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Title	**Interpolation effects for OFDM preamble**
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Re:	OFDM Preamble Ad-Hoc discussions
Abstract	The effects of interpolation on channel estimation accuracy for OFDM preamble are discussed.
Purpose	
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Effects of Interpolation on Channel Estimation Accuracy for OFDM Preamble

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1. Introduction

The proposed preamble for 802.16.3 OFDM PHY layer, is composed of two identical sequences, and a cyclic prefix. Each sequence is composed of 128 points. This structure is shown in Figure 1.

Cyclic prefix	128 point sequence	128 point sequence

Figure 1 Proposed preamble structure

The periodic structure of the preamble allows for accurate timing and frequency offset recovery, in the presence of unknown channel response. However, a difficulty associated with the periodicity, is that the preamble contains energy only in the *even* subcarriers, and no energy in the *odd* subcarriers. As a result, the channel response can be directly evaluated only at the even subcarriers. The channel response at the odd carriers needs be evaluated by some form of interpolation.

Recently, a new scheme proposed by Apruva Mody of Georgia Institute, uses 4 identical 64 points sequences. In this case more aggressive interpolation is required, since only every 4th subcarrier is energized.

The objective of this document is to study the effects of interpolation on the channel estimation accuracy, thereby to establish the validity of the proposed approach. Additionally, the 4x64 scheme is also analyzed.

2. The considered interpolation approach

We consider here the problem of interpolation/smoothing in the frequency domain. For each subcarrier, several neighboring subcarriers are combined to estimate the response of the subcarrier under study.

For odd subcarriers, the neighboring even subcarriers are used to estimate the response at that frequency. Thus interpolation is performed.
For even subcarriers, the neighboring subcarriers and the subcarrier under study are used to improve the channel estimation. Thus smoothing is performed.
In both cases, special care must be taken at the band edges, and also near the non-energizing DC carriers, where some if the neighboring subcarriers are missing.

Here, linear interpolation/filtering is used. The interpolation coefficients are derived by following a Minimum Mean Square Error (MMSE) approach.

Before applying the interpolation and filtering, fine timing estimation is applied. This was shown to be detrimental to the accuracy of the interpolation.

3. Definition of terms

Let us consider the 802.16.3 OFDM scheme. We need to estimate 200 spectral lines, half of which are located on either sides of the unused DC sub-carrier. The channel response is estimated from the preamble. We shall compare three approaches:

(a) The proposed scheme, discussed above, namely one OFDM symbol composed of two identical sequences of 128 points each. As discussed only 100 subcarriers are energized.

(b) Four Identical sequences of length 64. This is the scheme proposed by Apruva N. Mody. Every 4^{th} subcarrier is energized.

(c) Non-periodic FFT symbol, where all the 200 subcarriers are energized. This is used as a reference scheme.

For all cases, we shall assume that the power of the preamble is boosted by 3dB relative to the power of the data. This is made possible due to the fact the subcarrier phase loading is judiciously chosen to yield extremely low peak to average power ratio.

Here we shall use the following notations:

E_s – the average symbol power at FFT output. The average is over subcarriers and channel instances.
N_0 – thermal noise power at the FFT output.

• = E_s/N_0 – Thermal signal to noise at the FFT output.

•$_{pr}$ - Channel estimation signal to noise before smoothing interpolating.

•$_{est}$ - Channel estimation signal to noise after smoothing and interpolating.

G - Preamble power boosting.

D_{est}- Degradation due to channel estimation error.

For all cases, the estimation error, before smoothing is related to the signal to noise by:

$$\bullet_{pr} = \bullet \cdot G. \qquad (1)$$

Additionally, the degradation due to channel estimation error is roughly given by:

$$D_{est} \cong 10 \cdot \log_{10} \frac{\gamma_{est}^{-1} + \gamma^{-1}}{\gamma^{-1}} \quad \text{(dB)} \qquad (2)$$

4. Performance at 3.5MHz and SUI # 4

In this section we shall consider the case of 3.5 MHz channels, sampled at 4Ms/s.

The channel model considered was similar to SUI #4 with directional antennas. The length of the impulse response was scaled to 8uS (instead of 4uS) in order to test the system at extreme conditions.

Accurate knowledge of SNR value was assumed. Additionally, no ISI effects and no residual frequency error were considered.

4.1 Effects of interpolation

First the interpolating scheme (a) 2x128 was considered. The resulting estimation error per subcarrier, for various SNR's is shown in Figure 2.

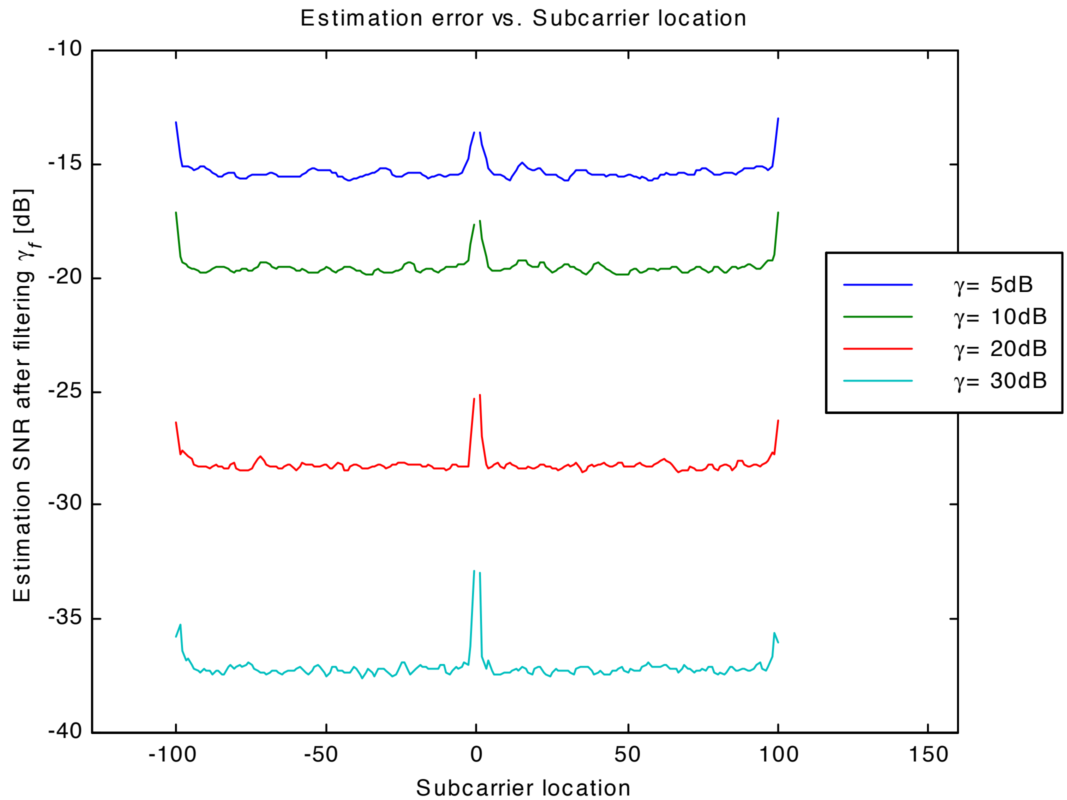

Figure 2 Estimation error vs. Subcarrier location for scheme (a)

From Figure 2, several observations can be made:
- The estimation errors are more severe at the band edges and near the DC carrier. In these cases, there are fewer neighboring subcarriers fro interpolation.
- The difference between thermal SNR, •, and estimation SNR depends upon the former.
- The SNR improvement for the •=5dB case is about 10 dB. This is partly related to the power boosting of 3 dB and partly to the interpolation/smoothing effect.
- For •=30dB the improvement is only 7dB.

The estimation error for the 4x64 is shown in Figure 1. In this case, and at high SNRs, the error varies significantly from energized subcarriers, to non-energized ones. The non-energized subcarriers have much higher estimation errors. This is related to the large separation between energized subcarriers, and the reduced correlation between them.

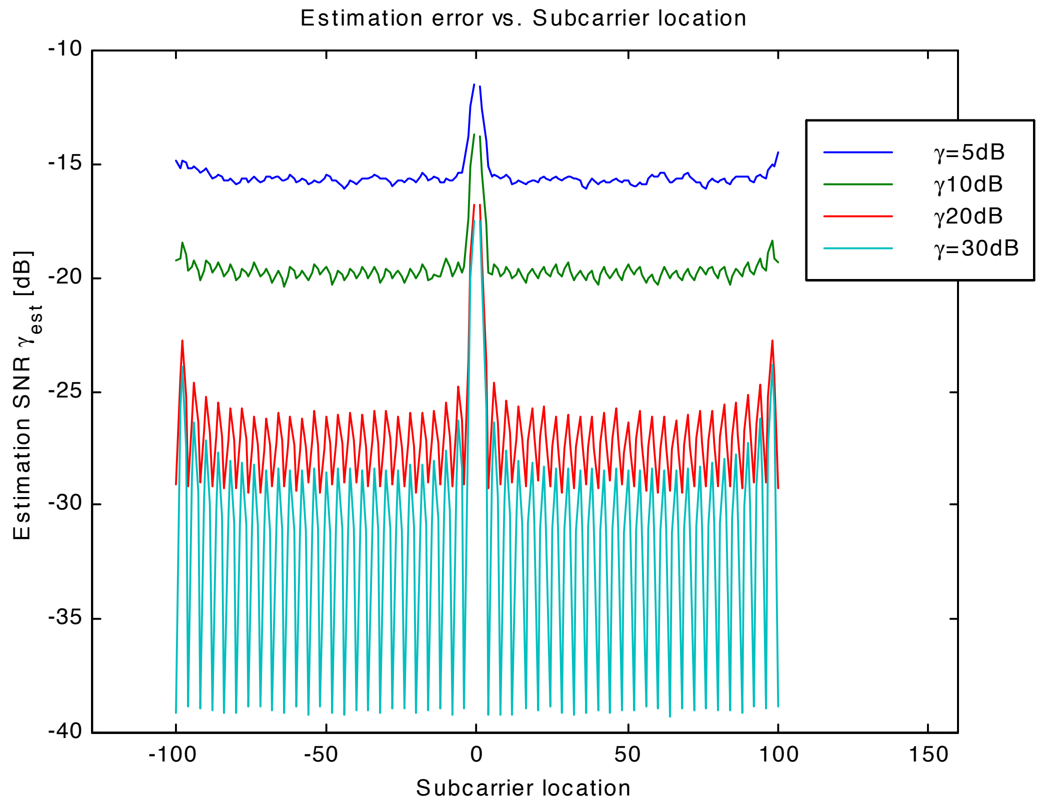

Figure 3 Estimation errors for the 4x64 scheme

4.2 Performance of all schemes.

In this section the three discussed schemes are compared.

First, they were compared in terms of Estimation SNR after filtering ($\bullet_{est}$). For all schemes, smoothing and interpolation were performed. The results are shown in Figure 4. For low SNR, the performance of the three schemes is almost identical. For high SNR, the 4x64 scheme is up to 7dB worse then the other two.

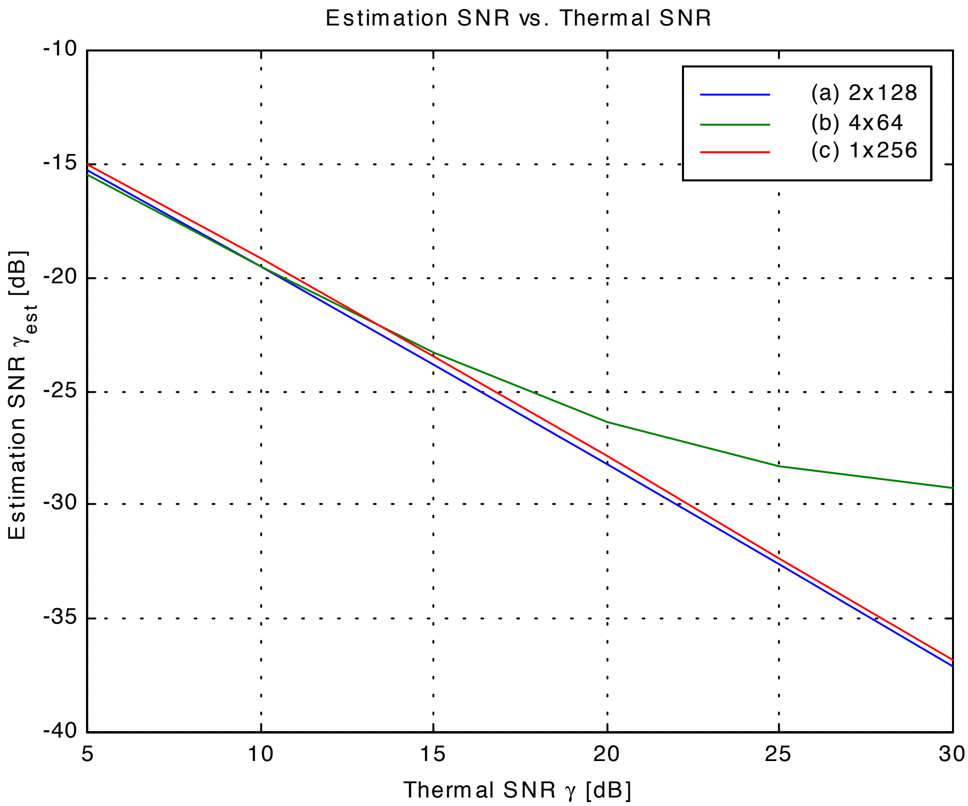

Figure 4 Estimation error vs. SNR

Next, the degradation due to estimation error was computed per equation (2). This is shown in Figure 5. The 1x256 and the 2x128 scheme incur roughly the same degradation, which is smaller than 0.7 dB. The 4x64 caused about 3.2 dB degradation.

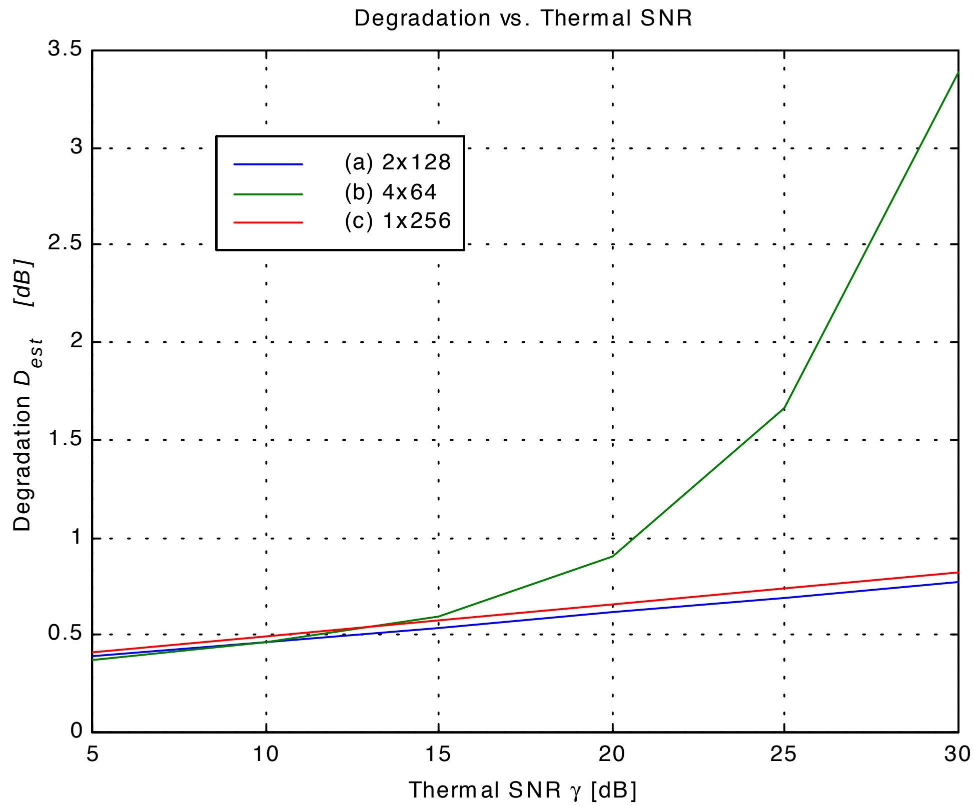

Figure 5 Degradation due to Estimation Errors.

5. Extension to the general case

In this section we extend the results to the general case where other bandwidths and longer impulse responses may be considered.

When we increase the delay spread or alternatively increase the bandwidth, (thereby increasing the subcarrier spacing), the interpolation method begins to fail. This is because the correlation between adjacent subcarrier is reduced. However, when the delay spread is increased other degradation factors may arise, most notably, the Inter Symbol Interference (ISI).

Our preamble will be properly designed if the degradation due to estimation errors for long impulse responses, will not be the dominating factor. Thus we need to compare the degradation due to estimation errors and due to the ISI.

We shall take the following approach. First we define a generic model for the impulse response that can easily scaled in time. Next, we give an approximate expression for the degradation due to ISI. This degradation is then compared with the degradation due to estimation. The comparison is performed at various delay spreads.

For the impulse response model we use an exponential decaying profile. This choice results in simple analytical expressions, and has a physical justification.

5.1 Model and performance expressions.

We consider here an exponentially decaying impulse response. The impulse response *h(t)* is such that

$$E\{ |h(t)|^2 \} = p(t), \qquad (3)$$

where E{} is the expectation operator, and *p(t)* is the channel profile given by

$$p(t) = 1/T_{rms}\, e^{-t/Trms} \qquad (4)$$

and *Trms* is the R.M.S delay spread.

Now let T_{FFT} denote the symbol duration (without cyclic prefix), and let T_{CP} denote the cyclic prefix length.

For the computation of the ISI we consider only the delay elements in h(t) for which $t>T_{CP}$. For some $t_0>T_{CP}$, the contribution of the delay elements $h(t_0)\ldots h(t_0+dt)$ is approximately given by:

$$\varepsilon(t_0)dt = p(t_0)\frac{t_0 - T_{cp}}{T_{FFT}}dt \qquad (5)$$

The SNR due to ISI, γ_{isi}, is given by:

$$\gamma_{isi} = \frac{\int_0^\infty p(t)dt}{\int_{T_{CP}}^\infty \varepsilon(t)dt} = \frac{T_{rms}}{T_{FFT}} e^{-T_{CP}/T_{rms}}\,. \qquad (6)$$

Like in (2) the degradation due to ISI alone is given by:

$$Disi = 10 \cdot \log_{10} \frac{\gamma^{-1} + \gamma_{isi}^{-1}}{\gamma^{-1}}, \quad \text{[dB]} \qquad (7)$$

and the degradation due to ISI and estimation is given by:

$$D_{isi+est} = 10 \cdot \log_{10} \frac{\gamma^{-1} + \gamma_{isi}^{-1} + \gamma_{est}^{-1}}{\gamma^{-1}}. \quad [\text{dB}] \quad (8)$$

5.2 Performance of the 2x128 scheme.

The degradation for the 2x128 scheme due to ISI (6) was compared against the degradation due to estimation and ISI (8). The conditions were:

- 256 points FFT
- Cyclic prefix equal to 1/8 and ¼ of the symbol duration.
- Delay spread varied in the range of $T_s \ldots 32T_s$, where T_s is the FFT sampling period.
- Thermal signal to noise is fixed at •=20dB.

The results are shown in Figure 6. For the 1/8 cyclic prefix, the estimation cause about 0.7dB additional degradation in the region of interest (where the total degradation is less then 2dB). For the ¼ case the degradation is about 1.5dB.

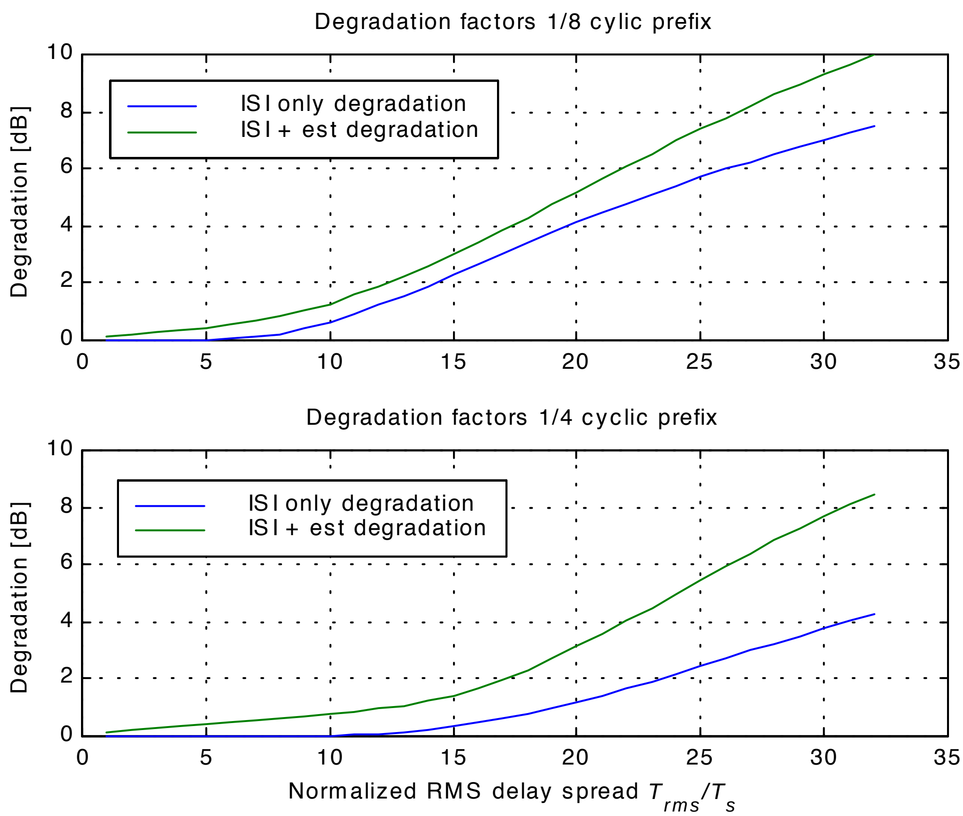

Figure 6 Degradation effects for the 2x128 scheme

5.3 Performance of the 4x64 scheme.

The 4X64 scheme was also evaluated. The results are shown in Figure 7. As can be seen the degradation are 5dB and 8 dB for the 1/8 and ¼ cyclic prefix cases.

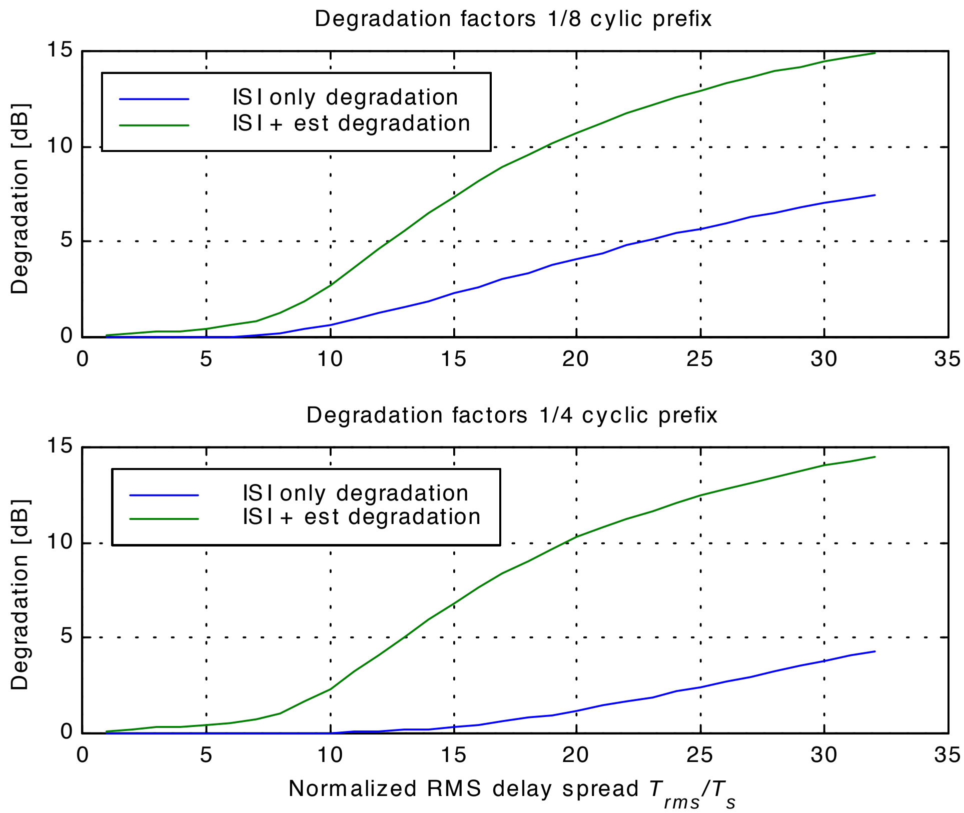

Figure 7 Degradation effects for the 4x64 scheme

6. Conclusions

For 3.5MHz channel and extended SUI #4 channel, the proposed 2x128 scheme performed almost as well as a reference 1x256 scheme. Very small degradation was observed for a wide range of SNRs. The 4x64 performed well at low SNRs but incurred a significant loss at high SNRs.

The higher delay spreads the degradation of the 2x128 relative to the 1x256 was 0.7 dB and 1.5 dB for cyclic prefixes of 1/8 and ¼ respectively. For the 4x64 scheme the degradation is 5dB and 8 dB under the same conditions.

The author's view is that 2x128 scheme performs sufficiently well. On the other hand, the 4x64 scheme takes things a bit too far, and incurs a non-negligible degradation.