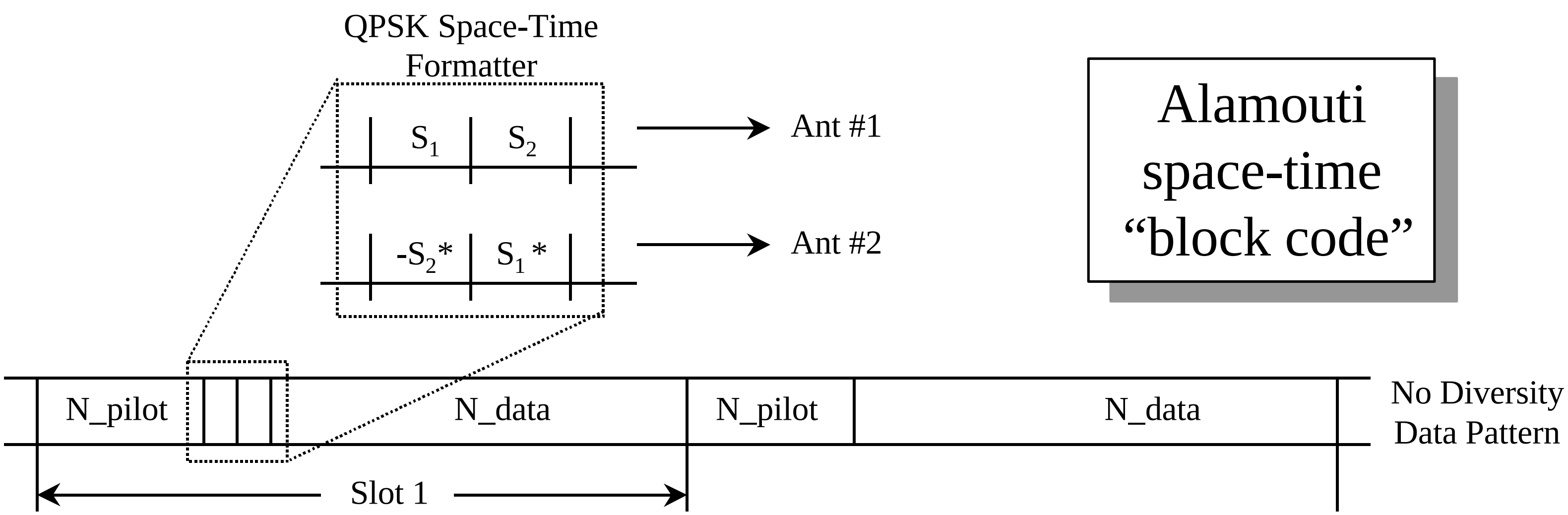
<u>Design Criteria [Fitz (Ohio State Univ.), Tarokh (AT&T)]</u>

- *Rank Criterion* : Maximize diversity advantage *r* over all distinct code word pairs **c** and **e**.
- *Product Distance Criterion* : Maximize coding advantage η over all distinct code word pairs **c** and **e.**

Space-Time Modulation Format Gives 3GPP “Open Loop” Transmit Diversity

Simplified Block Diagrams of 3GPP Transmitter with “Space-Time Transmit Diversity” (STTD)

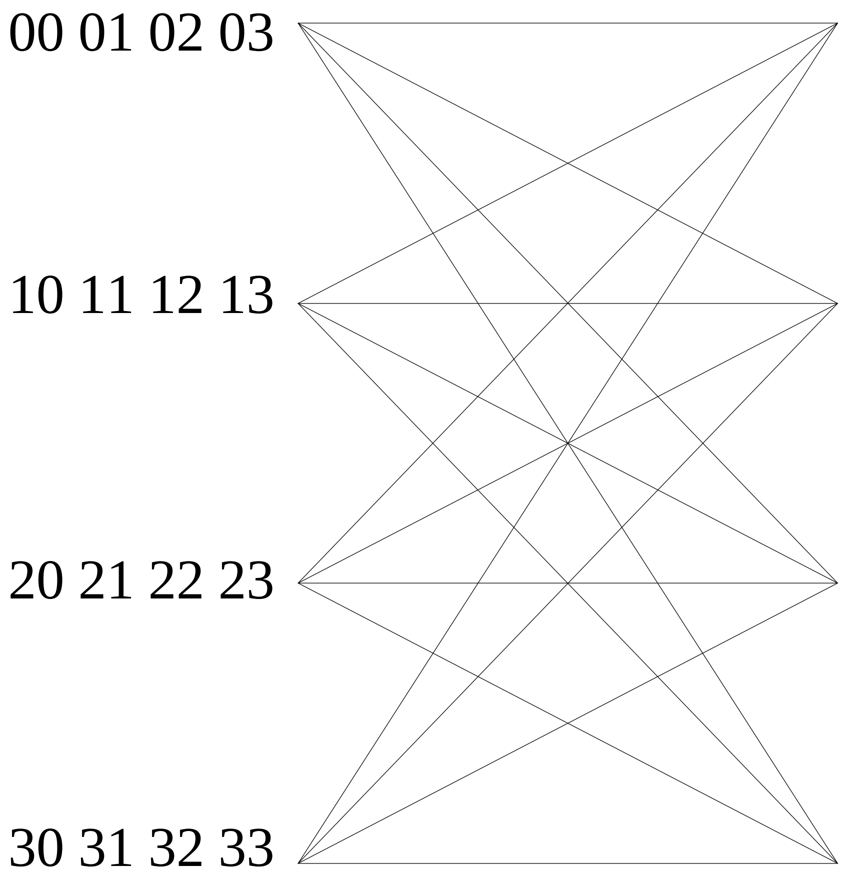




Alamouti space-time “block code”

Handcrafted Trellis Codes Achieving Full Spatial Diversity are Known

Tarokh-Seshadri-Calderbank (TSC) 4-State Trellis Code for QPSK Modulation

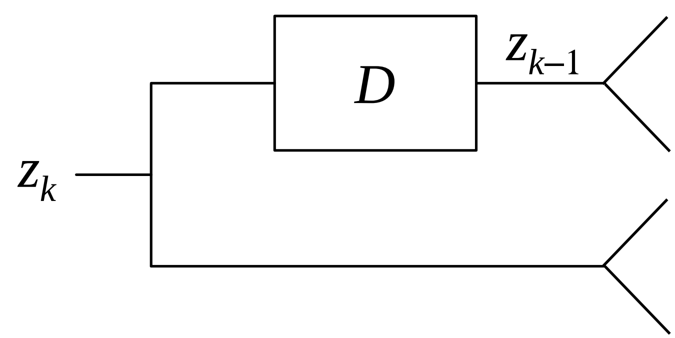


Binary Formulation

$$x_t^{(1)} = 2b_{t-1} + a_{t-1}$$

$$x_t^{(2)} = 2b_t + a_t$$

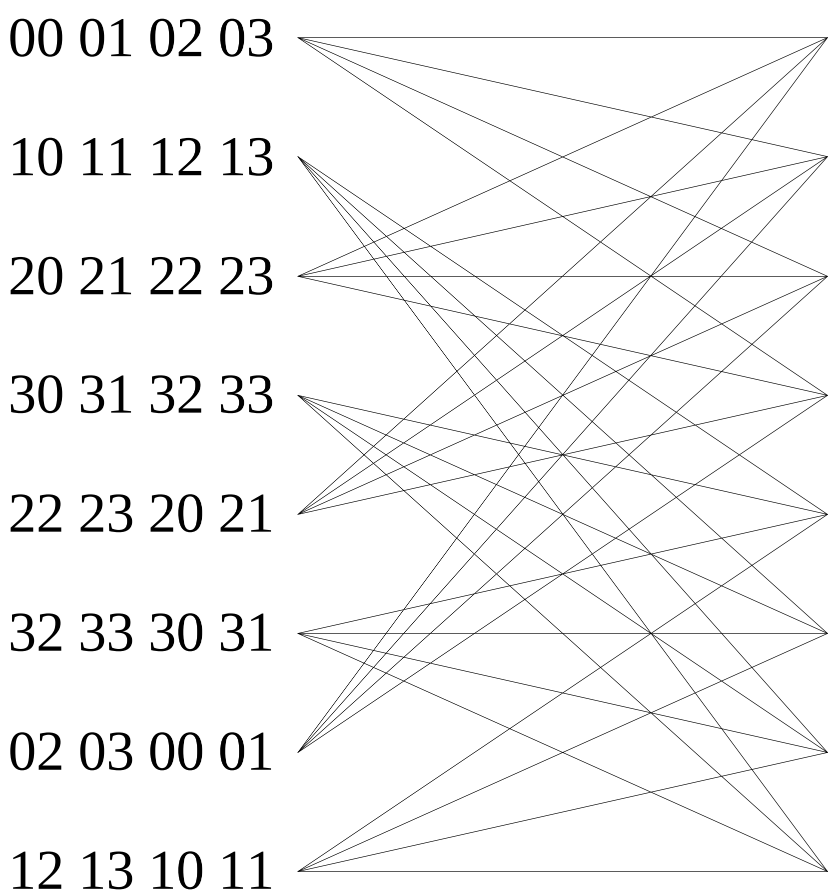
Z_4 Formulation



Achieves maximum 2-level spatial diversity.

Handcrafted Trellis Codes Achieving Full Spatial Diversity are Known

Tarokh-Seshadri-Calderbank (TSC) 8-State Trellis Code for QPSK Modulation

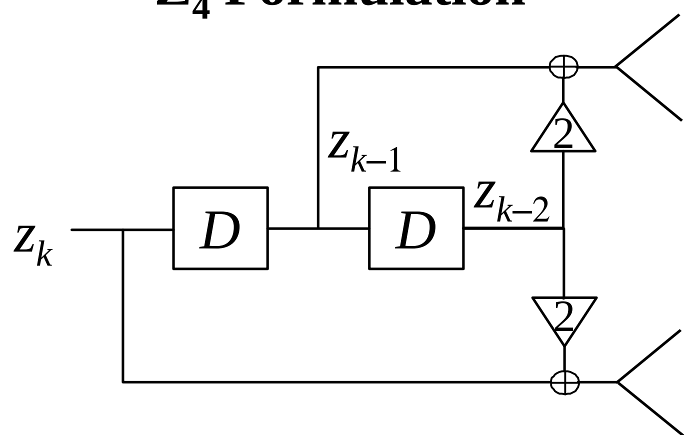


Binary Formulation

$$x_t^{(1)} = 2a_{t-2} + 2b_{t-1} + a_{t-1}$$

$$x_t^{(2)} = 2a_{t-2} + 2b_t + a_t$$

$\mathbf{Z_4}$ Formulation



Achieves maximum 2-level spatial diversity.

Binary Criteria Identify Full-Diversity Space-Time Codes

- **BPSK Binary Rank Criterion**: Let C be a linear $L \times n$ space-time code with $n \geq L$. Suppose that every non-zero binary code word $\mathbf{c} \in C$ is matrix of full rank over the binary field $\boldsymbol{F}$. Then, for BPSK transmission, the space-time code C satisfies the space-time rank criterion and achieves full spatial diversity L.

- **QPSK Binary Rank Criterion**: Let C be a linear $L \times n$ space-time code over $\boldsymbol{Z}_4$ with $n \geq L$. Suppose that, for every non-zero binary code word $\mathbf{c} \in C$, the row-based indicant $\Xi(\mathbf{c})$ or the column-based indicant $\Psi(\mathbf{c})$ has full rank L over $\boldsymbol{F}$. Then, for QPSK transmission, the space-time code C satisfies the space-time rank criterion and achieves full spatial diversity L.

- **Extensions to Higher-Order Modulation**: Use multi-level construction and apply binary rank criteria to the design of the constituent codes at each level.

“Stacking” Constructions Yield New Full-Diversity Space-Time Codes

Stacking Construction

$$\mathbf{c}(\bar{x}) = \begin{bmatrix} T_1(\bar{x}) \\ T_2(\bar{x}) \\ \vdots \\ T_L(\bar{x}) \end{bmatrix} = \begin{bmatrix} \triangle & \triangle & \triangle & \triangle & \cdots & \triangle \\ \Diamond & \Diamond & \Diamond & \Diamond & \cdots & \Diamond \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \pentagon & \pentagon & \pentagon & \pentagon & \cdots & \pentagon \end{bmatrix} \in \mathcal{C}$$

C satisfies BPSK binary rank criterion iff $T = a_1T_1 + a_2T_2 + \cdots + a_LT_L$ is non-singular unless $a_1 = a_2 = \cdots = a_L = 0$.

Multi-Stacking Construction

$$\begin{bmatrix} \triangle \underline{\pentagon} \pentagon & \triangle \underline{\pentagon} \pentagon & \cdots & \triangle \underline{\pentagon} \pentagon \\ \Diamond \underline{\triangle} \Diamond & \Diamond \underline{\triangle} \Diamond & \cdots & \Diamond \underline{\triangle} \Diamond \end{bmatrix} \in \mathcal{C}$$

Concatenations of space-time codes satisfying binary rank criterion form full-diversity space-time codes $\mathcal{C}$.

Transformation Theorem

Full-Diversity Space-Time Code

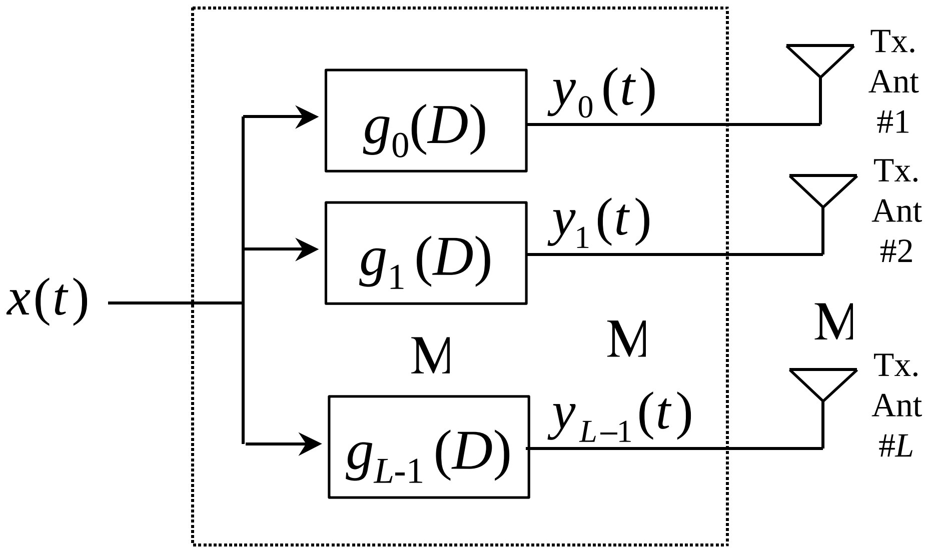
$$\begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \vdots \\ \bar{c}_L \end{bmatrix} = \begin{bmatrix} \triangle & \triangle & \triangle & \triangle & \cdots & \triangle \\ \Diamond & \Diamond & \Diamond & \Diamond & \cdots & \Diamond \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \pentagon & \pentagon & \pentagon & \pentagon & \cdots & \pentagon \end{bmatrix} \in \mathcal{C}$$

Linear transformation of full-rank $\Downarrow$

$$\begin{bmatrix} T(\bar{c}_1) \\ T(\bar{c}_2) \\ \vdots \\ T(\bar{c}_L) \end{bmatrix} = \begin{bmatrix} \underline{\triangle} & \underline{\triangle} & \underline{\triangle} & \underline{\triangle} & \cdots & \underline{\triangle} \\ \underline{\Diamond} & \underline{\Diamond} & \underline{\Diamond} & \underline{\Diamond} & \cdots & \underline{\Diamond} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\pentagon} & \underline{\pentagon} & \underline{\pentagon} & \underline{\pentagon} & \cdots & \underline{\pentagon} \end{bmatrix} \in T(\mathcal{C})$$

Full-Diversity Space-Time Code

Convolutional Codes with Optimal d_{free} Yield Full-Diversity Space-Time Codes



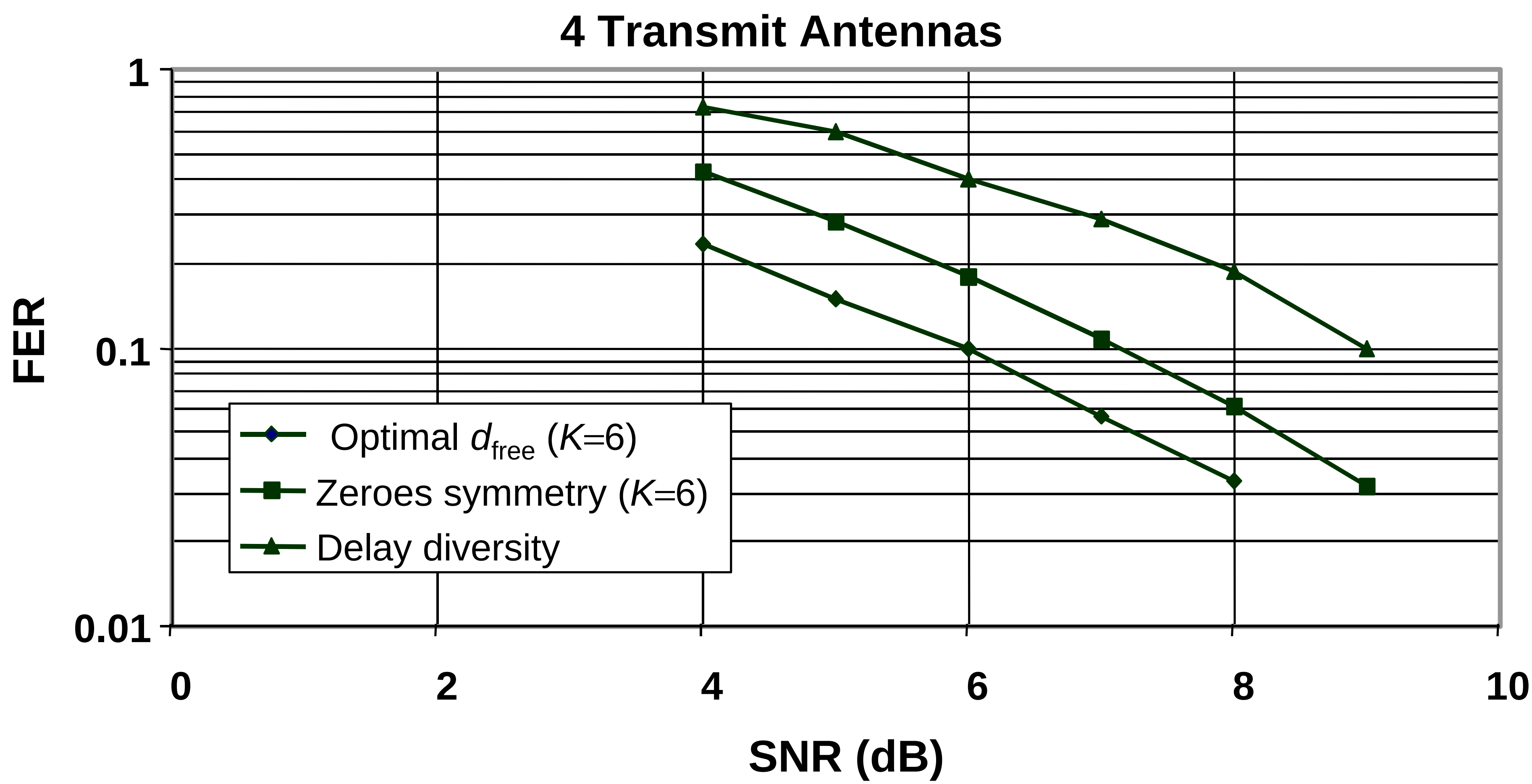
Convolutional Encoder

L	n	Connection Polynomials	d_{free}
2	2	5, 7	5
	3	64, 74	6
	4	46, 72	7
	5	65, 57	8
	6	554, 744	10
	7	712, 476	10
	8	561, 753	12
3	3	54, 64, 74	10
	4	52, 66, 76	12
	5	47, 53, 75	13
	6	554, 624, 764	15
	7	452, 662, 756	16
	8	557, 663, 711	18
4	4	52, 56, 66, 76	16
	5	53, 67, 71, 75	18
	7	472, 572, 626, 736	22
	8	463, 535, 733, 745	24
5	5	75, 71, 73, 65, 57	22
	7	536, 466, 646, 562, 736	28

“Natural” space-time code associated with rate 1/*L* convolutional code: *Multiplex L output coded bits in space (among antennas) rather than time.*

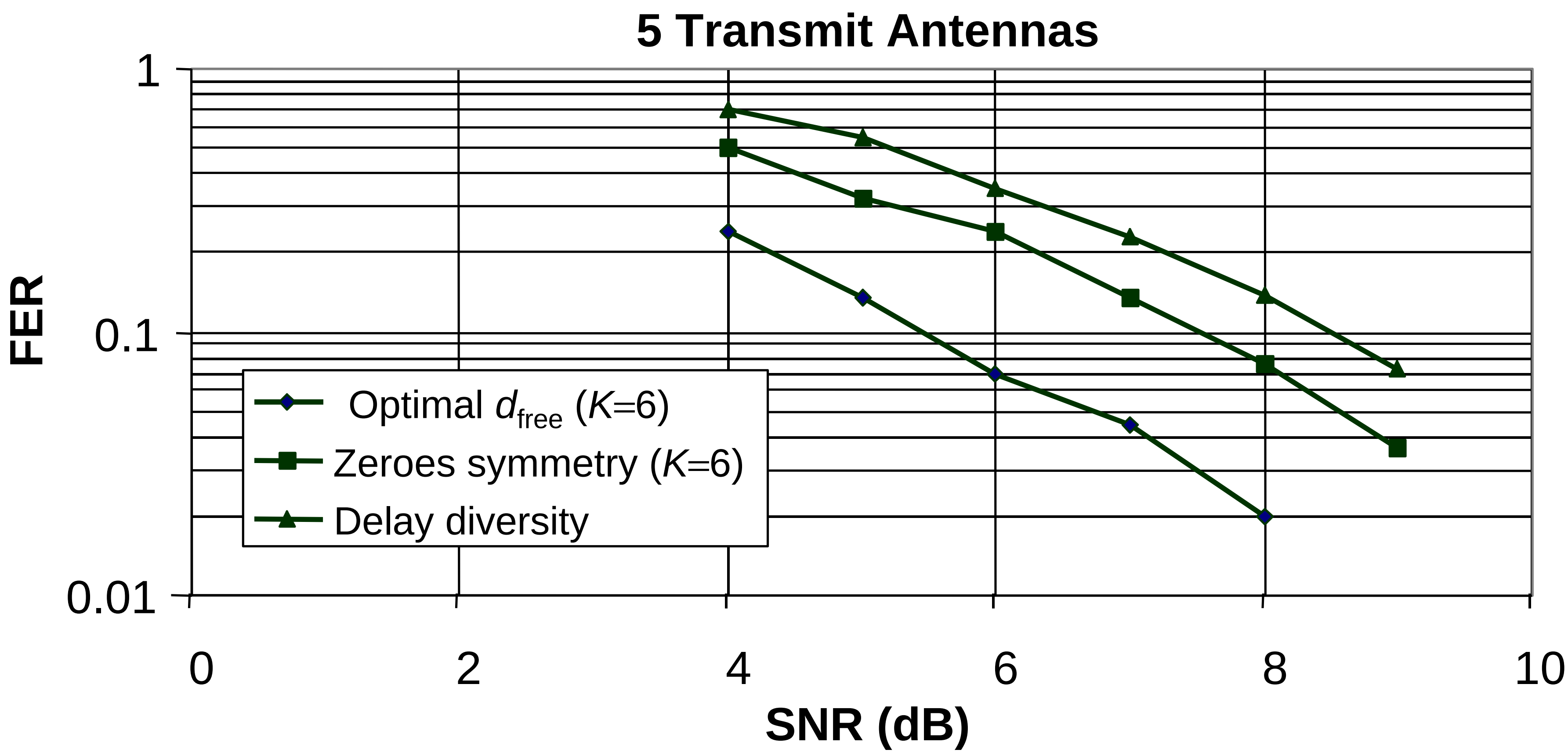
Practical examples of our general space-time “Stacking Constructions.”

Performance of BPSK Space-Time Codes with 4 Transmit Antennas



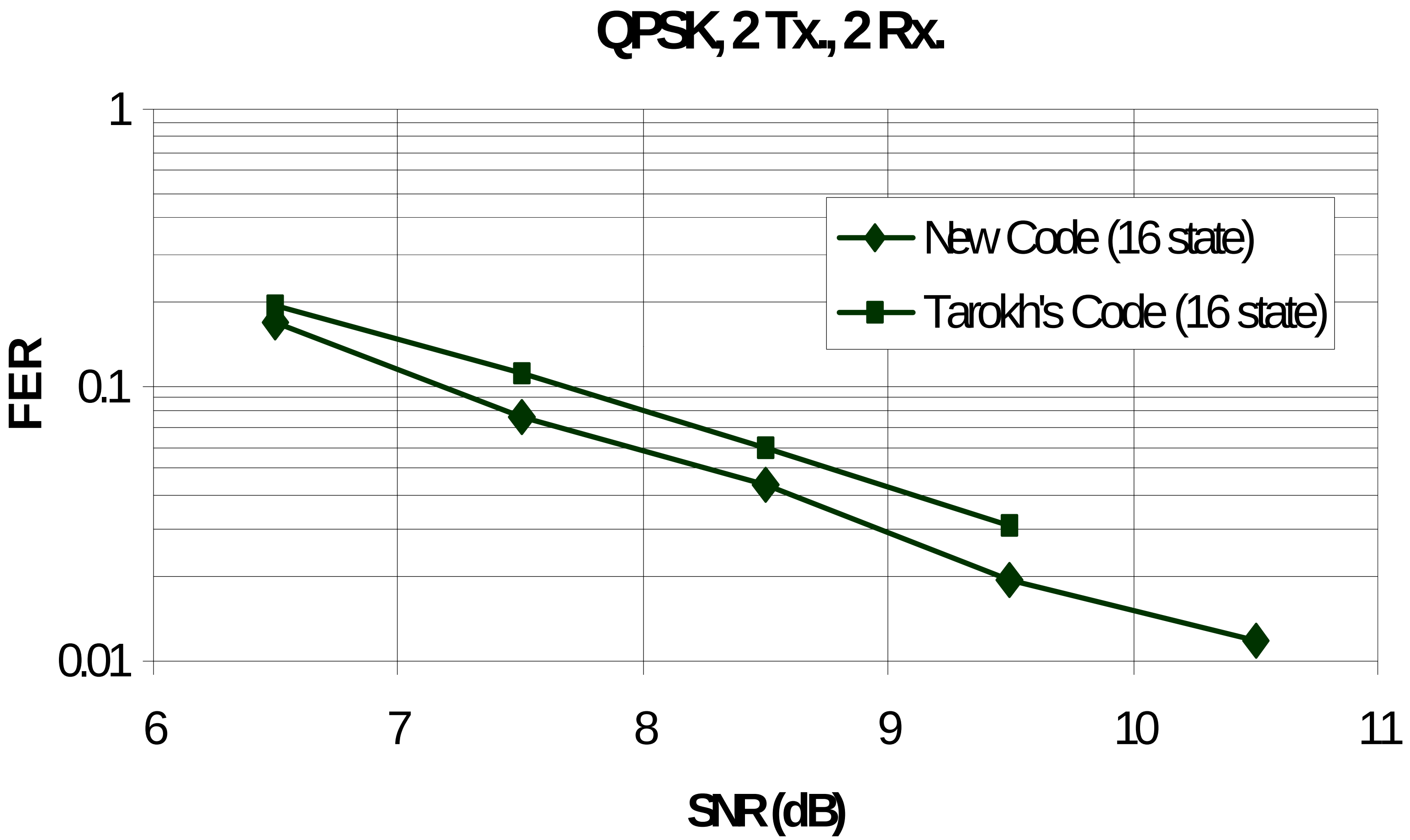
- Optimal d_{free} code is 3 dB better than the delay diversity scheme.
- Optimal d_{free} code is 1 dB better than Fitz-Grimm zeroes symmetry code.

Performance of BPSK Space-Time Codes with 5 Transmit Antennas

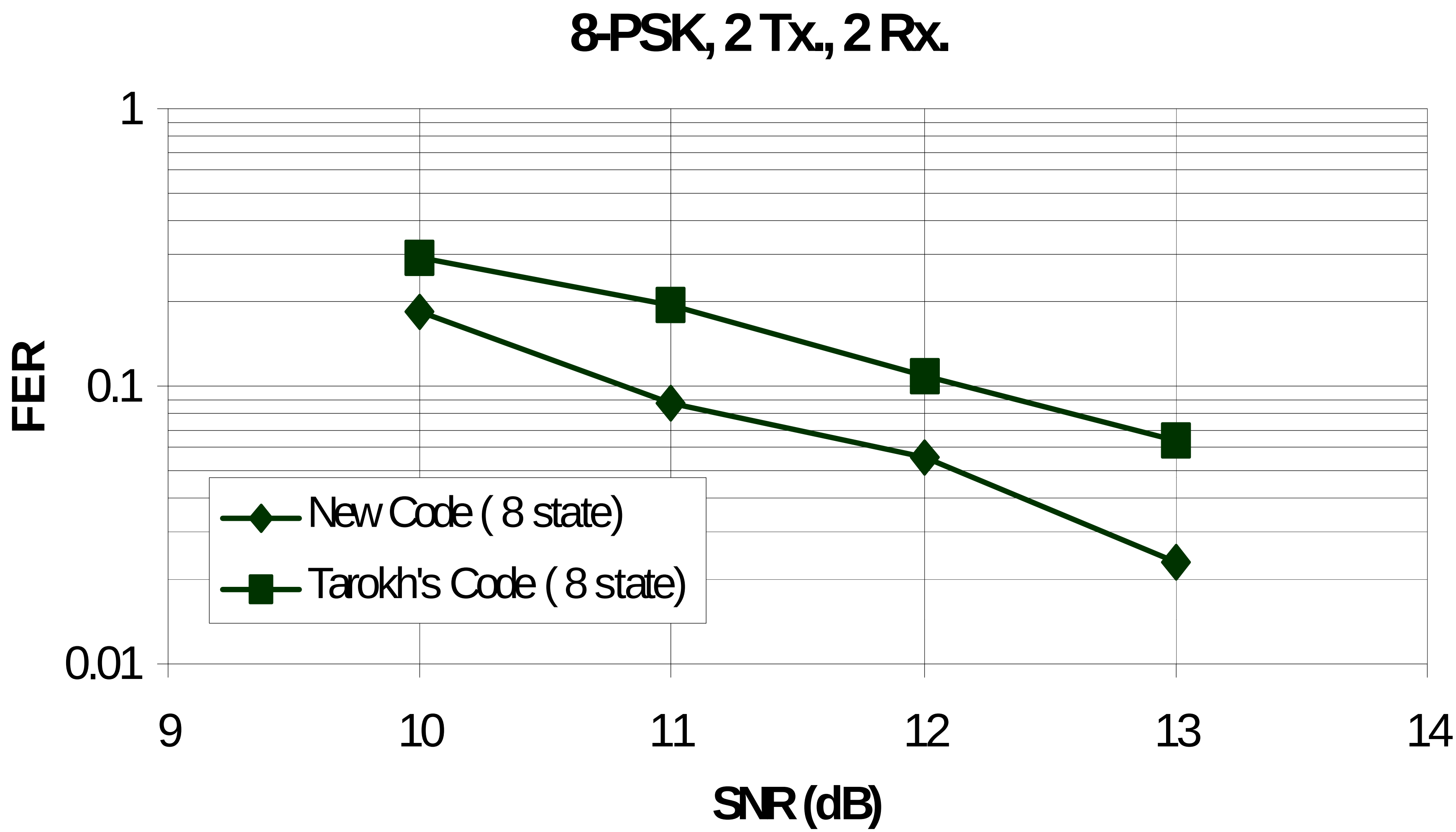


- Optimal d_{free} code is 3 dB better than the delay diversity scheme.
- Optimal d_{free} code is 2 dB better than Fitz-Grimm zeroes symmetry code.

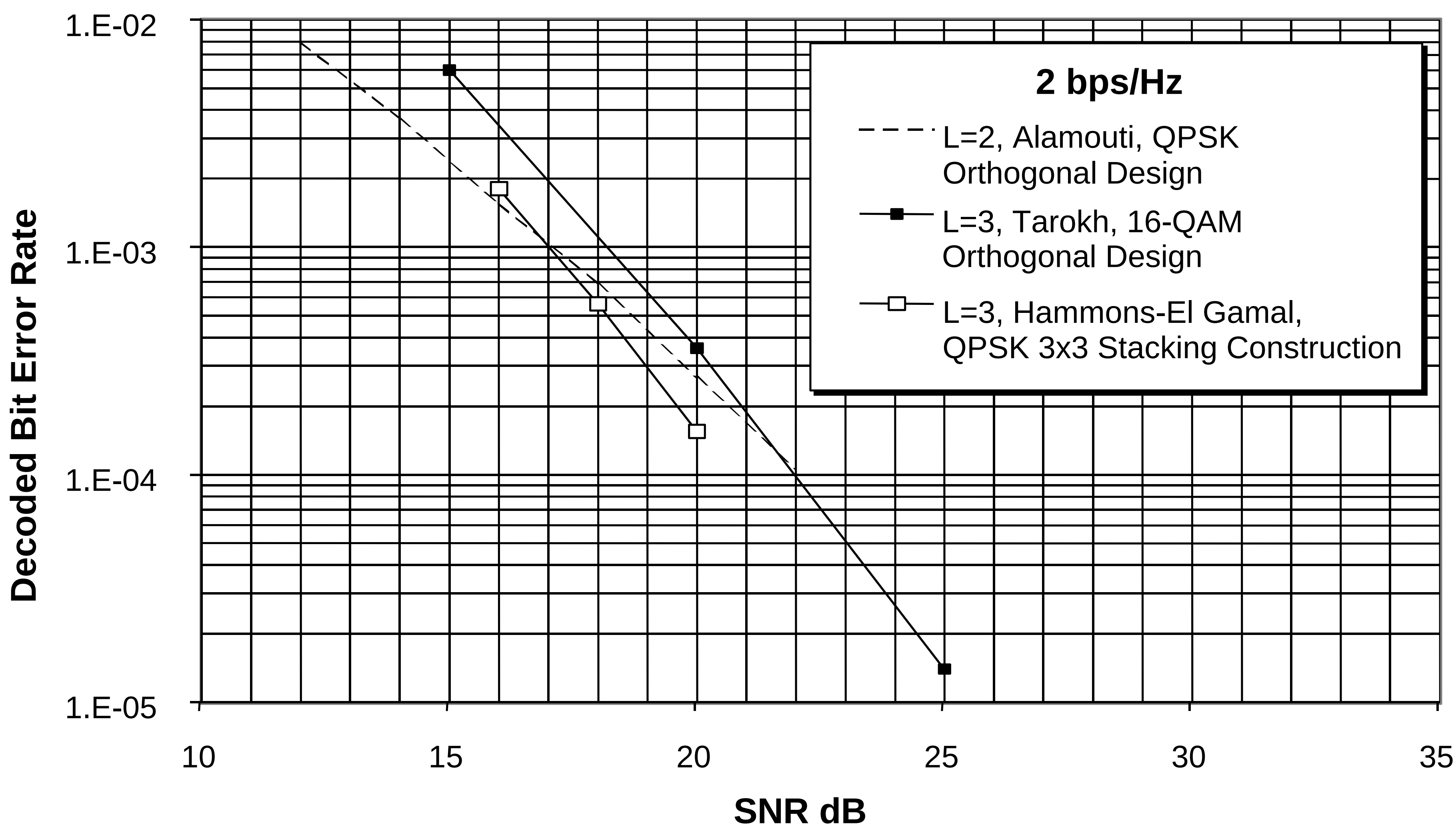
Performance of QPSK Space-Time Codes in Quasi-Static Fading Channels



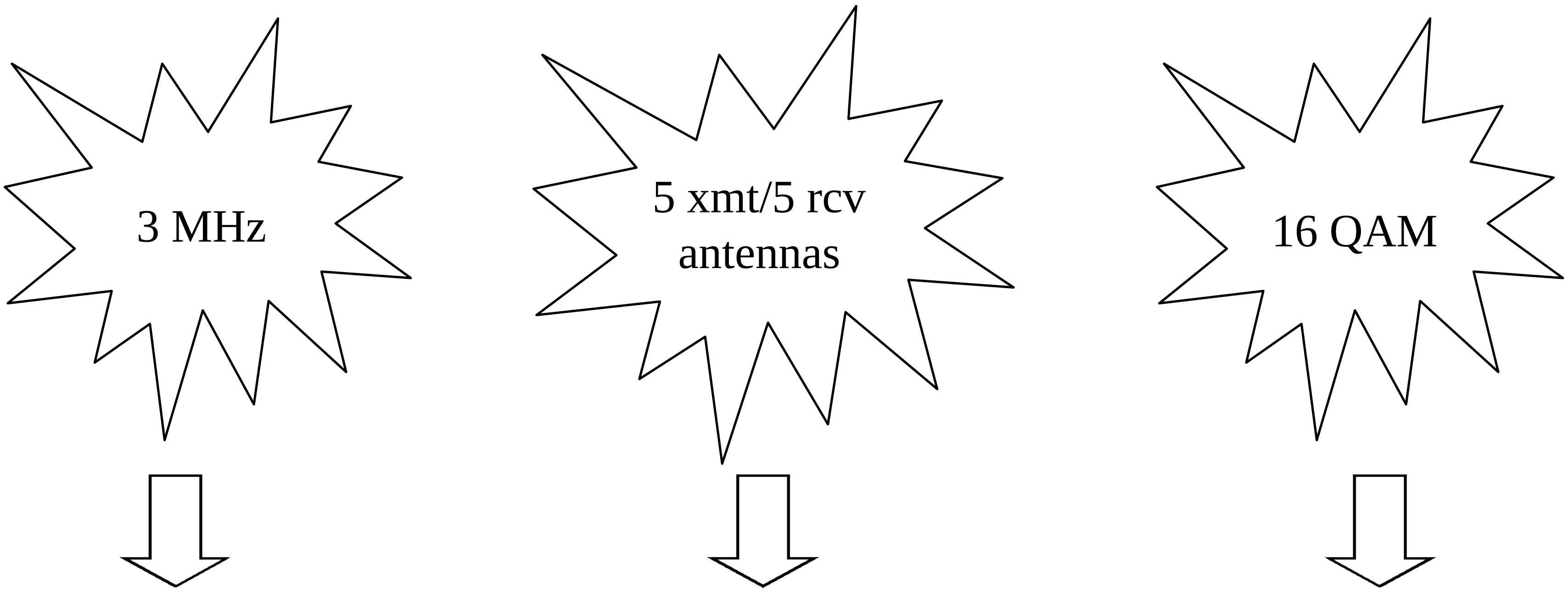
Performance of 8-PSK Space-Time Codes in Quasi-Static Fading Channels



Stacking Construction Yields New Codes for Space-Time Appliques



Foschini Showed that Outage Capacity Increases Linearly When L_r Equals L_t



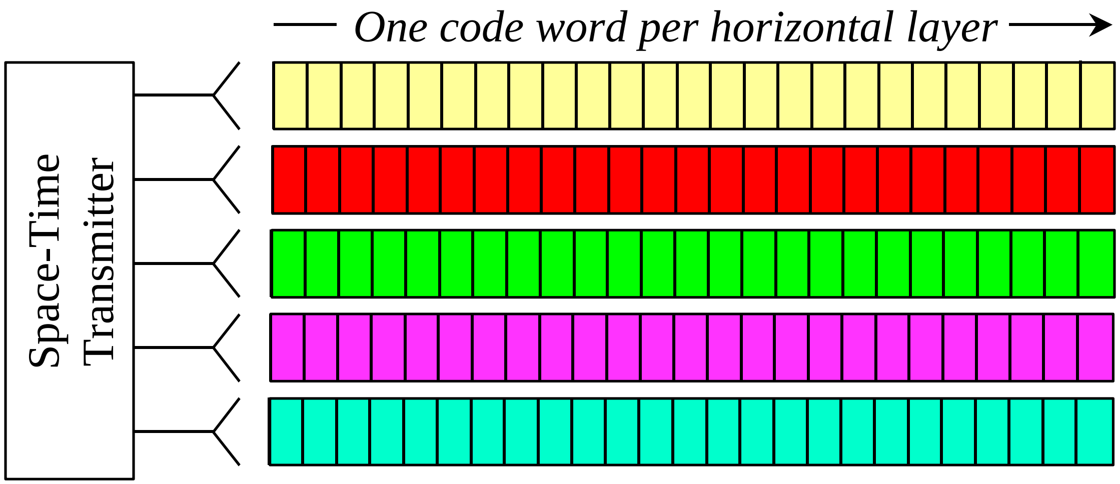
Bit Rate = Bandwidth · Number of antennas · Bit rate/antenna · Coding rate

⇩

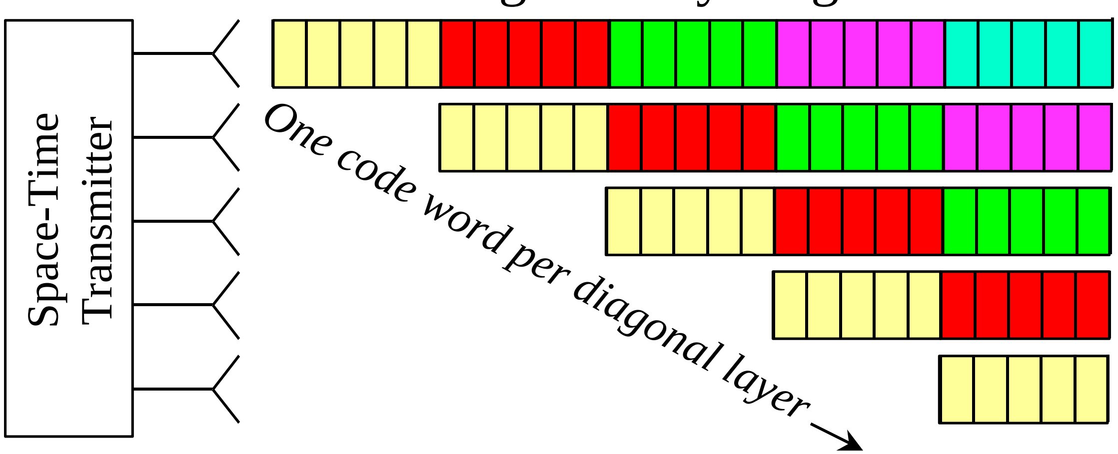
30 Mbps = 3 · 5 · 4 · 1/2

Layered Space-Time Architectures

Horizontal Layering



Diagonal Layering

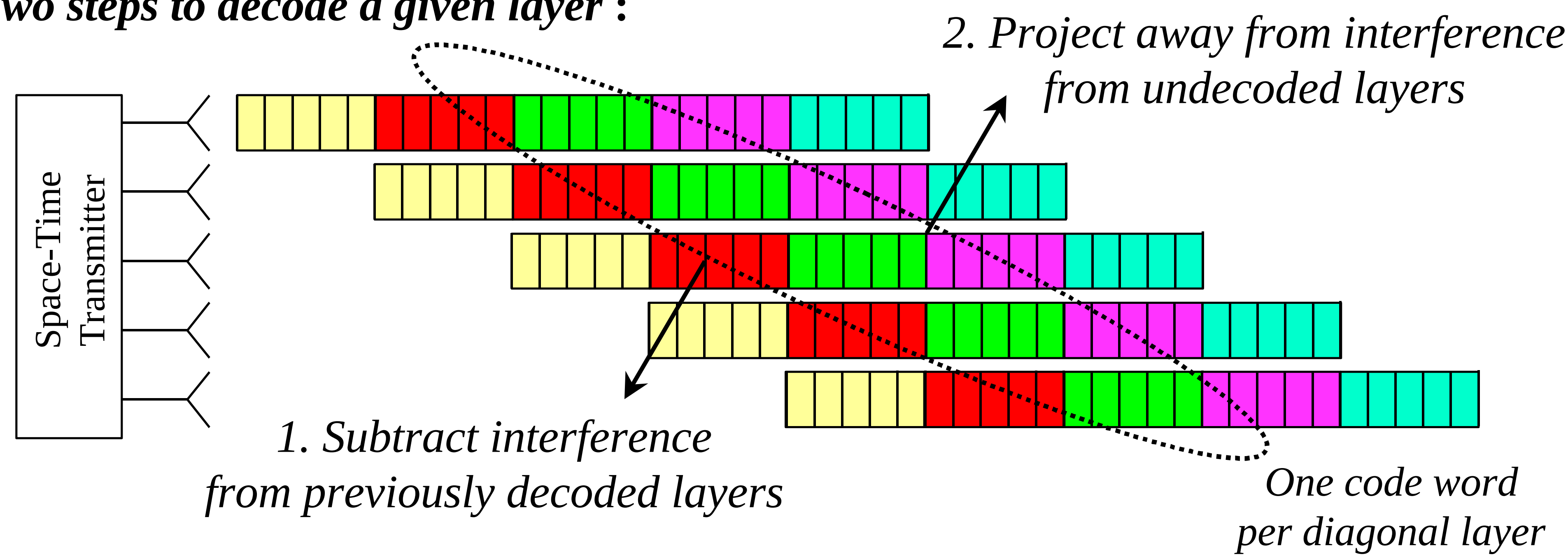


Definition: A *layer* is an assignment of space-time transmission resources to a component channel encoder in which at most one antenna is available each transmitted symbol interval.

Properties:
- No spatial interference within a layer.
- Decoding is performed layer by layer.
- Conventional channel codes can be used.

Lucent's BLAST Technology

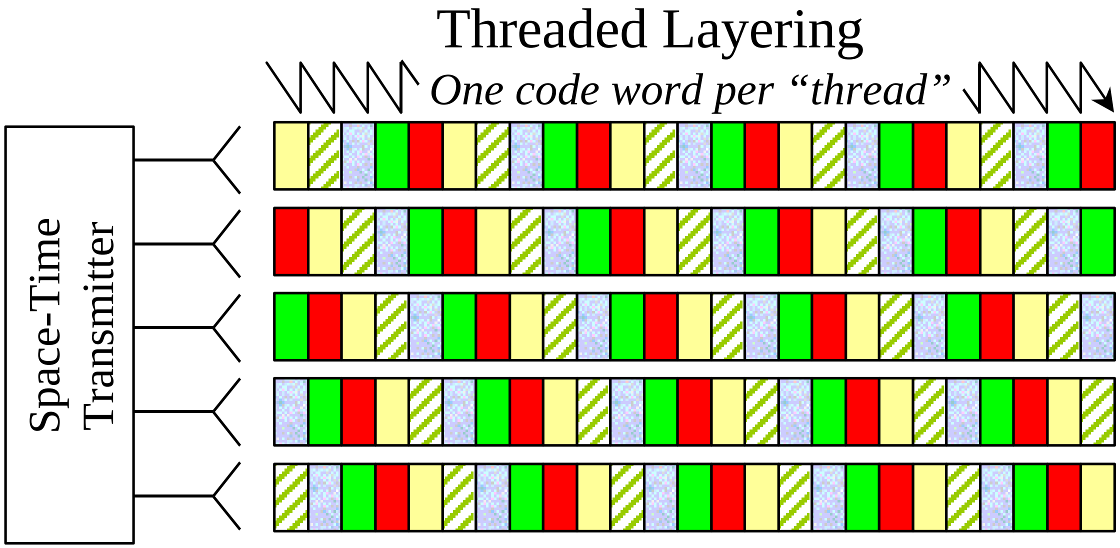
Two steps to decode a given layer :



Potential Limitations of BLAST Signal Processing

- Requires equal number of transmit and receive antennas
- Spatial diversity varies within a code word
- Errors can propagate both spatially and temporally
- Limited ability to interleave for temporal diversity
- Loss in throughput due to diagonal layering

Hughes Offers Threaded Space-Time Architecture

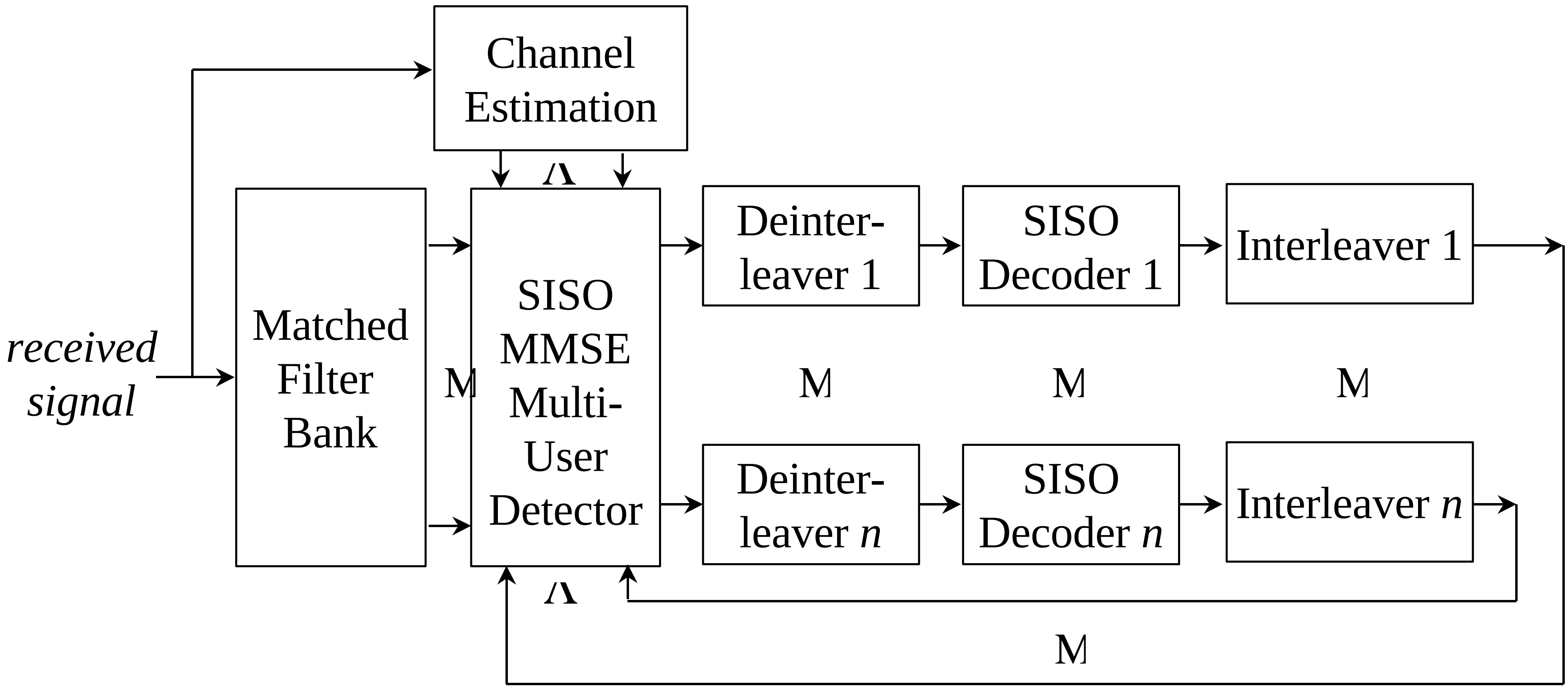


Thread = Layer whose temporal span is maximal for each antenna

Characteristics of Threaded Space-Time Architecture

- Generalized layering exploits spatial and temporal diversity
- Threaded space-time channel codes ensure full-diversity
- Receiver uses new, efficient, multi-user detection techniques
  - Soft-decision feedback reduces spatial error propagation.
  - Iterative MMSE processing results in a symmetrical performance.

Iterative MMSE Space-Time Receiver



- Iterative multi-user detection (MUD) is one key to threaded space-time architecture.
- Threaded channel codes, based on space-time principles, are optimized for MUD.

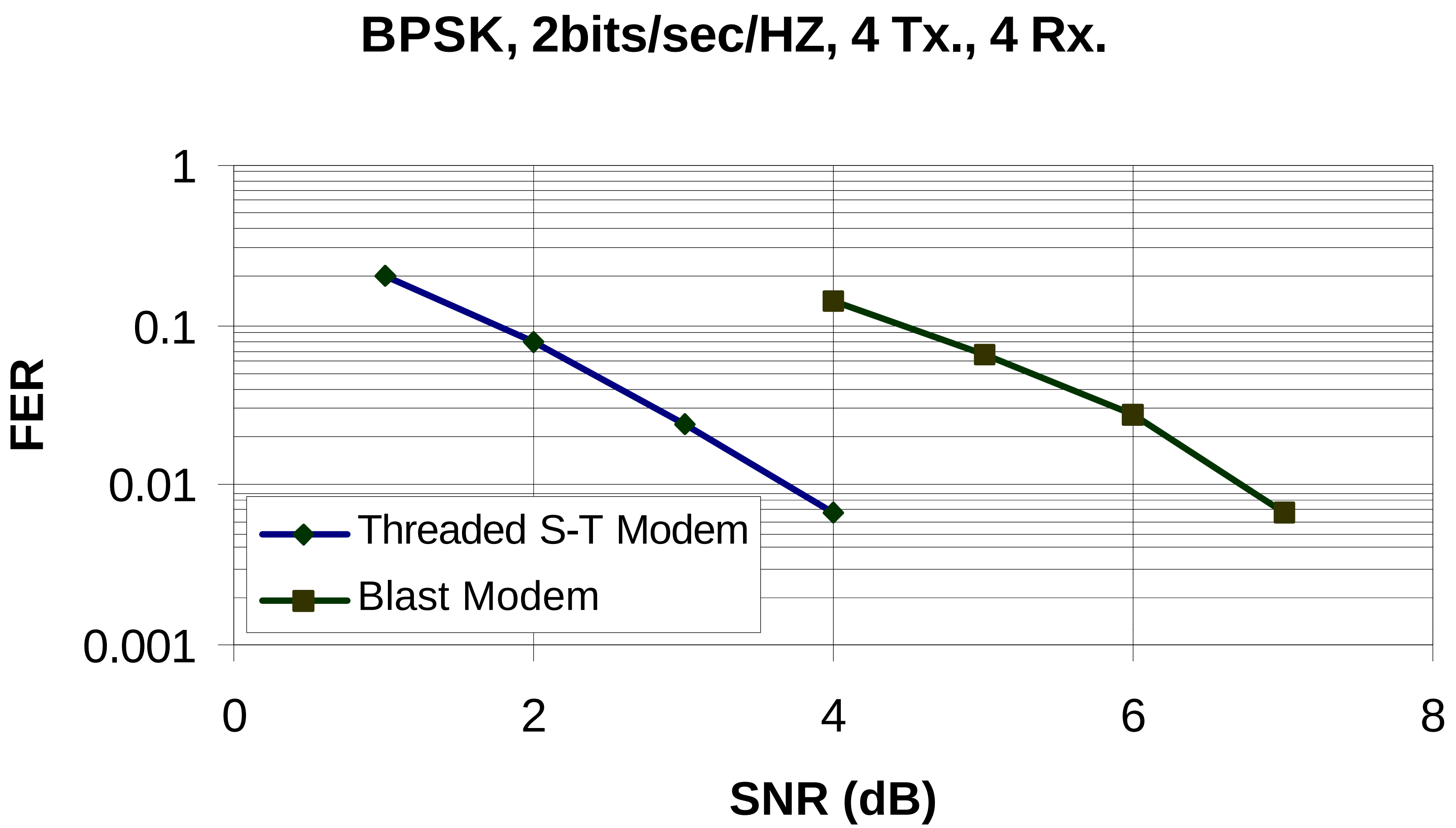
Threaded Space-Time Code Design for Quasi-static Fading Channels

Theorem (Threaded Stacking Construction):
Let L be a layer of spatial span n. Given binary matrices $\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_n$ of dimension $k \times \lambda$, let C be the binary code of dimension k consisting of all code words $g(\bar{x}) = \bar{x}\mathbf{M}_1 | \bar{x}\mathbf{M}_2 | \ldots | \bar{x}\mathbf{M}_n$, where $\bar{x}$ denotes an arbitrary k-tuple of information bits. Let $\mathbf{f}_L$ denote the spatial modulator having the property that the modulated symbols $m(\bar{x}\mathbf{M}_j)$ are transmitted in the symbol intervals of L that are assigned to antenna j.

Then, as the space-time code in a communication system with n transmit antennas and m receive antennas, the space-time code $\mathcal{C}$ consisting of C and $\mathbf{f}_L$ achieves spatial diversity dm in a quasi-static fading channel if and only if d is the largest integer such that $\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_n$ have the property that

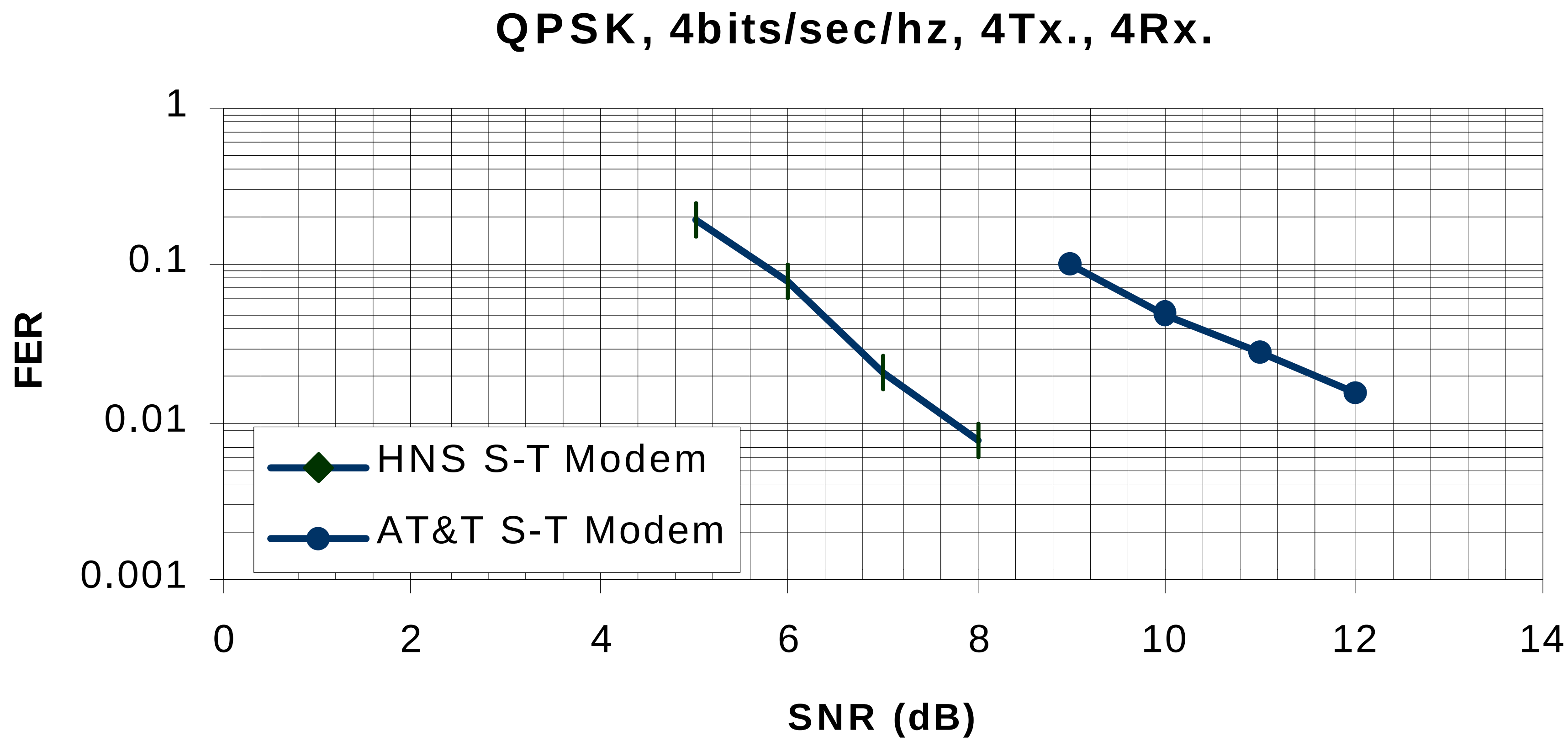
$$\forall a_1, a_2, \ldots, a_n \in \mathbf{F}, \quad a_1 + a_2 + \cdots + a_n = n - d + 1:$$
$$\mathbf{M} = \begin{bmatrix} a_1\mathbf{M}_1 & a_2\mathbf{M}_2 & \cdots & a_n\mathbf{M}_n \end{bmatrix} \text{ is of rank } k \text{ over } \mathbf{F}.$$

Threaded-STC Yields Bigger Bang than the BLAST Technology



At 1% FER, advantage is more than 3 dB.

Threaded Space-Time Outperforms AT&T's Group Suppression Approach



At 1% FER, advantage is more than 4 dB.

Conclusions

- MMDS must contend with slow fading channels
- Space-time technology offers potentially large gains in this environment
- 802.16.3 should be aggressive in study and adoption of best space-time solutions

References

Journal Papers

- Hammons and El Gamal, “On the theory of space-time codes for PSK constellations,” *IEEE Transactions on Information Theory*, March 2000.
- El Gamal and Hammons, “A new approach to layered space-time signal processing and code design,” accepted for publication (pending revision) in *IEEE Transactions on Information Theory*.

Conference Papers

- Hammons and El Gamal, “Space-Time Codes: New Design Criteria and General Constructions (Invited Paper)," Sixth Annual Workshop on Smart Antennas in Wireless Mobile Communications, Stanford University, 1999.
- El Gamal and Hammons, “On the Design of Space-Time Codes," In Proceedings of the *IEEE 1999 Information Theory Workshop*, Metsovo, Greece.
- El Gamal and Hammons, “Binary Design Criteria for Space-Time Codes," In Proceedings of *Thirty-Seventh Annual Allerton Conference on Communication, Control, and Computing*, University of Illinois, Urbana-Champaign, 1999.
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- El Gamal and Hammons, “Space-Time Coding for the Block Fading Channel,'‘ In Proceedings of *CISS'2000*, Princeton University.
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