Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Comments on OFDMA Ranging Scheme described in IEEE 802.16ab-01/01r1**
Date Submitted	**2001-08-28**
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Re:	Ranging Scheme in IEEE 802.16a OFDMA Mode
Abstract	This document presents the results of a detailed simulation and analytical study of the ranging scheme proposed in IEEE 802.16ab-01/01r1.
Purpose	Improving current 802.16 OFDMA PHY standard
Notice	
Release	
Patent Policy and Procedures	

Comments on OFDMA Ranging Scheme described in IEEE 802.16ab-01/01r1

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1. MOTIVATION FOR THIS STUDY

In contributions [1]-[2] to the IEEE, a bandwidth request and ranging mechanism has been described that relies on frequency domain "codes" for its operation. These contributions state that the proposed scheme can achieve over 90% success rate with 20 contending users and 10 contention slots [1]. A simulation curve is provided in support of this stated performance.

In this contribution, we examine the performance of the above ranging scheme more rigorously through both theoretical analysis and extensive simulations. Our results indicate that, under realistic channel and traffic conditions, and with the system parameters specified in [3], at most 4-5 users can be accommodated by the scheme at any time, which is far below the results provided in [1]-[2]. We explain the reasons for this observed multiuser performance.

2. DESCRIPTION OF OFDMA RANGING

A typical scenario of OFDMA Ranging transmissions is shown in Figure 1. The figure shows two users transmitting Long Ranging symbols and two users transmitting Short Ranging symbols. Actually, at any given time, there may be more than two users transmitting Long and/or Short Ranging codes, or there may be none.

Two 53-tone "subchannels" are dedicated as "Ranging Subchannels" at all times [3]. All Short Ranging symbols are transmitted on one or the other of the two 53-tone "Ranging Subchannels" assigned by the MAC. All Long Ranging symbols are transmitted on both of these "Ranging" subchannels (106 tones).

The "Frequency" axes in Figure 1 show only tones in the "Ranging" subchannels. In fact, these tones are not contiguous; they are interspersed within the much larger number of subchannels that are carrying payload data.

As shown, the Long Ranging symbols from users #1 and #2 use different codes. There are sixteen codes available for Long Ranging, and each user selects one of them at random. Therefore, sometimes two users will transmit with the same Long Ranging code. Time Offsets from the users transmitting Long Ranging codes, in this case T_1 and T_2, are arbitrary. Users have no symbol time reference or ranging information before Long Ranging, and therefore may transmit these symbols at any time.
The reason that the Long Ranging symbols are lengthened to span two FFT periods is so that the Base Station will get at least one uninterrupted FFT of the symbol.

From each FFT output, the base station must separate and detect each transmitted code from an unknown number of users sending Long Ranging and/or Short Ranging symbols. While detecting the Long Ranging symbols, it must jointly estimate the unknown time offset (i.e. T_1 and T_2), and the 106 complex channel gains from each user. The first, and most difficult step is to filter out (by correlating) and detect each Long Ranging code. To do this, it must sweep the time offset (i.e. T_j) used to derotate the phase of each tone to maximize the correlation, while searching for each of the 16 Long Ranging codes. After thus detecting the codes and estimating the time offsets, it must calculate the channel gains, assuming that the detected codes were in fact transmitted. Similarly, from the Short Ranging signals, it must jointly update the previously estimated time offset (i.e. T_3 and T_4) and the 53 complex channel gains from each user. (These should be much easier than the Long Ranging codes due to the previous available estimates.) In any event, successful detection and estimation of all this data will only be possible when there are a limited number of users simultaneously ranging. The limit on the number of simultaneous users is determined primarily by channel distortion, that reduces code autocorrelation, and "*angular*" code cross correlation properties (as opposed to their time cross-correlations), that determine the interference floor introduced from one user to another (See Section 5).

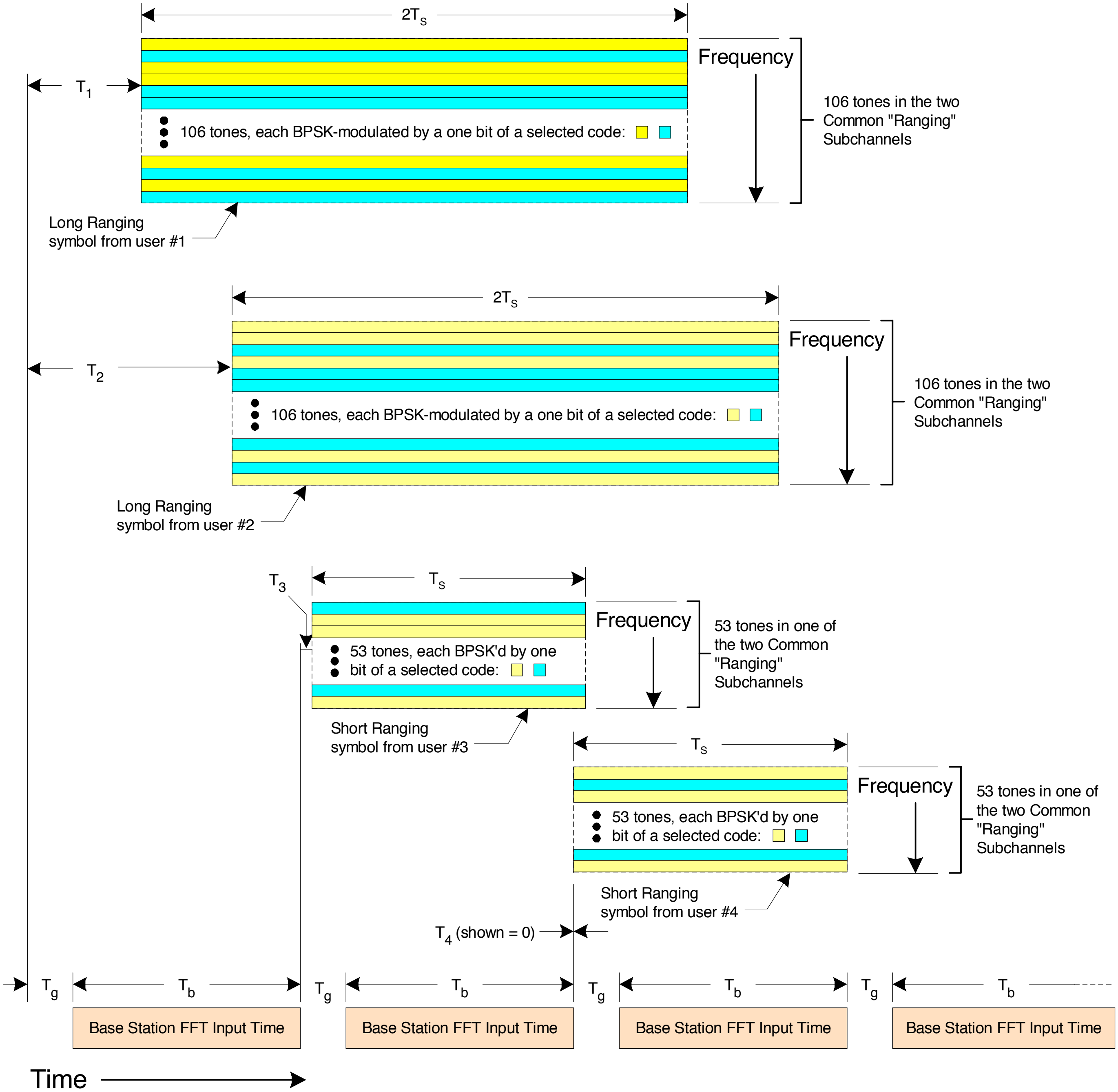

Figure 1. Typical Scenario of OFDMA Ranging

In order to avoid overpowering the system, Long Ranging users may try to transmit first at their lowest power level, gradually increasing until acknowledged by the base station's MAC. The optimum power step will depend on traffic volume. For fixed subscriber stations, the correct power level will vary much less than the available range, and therefore some memory of past power settings in the subscriber station should help reduce the number of these Long-Ranging retries significantly.

Two users may randomly select the *same* Long Ranging code, and, if the resulting symbols arrive during the same FFT, a difficult collision will occur. This collision may be impossible to resolve, since, depending on the distance between the two users, they may be erroneously detected as two multipath transmissions from the same user.

Because the code bits are mapped onto subcarriers, and because subscriber stations' subcarrier frequencies are locked to the base station, there is no possibility of code bits being offset to different subcarriers among different transmitters, or between transmitter and receiver. (This is unlike unsynchronized single-carrier CDMA, where the unknown symbol time offsets will directly offset the code bits.) For the same reason, the (time) autocorrelation properties of the selected codes are not significant. Time offsets between BS and CPE will be result in *phase rotation* of the ranging subcarriers. Ranging codes should therefore be chosen which are orthogonal to one another with respect to "angle" as opposed to time shifts. Similarly, the "time-offset" cross-correlation properties of the codes are not relevant here. The codes proposed to be used in the present draft standard (cf. Fig.234, pp. 170, [3]) do not have this property.

3. DETAILS OF THE SIMULATION

In all cases, the system model is a 2K OFDMA system as specified in the draft standard, that is:

- 1 DC carrier not used
- 176 lower guard tones
- 175 upper guard tones
- 1696 used subcarriers.

Subcarrier frequency spacing is (8/7)*6 MHz/2048 = 3.35 KHz.

32 subchannels of 53 subcarriers have carriers assigned with the "permutation" algorithm in the draft standard [3].

3.1 Transmission Scheduling

To begin the simulation, a schedule of long-ranging and short-ranging transmissions is generated, assuming a Poisson distribution of originating requests per OFDM symbol (see appendix-A). At each time step, a long-ranging or short-ranging transmission may or may not occur (geometric random variable). However, because there are 2048 time steps in the schedule per OFDM symbol, and only a few transmissions per OFDM symbol, the probability of more than one transmission at a time step is negligible and ignored. In accordance with the current draft IEEE 802.16ab-01/01r1, July 2001, sec. 8.3.6.3.3.4.3.1, subscriber stations are allowed to choose the same code on the same subchannel(s) and collide.

It was found that spacing the correlator time steps equal to (T_s/N_{FFT}) causes a degradation of up to 2.5 dB in the detected peaks. (The maximum degradation occurs when the actual time is halfway between two correlator time steps). Therefore, an actual receiver must sample at several times this rate. However, because the simulation time was dominated by these correlators, for expediency a time step of (T_s/N_{FFT}) was used, and all transmissions were constrained to begin coincident with one of these samples.

Because short-ranging transmissions are synchronized with the BS receiver, their time is modified, moved to the beginning of the OFDM symbol in which they occurred. For each transmission, a pseudonoise code is selected at random from the long- and short-ranging codebooks specified by the scenario.

For each OFDM symbol in the simulation, the following are chosen at random:

- cellID, an integer from 0 to 11
- rangingSubchannel1, an integer from 0 to 31
- rangingSubchannel2, another integer from 0 to 31, except not allowed to be the same as rangingSubchannel1

3.2 Construction of the Received Signal

The simulation then proceeds on a symbol-by-symbol basis. At the beginning of each symbol, the ranging transmissions that will be received during this symbol are found from the schedule. Because short-ranging transmissions are synchronized with the BS receiver and last for one symbol, their contributions are simply added to each FFT bin, all with the same amplitude. The long-ranging transmissions are more complicated because they may begin at any time and last for a duration of two symbols. It is apparent, however, that although these long-ranging signals will, in general, be present during three consecutive OFDM symbols, their contribution will be reduced during the first and last symbol (cf. Fig.1).

For each signal, complex channel coefficients are selected from the two-dimensional Gaussian distributions specified by the channel model (latest SUI models [3]) assumed in the scenario to be simulated. The channel's frequency-domain gains (H) are obtained by FFT of the impulse response.

To summarize the simulation of the transmissions, each bit in each Long-Ranging and Short-Ranging signal is phase-shifted by the channel "H" gain corresponding to its particular tone frequency in the randomly-selected subchannels. Each bit in each Long-Ranging is further phase-shifted according to its selected start time, and as described above, its amplitude is reduced if it does not all fall within the current FFT window.

All the Long-Ranging and Short-Ranging signals are then added together to form the signal received at the Base Station.

3.3 Receiver

This received signal is applied to a bank of frequency domain correlators, which look respectively for each of the codes allowed in the particular scenario. The "slider" variable in each correlator is "time", but this is implemented by appropriate phase rotation, depending on the subcarrier frequency, of each tone in the correlator reference code. Thus, the correlator outputs look as though they are time-domain correlators, although they are actually implemented in the frequency domain.

Some typical correlator outputs are shown in Figure 2 through Figure 7, where the desired transmission has been timed to occur at the middle of the correlator output. In these figures, the outputs are colored red for a time duration equal to the channel impulse response, beginning with the time the code would have been received over a distortionless channel. The magenta horizontal lines show the rms value of each plot. The interfering transmissions are other long-ranging transmissions, all using different codes from the set of sixteen.

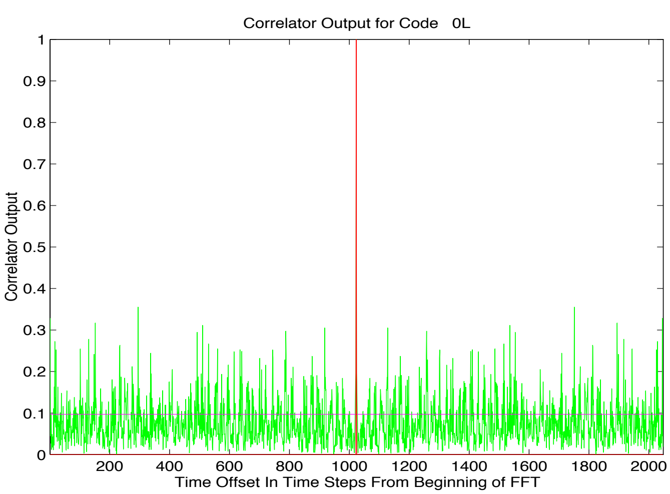

Figure 2. Typical correlator output for a long-ranging user over a distortionless channel with no inteferference.

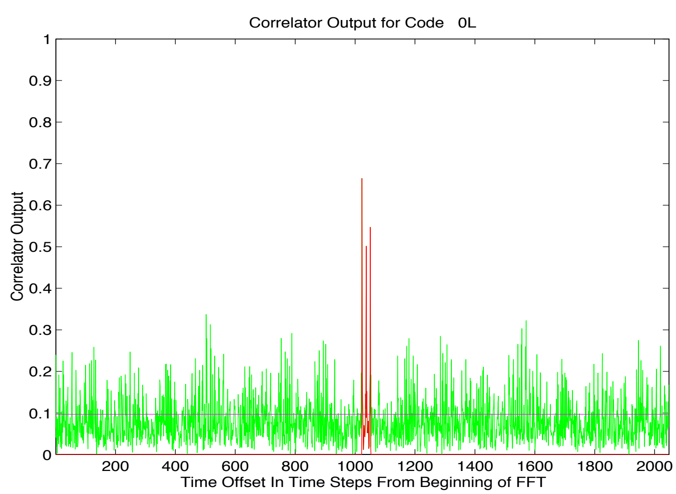

Figure 3. Typical correlator output for a long-ranging user over a SUI-4 channel with no interference.

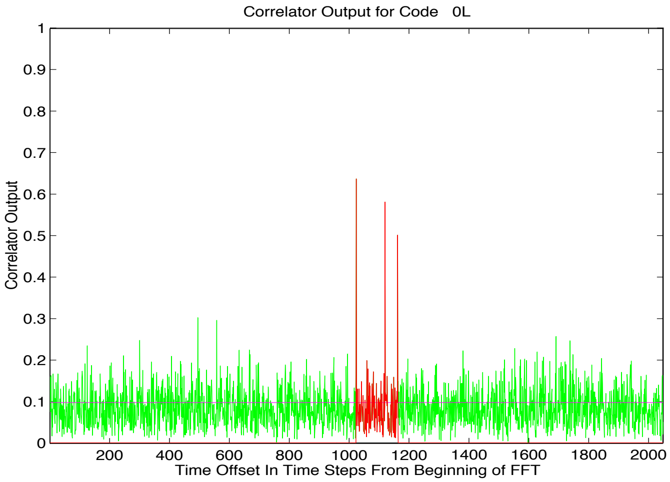

Figure 4. Typical correlator output for a long-ranging user over a SUI-6 channel with no interference.

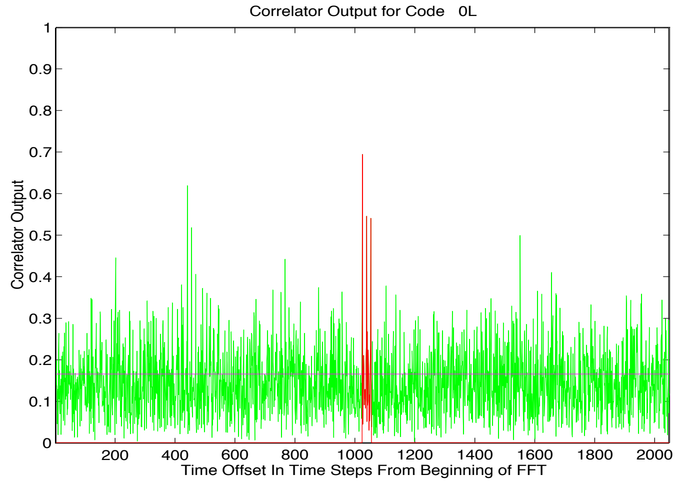

Figure 6. Typical correlator output for a long-ranging user over a SUI-4 channel with two (2) interfering transmissions.

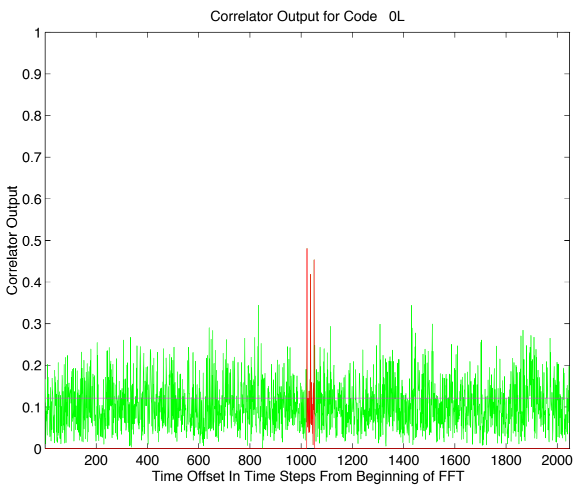

Figure 5. Typical correlator output for a long-ranging user over a SUI-4 channel with one (1) interfering transmission.

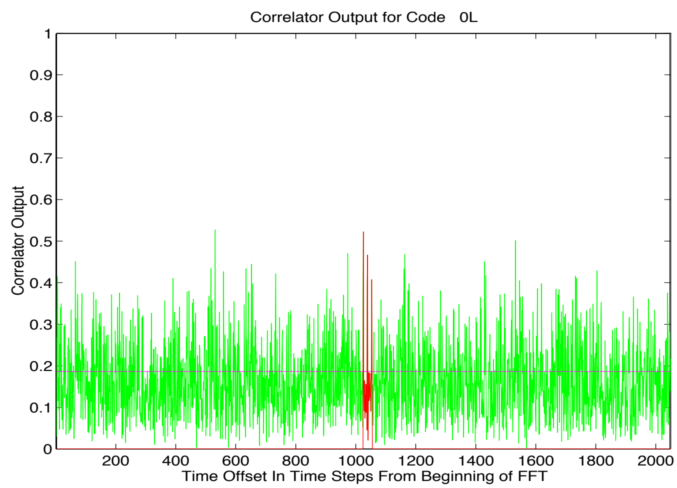

Figure 7. Typical correlator output for a long-ranging user over a SUI-4 channel with three (3) interfering transmissions.

For each correlator, the rms value of its outputs is found, and each output is normalized to this rms value. The result is in dB.

3.4 Detection

Detection of a ranging transmission when one was in fact transmitted using the detected code near the detected time is called a "hit". Because Long-Ranging transmissions can occur at any time, hits of Long-Ranging transmissions are found by examining the output values of the appropriate correlator during a time window beginning at the actual start time of the transmitted symbol, and lasting as long as the channel impulse response.

Hits of Short-Ranging transmissions are found similarly, but in this case the length of the time window is equal to the time specification in the proposed standard, namely 30% of the maximum guard interval of 1/4 OFDM symbol. In fact, these time intervals are arbitrary base-station receiver design parameters. Other reasonable values were tried, but no substantial change in performance was obtained.

In either case, the maximum correlator output value in the prescribed time window is obtained. Values less than 2 dB (above rms) are ignored. Higher values are rounded down to the nearest dB, and then tallied into bins 1 dB wide for further analysis.

False Detections ("false alarms") are found in the Long-Ranging code correlator outputs by examining each of the output values during each OFDM symbol. Also, each of points is examined to determine if a legitimate correlation peak could be expected at this time. Legitimate peaks are expected if the time is within a window beginning with the transmission of a Long-Ranging code and lasting as long as the channel impulse response. Legitimate peaks are also expected if the time is within the time interval described above for Short-Ranging transmissions, 30% of 1/4-OFDM-symbol, same as for detecting "hits". Again, values less than 2 dB above rms are ignored, and values within a time period when legitimate peaks are expected, are also ignored. Again, higher values are rounded down to the nearest dB, and then tallied into bins 1 dB wide. False Detections are found in the Short-Ranging code correlator outputs similarly, except instead of all 2K samples, the window of Expected transmissions begins at the beginning of the OFDM symbol and lasts only to the 30% of 1/4-OFDM-symbol period described above. This is because the Short-Ranging transmissions should all be time-aligned to the beginning of the base station receiver's FFT, and the Short-Ranging code correlators therefore need not search for transmissions at other times. Again, values of less than 2 dB above rms are ignored, and others are quantized to the nearest dB and tallied.

If the transmission part of the simulator flagged a collision (same codes used), then both transmissions will fail to be received. To correctly model this behavior, the simulation tallies a guaranteed Miss, regardless of correlator threshold setting.

Whenever a bona fide long-ranging transmission occurs, the false-alarm detection in the BS receiver correlator for that code is immediately disabled, and remains disabled for a duration equal to the channel impulse response.

Finally, the number of long-ranging transmissions which is allowed to be received in any OFDM symbol is limited to an arbitrary value. Only the highest correlator outputs are retained.

4. SYSTEM PARAMETERS

We have chosen four System Parameter Sets to simulate. The first two are baseline cases which are allowed under the current draft IEEE 802.16ab-01/01r1, July 2001, sec. 8.3.6.3.3.4.3.1. The latter two involve modifications of the current draft.

In order to keep the size of this contribution somewhat reasonable, all System Parameter Sets are based on a single, typical set of OFDMA uplink shows parameters. These common parameters are shown in Table 1.

Number of points in FFT	2048
Number of lower-frequency guard tones	176
Number of higher-frequency guard tones	175
Channel Width	6.0 MHz
Subcarrier Frequency Spacing	(8/7)*(6.0MHz/2048) = 3348 Hz
OFDMA Subcarrier Permutation Base	[3 18 2 8 16 10 11 15 26 22 6 9 27 20 25 1 29 7 21 5 28 31 23 17 4 24 0 13 12 19 14 30]
Number of Short-Ranging Codes	32
Number of Subcarriers Per Subchannel	53
Receiver Correlation Time	Yes

Table 1. Parameters which are common to all System Parameter Sets.

The four System Parameter Sets are named Baseline2, Baseline4, RareLong and Short106. The differences among them, and the motivation for each, are now described.

Baseline2 is the base line system as currently proposed in IEEE 802.16ab-01/01r1, July 2001, sec. 8.3.6.3.3.4.3.1, using the default allocation of two subchannels (6.25% of system capacity) for ranging. Long-ranging is assumed to be a frequent process, which is initiated by a subscriber station each time it enters the system, presumably this is at least once each time the modem is powered on. Accordingly, there are 16 codes allocated to long-ranging and 32 codes allocated to short-ranging. The base station receiver is allowed to detect up to four long-ranging transmissions per OFDM symbol.

Baseline4 is the same as Baseline2, except the base station has allocated four subchannels (12.5% of system capacity) to ranging instead of two. It is assumed that the base station will do this whenever it detects too much congestion on the ranging channel when operating with the default allocation (Baseline2). (Note: In the current draft IEEE 802.16ab-01/01r1, July 2001, sec. 8.3.6.3.3.4.3.1, other allocations such as 1, 3, 5, 6, 7, 8… subchannels are also allowed.)

RareLong is a modification of Baseline2. In this case, the system design has been changed to reflect the assumption that long-ranging will be rarely done, only necessary when a subscriber station installed or reinstalled in a new location which changes its time-of-flight to and from the base station enough that its ranging parameters must be updated. This should be the case if the ranging parameters are stored in nonvolatile memory in the subscriber station. With this assumption, it is no longer necessary to have 16 long-ranging codes. It is proposed that the base station be able to dynamically declare how many long-ranging codes are active, with the default being one (1) code. All simulations of this system are done assuming one (1) active long-ranging code, and consequently the base station receiver is allowed to detect only one (1) long-ranging transmission per OFDM symbol.

Short106 is the same as RareLong, except the structure of short-ranging transmissions was modified. In Baseline2, as currently proposed in IEEE 802.16ab-01/01r1, July 2001, sec. 8.3.6.3.3.4.3.1, short-ranging transmissions are 53 bits in length use only one subchannel. Because there are two subchannels allocated to ranging (in Baseline2), the subscriber station is free to choose either of these two subchannels. This has the advantage that, even if two subscribers choose the same code in the same OFDM symbol, if they choose different subchannels, there will be no collision and the base station could receive both. In Short106, this has been modified to see if code division is more advantageous than this frequency division. Instead of 53 bits, the short-ranging code length has been extended to 106 bits, and both subchannels are used for all short-ranging transmissions. There is no choice of subchannel. The number of codes is still 16. Instead of searching with 32 53-bit correlators on two subchannels per OFDM symbol, the base station searches with 16 106-bit correlators on one subchannel.

The differences among the Sytem Parameter Sets are summarized in Table 2.

Parameter ↓	Baseline2	Baseline4	RareLong	Short106
Ranging Code Type	PN	PN	PN	PN
Length of long-ranging code (bits)	106	212	106	106
Number of subchannels allocated to ranging	2	4	2	2
Fraction of system capacity allocated to ranging	6.25%	12.5%	6.25%	6.25%
Length of short-ranging code (bits)	53	53	53	106
Probable frequency of long-ranging transmissions by the aggregate of subscriber stations, in number per OFDM symbol	High,1- 2	High, 1-2	Rare, .02	Rare, .02
Number of long-ranging codes, and consequent number of long-ranging correlators in BS receiver	16	16	1	1
Number of long-ranging transmissions which the BS receiver is allowed to detect, per OFDM symbol.	4	4	1	1

Table 2. System Parameters for Different Sets.

5. SIMULATION RESULTS

The tallies of Hits and False Detections from the simulator outputs are each integrated and normalized by the expected number of transmissions per OFDM symbol. The integrated and normalized Long- and Short-Ranging Hits produce the Probability of Missing a Ranging Transmission. The integrated and normalized Long- and Short-Ranging False Detections produce the number of False Detections per OFDM Symbol. Both of these curves depend on the selected receiver threshold; lower thresholds give a lower probability of Missing but also more False Detections; higher thresholds give fewer False Detections but a higher probability of Missing.

The resulting Miss Probability and False Detection curves are plotted on the same graph for each scenario with each System Parameter Set in Figure 8 through Figure 17.

For each System Parameter set, results are given for several different scenarios, involving different expected values of long- and short-ranging transmissions per OFDM symbol, and different channel models. To get started, note that the top graphs in each column are results with the SUI-1 channel model, the middle graph is with SUI-4, and the bottom graph is with SUI-6. Also note that separate results, on separate pages, are giving for long-ranging and short-ranging correlator outputs. These separate results are not to be confused with the expected numbers of long- and short-ranging transmitters. For example, the graphs on the left column of Figure 8 show the performance of the long-ranging correlator outputs with an expected value of nominally one long-ranging transmission and two short-ranging transmissions per OFDM symbol. In this case, all of the short-ranging transmissions are interference to the desired long-ranging transmission. On the following page, Figure 9 shows the performance of the short-ranging correlator outputs under the same conditions. In this case, the long-ranging transmissions are interference to the desired short-ranging transmission. Of course, long-ranging and short-ranging transmissions also interfere with one another.

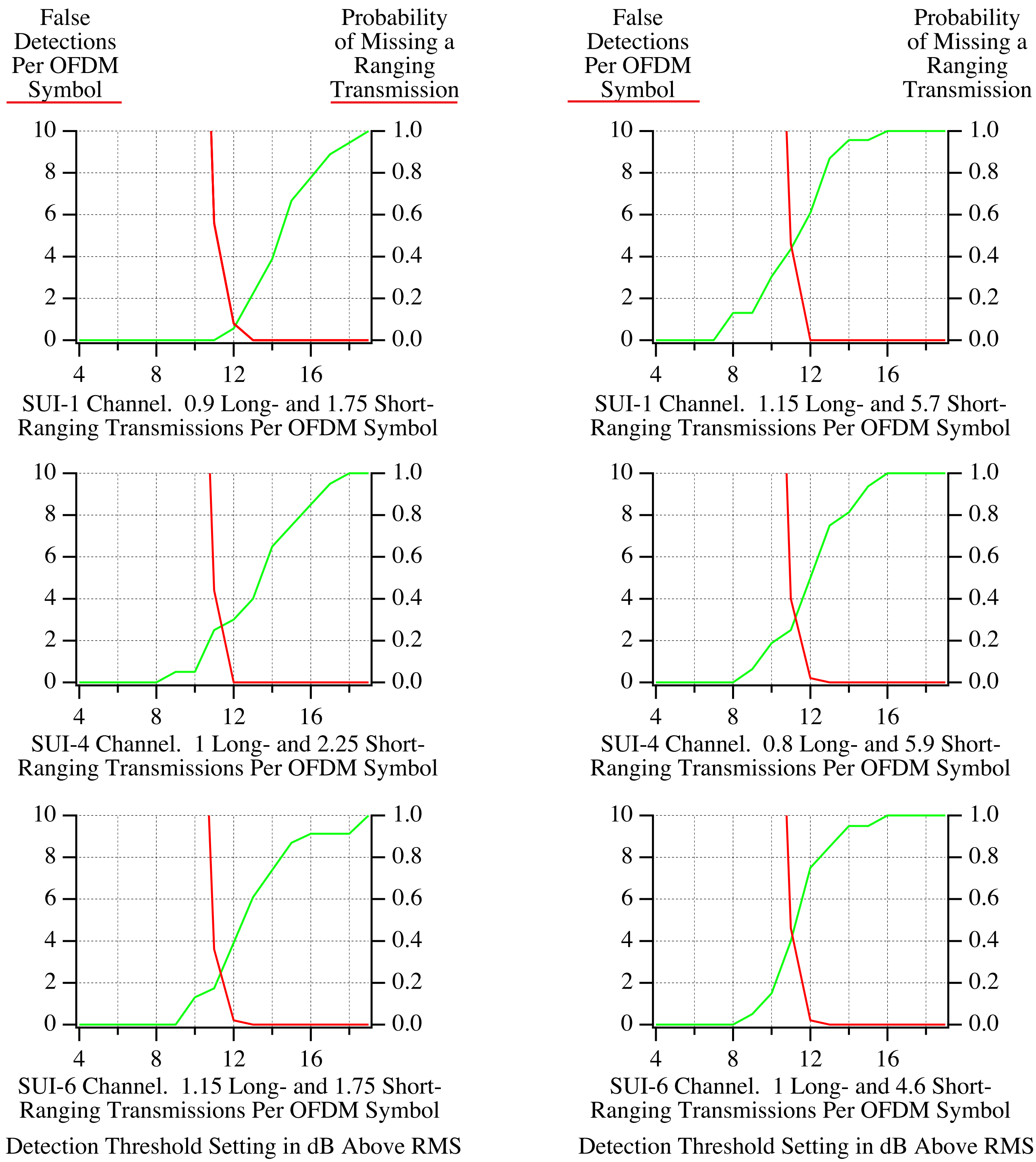

Figure 8. Long-Ranging Performance with System Parameter Set Baseline2, with nominally one (1) long-ranging transmission per OFDM Symbol.

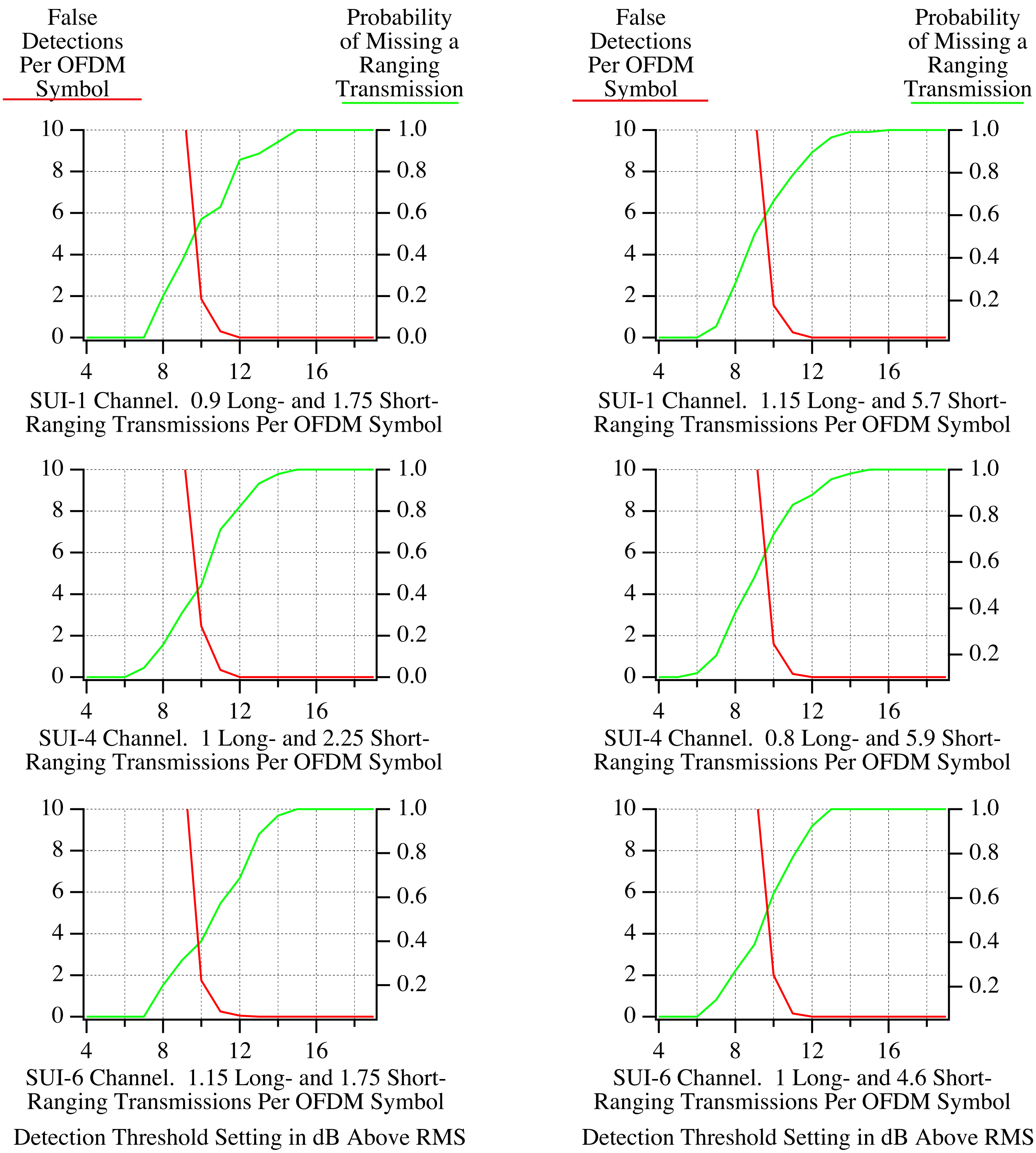

Figure 9. Short-Ranging Performance with System Parameter Set Baseline2, with nominally one (1) long-ranging transmission per OFDM Symbol.

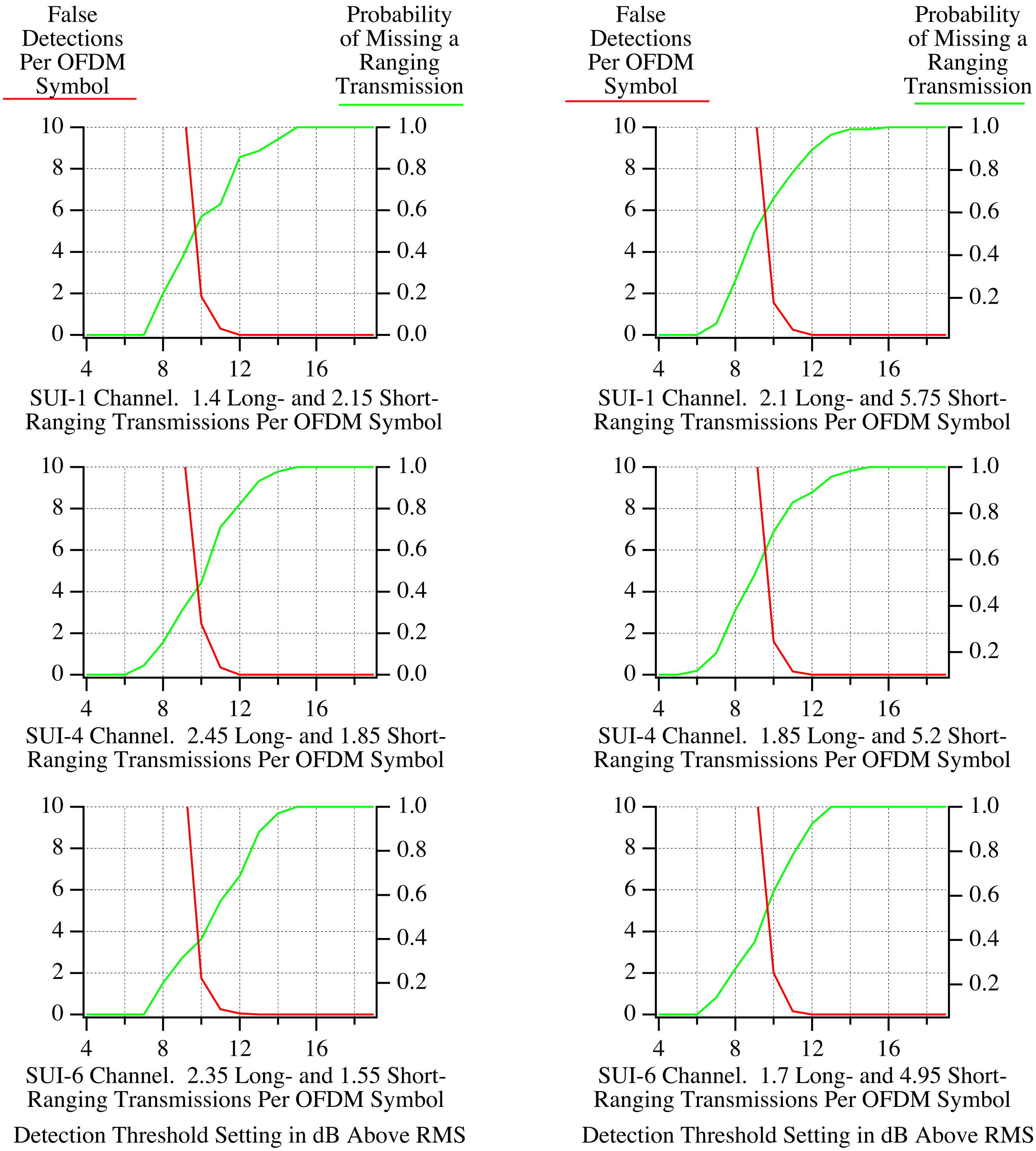

Figure 10. Long-Ranging Performance with System Parameter Set Baseline2, with nominally two (2) long-ranging transmission per OFDM Symbol.

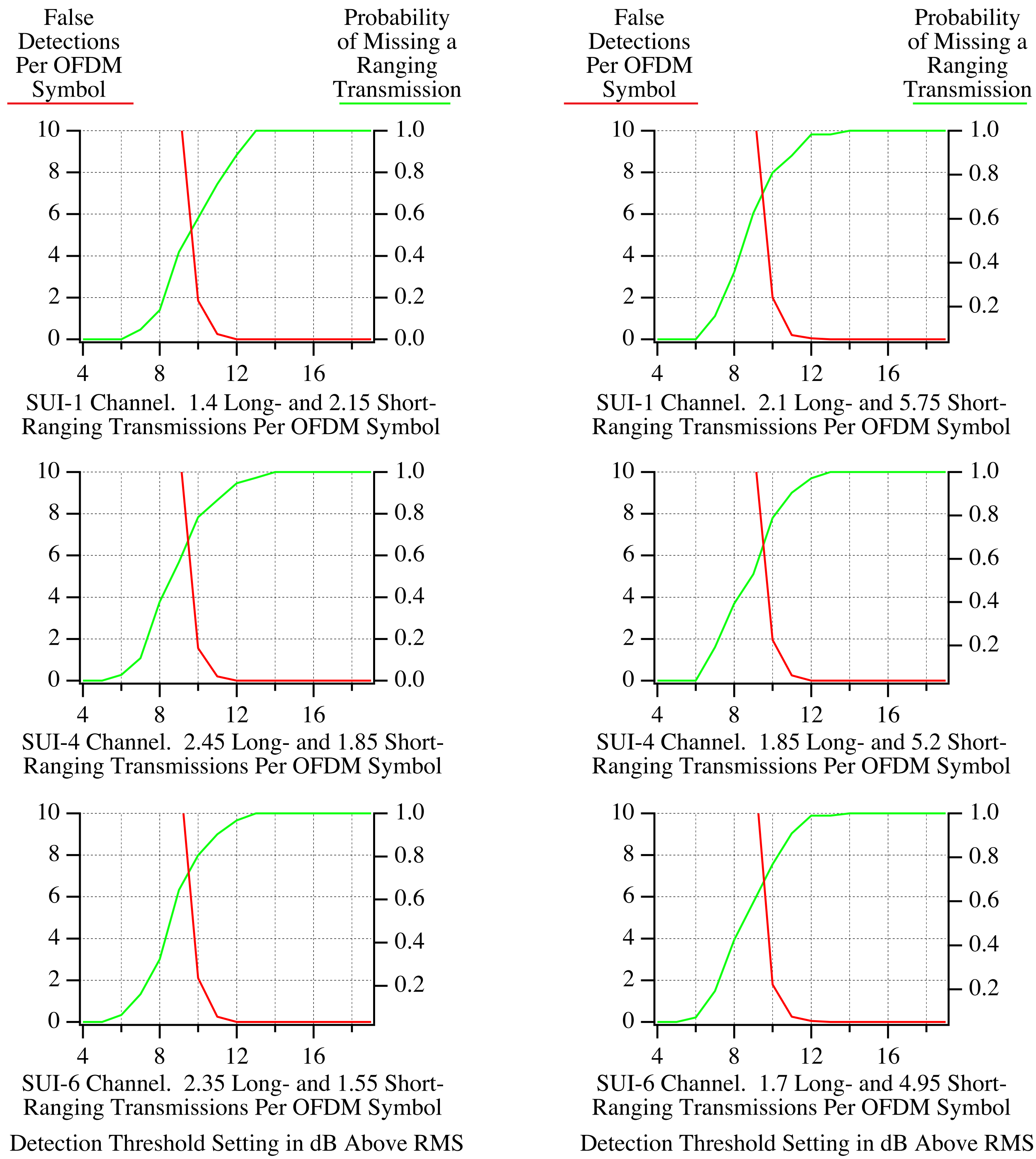

Figure 11. Short-Ranging Performance with System Parameter Set Baseline2, with nominally two (2) long-ranging transmission per OFDM Symbol.

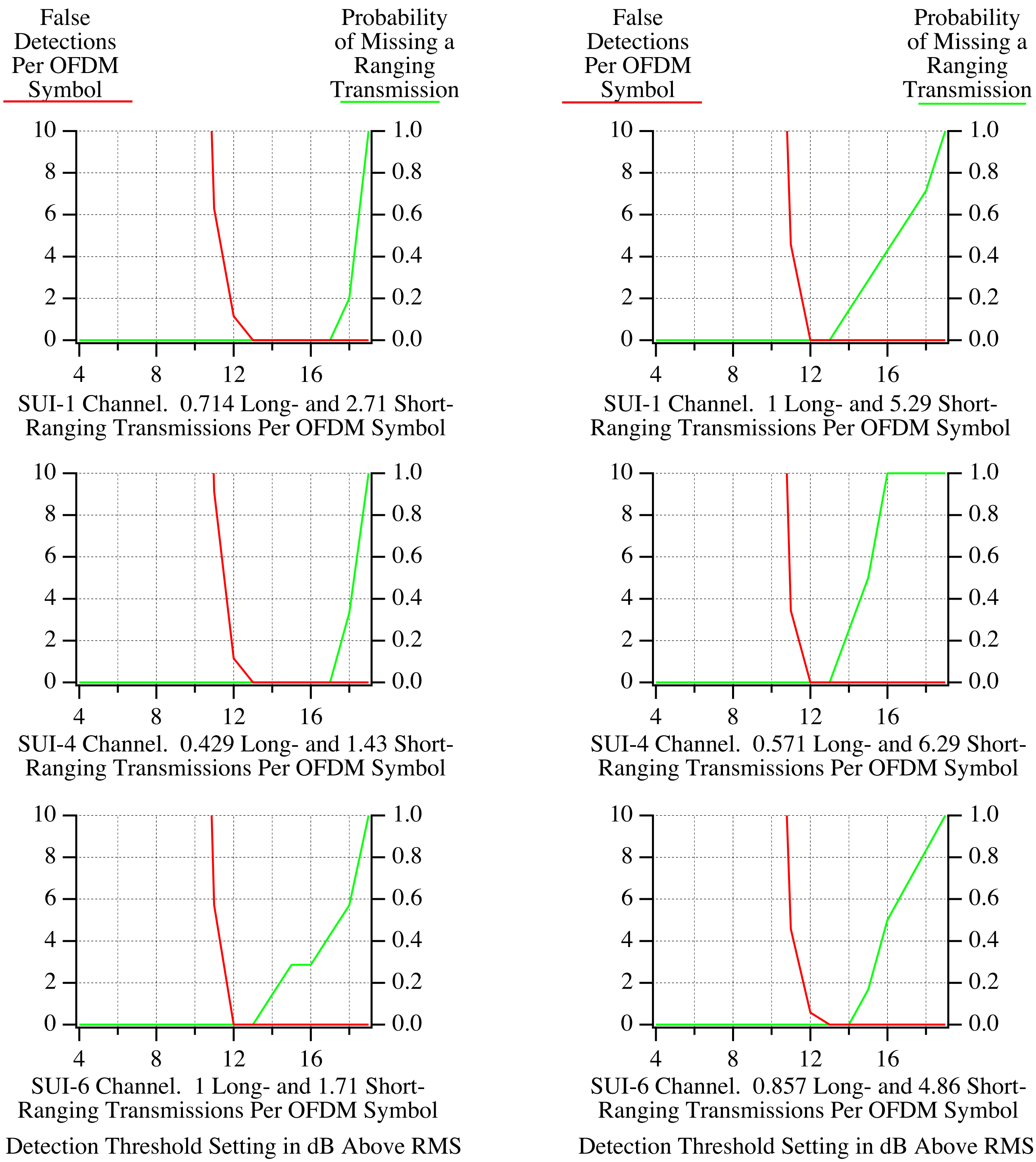

Figure 12. Long-Ranging Performance with System Parameter Set Baseline4, with nominally one (1) long-ranging transmission per OFDM Symbol.

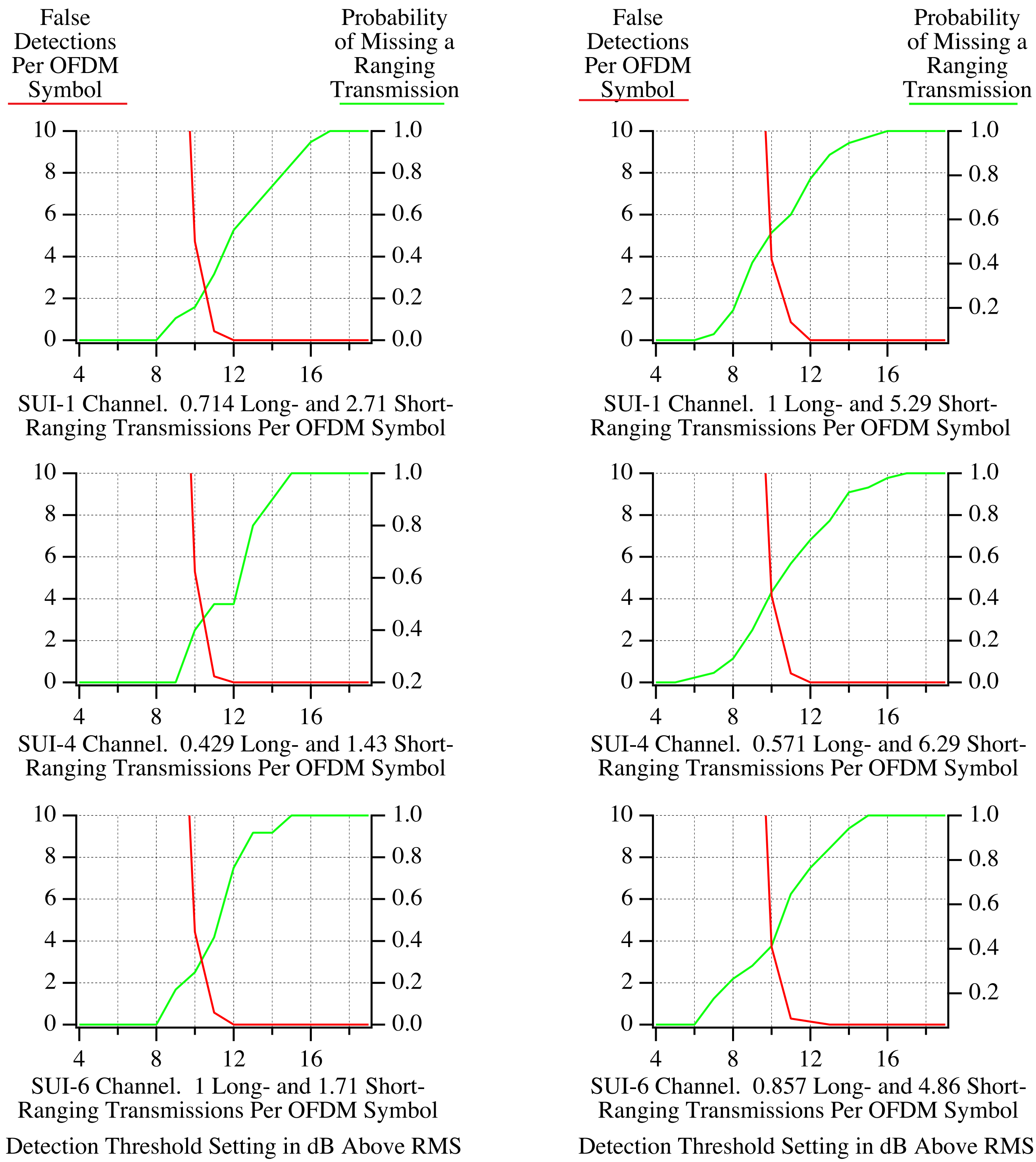

Figure 13. Short-Ranging Performance with System Parameter Set Baseline4, with nominally one (1) long-ranging transmission per OFDM Symbol.

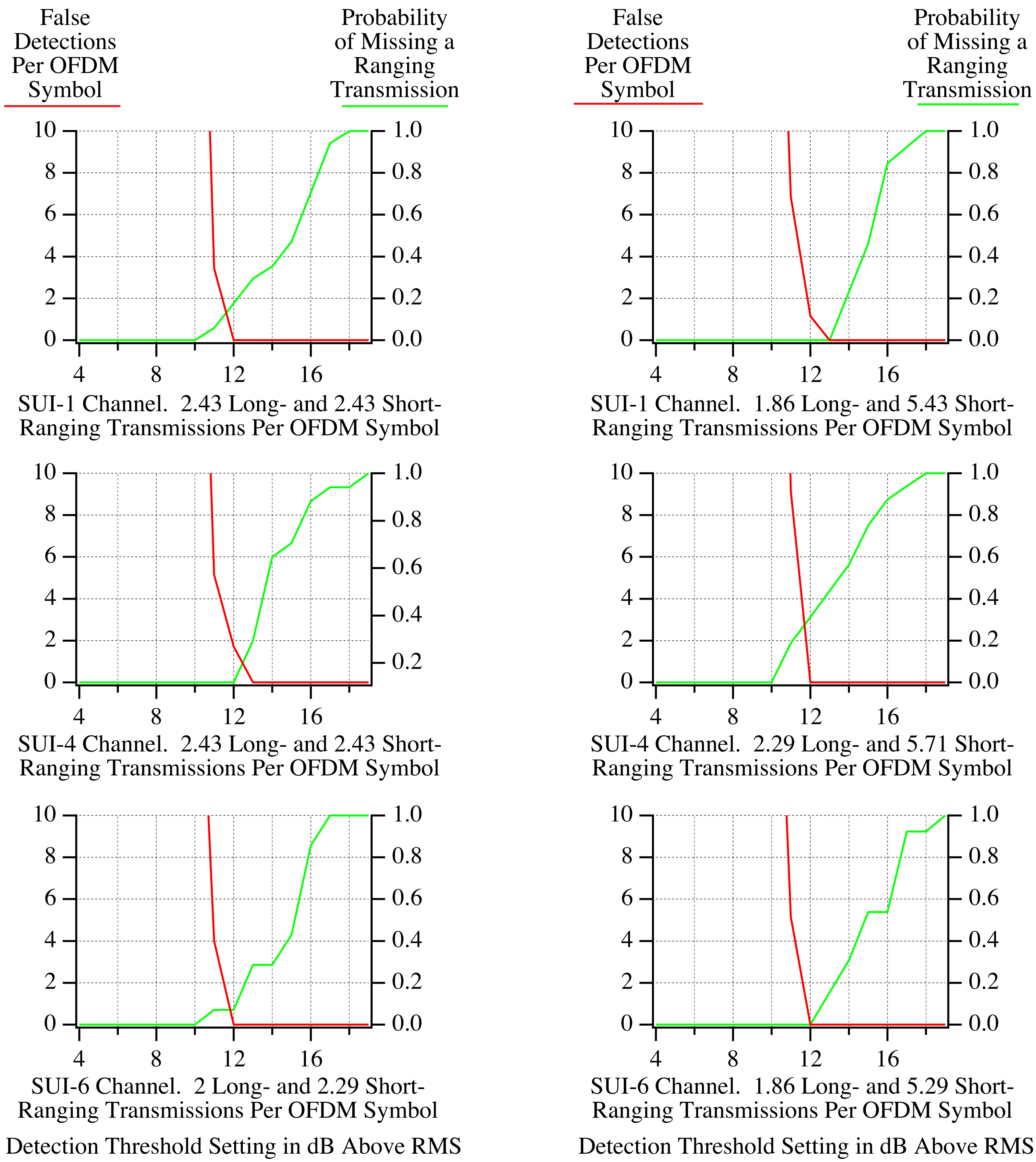

Figure 14. Long-Ranging Performance with System Parameter Set Baseline4, with nominally two (2) long-ranging transmission per OFDM Symbol.

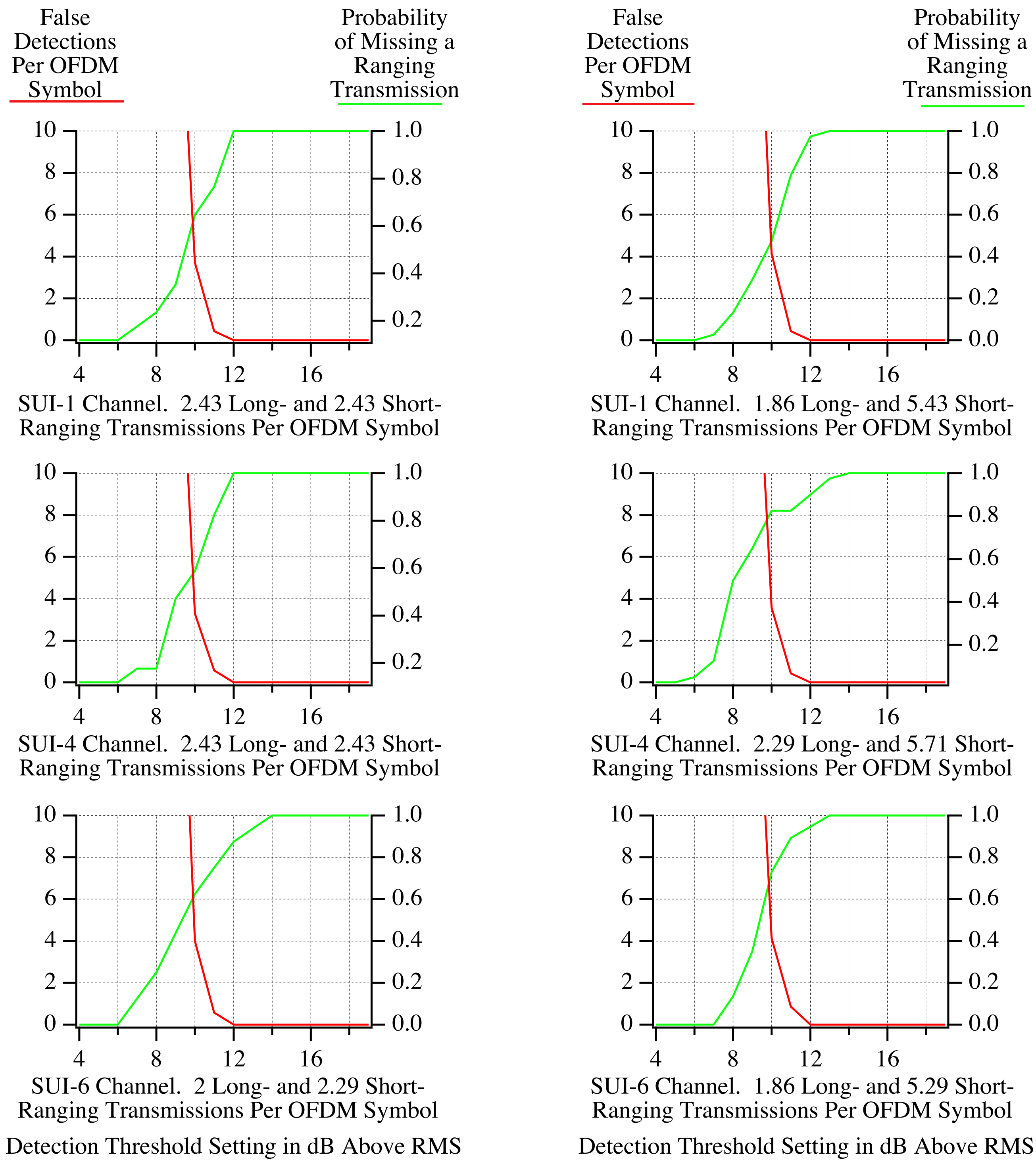

Figure 15. Short-Ranging Performance with System Parameter Set Baseline4, with nominally two (2) long-ranging transmission per OFDM Symbol.

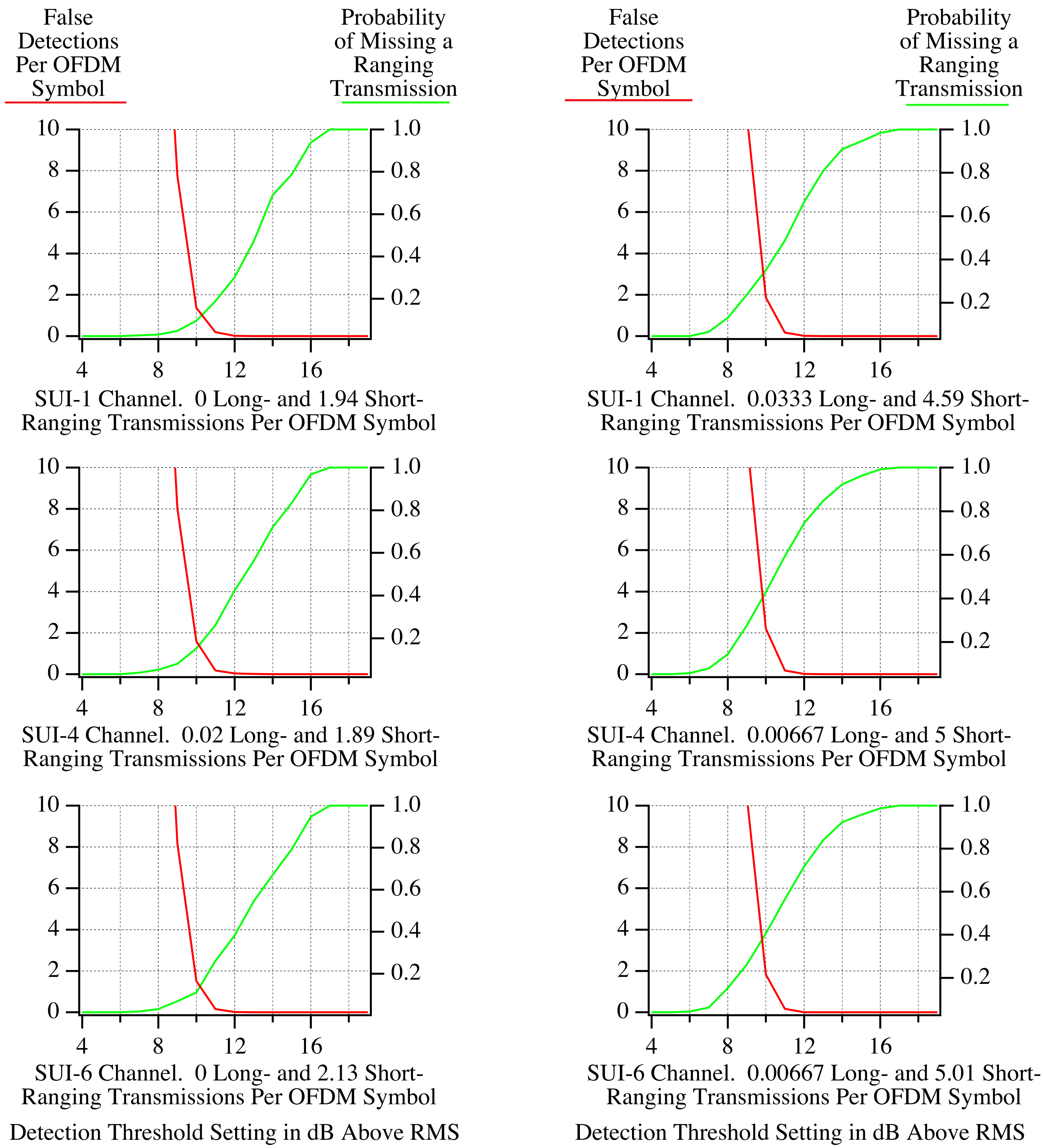

Figure 16. Short-ranging performane with System Parameter Set RareLong.

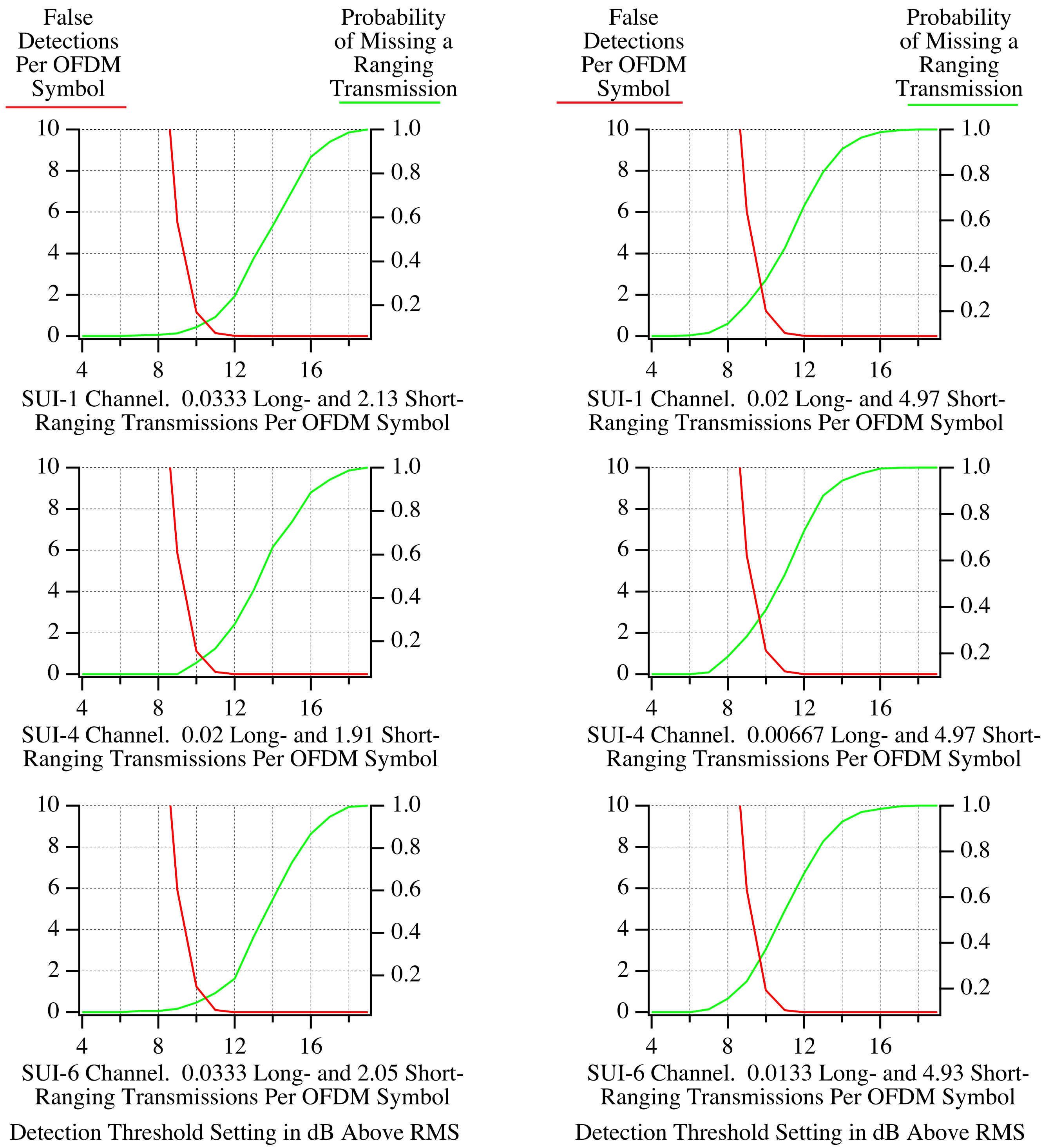

Figure 17. Short-ranging performance with System Parameter Set Short106.

6. CONCLUSIONS AND REMARKS

The ranging mechanism described in [1]-[3] is conceptually interesting but suffers from a lack of robustness in an actual multiuser, multipath environment. Two factors limit the performance: (i) the presence of a frequency selective (multipath) channel, which introduces a different complex scaling on every code "bit", and (ii) Cross-correlation properties of the ranging codes, which determine the interference floor in the detection process.

We should also point out here that comparisons of this scheme with conventional (time-domain) CDMA are somewhat misleading. Neither does this scheme have the wideband anti-multipath (raking) capability of a conventional spread spectrum system, nor does its capacity degrade gracefully as the number of ranging codes is increased (due to fundamental difference between the nature of time and frequency domain codes). Further work is thus required to improve the performance of the proposed scheme.

7. REFERENCES

[1] Itzik Kitroser, Yossi Segal and Zion Hadad, "Bandwidth Request Using CDMA Codes in OFDMA(OFDM) Base PHY for TG3 and TG4", IEEE802.16.3c-01/55, April 2001.
[2] Itzik Kitroser, Yossi Segal and Zion Hadad, "OFDM based Ranging Enhancement for the TG3 and TG4", IEEE802.16.3c-01/54, April 2001.
[3] IEEE 802.16ab-01/01r1, Air Interface for Fixed Broadband Wireless Access Systems, July 2001, Available WWW: http://grouper.ieee.org/groups/802/16/index.html
[4] C.R. Baugh et al., "Traffic Model for 802.16 TG3 MAC/PHY Simulations", IEEE 802.16.3c-01/30r1, March 2001.

Appendix A. Single User Theoretical Performance

Consider an L path channel with delay $\tau_{l,m}$ and complex gain $h_{l,m} = h_{l,m}{}^{I} + j.h_{l,m}{}^{Q}$ associated with the l th path during the mth OFDM symbol interval. Then the Channel Impulse Response is given by

$$h_m(t) = \sum_{l=1}^{L} h_{l,m}\delta(t - \tau_{l,m})$$

We can also absorb the (unknown) time of flight from the CPE to the Base Station into this Channel Impulse response by introducing the additional delay parameter T_{TOF}. Note that one of the functions of the Ranging protocol is to estimate this "T_{TOF} '' parameter in the presence of other contending users, and under unknown channel conditions.

The channel response at the k th tone (QAM symbol) is now given by

$$H_{m,k}(f) = \sum_{l=1}^{L} h_{l,m} e^{-j2\pi.f.\tau_{l,m}} e^{-j2\pi.f.T_{TOF}} \Big|_{f=k/T_s}$$

In the remainder of this document we will assume that the cyclic prefix is longer than the maximum multipath spread of the channel so that we can drop the symbol index m, and use the well-known parallel Gaussian model for Multicarrier transmission

$$Y_k = X_k H_k + \eta_k$$

Here X_k is the QAM symbol transmitted on the k th tone in any symbol interval; H_k is the channel frequency response on that tone (specified above), and η_k represents uncorrelated, additive, Gaussian noise; and Y_k represents the symbol as it is received at the Base Station after being distorted through the frequency-selective channel.

We are now ready to deal with the ranging process itself. The scheme described in [1] works as follows: A CPE that wants to obtain system access contends by transmitting a "code" on certain subcarriers for an interval equal to two OFDMA symbols; essentially this is a BPSK modulation of certain tones at the Transmitter IFFT. In other words, we can express any ranging code by the set $C_k \in \{-1,+1\}$ for $k \in \upsilon$, where υ is the set of subcarriers included under the "Ranging subchannels" (the cardinality of this set, Γ, is in multiples of 53 in the present IEEE standard). The receiver then uses a set of correlators matched to all possible transmitted codes, along with an additional phase exponent on each subcarrier to estimate the CPE's Time of flight. For example, let us consider a Receiver correlator ``tuned'' to the code symbols C_k, scanning with variable phase parameter θ. The output of the correlator is then

$$\begin{aligned}\Re(\theta) &= \sum_{k\in\upsilon} Y_k C_k e^{j2\pi.k\theta} \\ &= \sum_{k\in\upsilon} X_k C_k H_k e^{j2\pi.k\theta} + \sum_{k\in\upsilon} C_k \eta_k e^{j2\pi.k\theta}\end{aligned}$$

When $\{X_k\} = \{C_k\} \forall k \in \upsilon \Rightarrow X_k C_k = |C_k|^2$, the correlator is "matched" to the transmitted code, hence

$$\begin{aligned}\Re(\theta) &= \sum_{k\in\upsilon}\sum_{l=1}^{L} h_l e^{-j2\pi.k\tau_l/T_s} e^{-j2\pi.kT_{TOF}/T_s} e^{j2\pi.k\theta} + \sum_{k\in\upsilon} C_k \eta_k e^{j2\pi.k\theta} \\ &= h_{l'}\Gamma + \sum_{l\neq l'} h_l \sum_{k\in\upsilon} e^{-j2\pi.k.(\tau_l-\tau_{l'})/T_s} + \sum_{k\in\upsilon} C_k\eta_k e^{j2\pi.k\theta} \qquad \text{for } \theta = \frac{T_{TOF}+\tau_{l'}}{T_s}\end{aligned}$$

The first term in the above expansion represents the useful Autocorrelation "peak" from the correlator; note that this peak is linearly related to the number of subcarriers reserved for ranging (given by the cardinality Γ of the ranging set υ). Also note that the effect of the channel on the peak is to diminish its amplitude by a value proportional to its corresponding channel tap. The number of peaks seen in the autocorrelation is theoretically equal to the "number of taps" in the channel. Each peak is scaled proportionately by its respective channel tap.

The second and third terms in the autocorrelation represent the noise floor observed in the autocorrelation function Denoting this noise as $n(\theta)$, we can see that for a random enough selection of subcarriers k, $E\langle n(\theta)\rangle = 0$ and, for large enough Γ, $n(\theta) \sim N(0,\sigma_n^2)$ (by the Central Limit Theorem).

For <u>mismatched</u> codes, however, $\{X_k\} \neq \{C_k\} \forall k \in \upsilon$, so that

$$\Re(\theta) = \sum_{k\in\upsilon}\sum_{l=1}^{L} X_k C_k h_l e^{-j2\pi.k(\tau_l+T_{TOF}-\theta.T_s)/T_s} + \sum_{k\in\upsilon} C_k\eta_k e^{j2\pi.k\theta}$$

and hence the behavior of the correlation function is dominated by the "cross-correlation properties" of the codes X_k and C_k. Good codes would be those that would ideally have zero cross-correlation, or, more realistically, for some detector threshold ξ,

$$\sup_\alpha \sum_{k\in\upsilon} X_k C_k e^{j2\pi.k\alpha} \leq \xi$$

In case of multiple users, several codes will be transmitted concomitantly and hence the above property has to hold for all pairs of codes as well as all sets of their linear combinations. The design of codes having such a property is an open problem.

Appendix B. Traffic Model

At each time step of the simulation, the probability of originating a long range transmission is given by p (say). The probability mass function of having k long ranging transmissions originating during an OFDMA symbol is then given by

$$p_K = \binom{N}{k} p^k (1-p)^{N-k} .$$

As $N \to \infty$, and keeping $Np = \lambda$ = constant, we get

$$Lim_{N\to\infty} p_K = Lim_{N\to\infty} \binom{N}{k} p^k (1-p)^{N-k}$$

$$= Lim_{N\to\infty} \frac{N!}{(N-k)!k!} p^k (1-p)^{N-k}$$

$$= Lim_{N\to\infty} \frac{p^k}{k!} N^k \left\{ \frac{1}{N^k} (1-p)^{N-k} \frac{N!}{(N-k)!} \right\}$$

$$= \frac{\lambda^k}{k!} Lim_{N\to\infty} \left\{ \frac{N.(N-1)....(N-k+1)}{N^k} (1-p)^{N-k} \right\}$$

$$= \frac{\lambda^k}{k!} Lim_{N\to\infty} \left\{ (1-\frac{pN}{N})^N \frac{1}{(1-p)^k} \right\}$$

$$= \frac{\lambda^k}{k!} e^{-\lambda}$$

which represents a Poisson characterization of the call arrival process. The parameter • is a variable of the simulation and determines how many users are simultaneously ranging during any given OFDMA symbol.

Note that the Poisson model offers a very faithful characterization for call arrivals, even though this model has been proved to be optimistic while considering the bursty nature of data traffic itself (see e.g., [4]). That traffic is better approximated using Interrupted Poisson processes or Markov-modulated Bernoulli processes. Nevertheless we have chosen a Poisson model in this work since we are concerned with call arrival as opposed to actual data traffic itself. Also, introducing a burstier model for the traffic generation will only degrade the performance results presented in this work.