Project	**IEEE 802.16 Broadband Wireless Access Working Group < http://ieee802.org/16 >**
Title	**The Effect of Bandwidth on Adaptive Arrays Using the SUI Channel Models**
Date Submitted	2002-03-12
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Re:	In support of the optional sub-channel carrier spacing in 802.16.a draft standard in Section 8.3.5.7 OFDMA 2 PHY (optional)
Abstract	It is demonstrated that the performance of the classical Wiener spatial filter degrades dramatically as the bandwidth of the channel increases for fixed wireless channels. The degradation is most pronounced in channels that contain multiple users sharing the same frequencies. An optimal bandwidth that trades off dispersion effects due to channel time variation versus dispersion effects due to frequency variation can be found in the vicinity of 50 kHz. The performance effects are demonstrated by simulating the Stanford University Interim (SUI) channel models.
Purpose	This document supports the contention that allowing carriers from a single sub-channel to occupy adjacent frequency bins is advantageous to support classical linear beamforming for multiple antennae systems. Allowing such a mode permits the use of narrow-band beamforming and avoids the problem of array overloading due to multipath induced by co-channel interference from wide-band signals. Such a mode is currently proposed in the 802.16.a draft standard in Section 8.3.5.7 OFDMA 2 as an optional PHY.
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1. Channel Model and Multipath Loading

Classical Wiener spatial beamforming depends on the narrow band antenna assumption to hold. This assumption specifies that the received complex data vector $\mathbf{x}(t)$ for an M element antenna array can be written as [1],

$$\mathbf{x}(t) = \mathbf{a}s(t) + \boldsymbol{\varepsilon}(t), \tag{1.1}$$

where $\mathbf{x}(t)$ is an $M \times 1$ complex receive data vector at time t, $\boldsymbol{\varepsilon}$ is an $M \times 1$ complex noise vector, $\mathbf{a}$ is the complex $M \times 1$ array vector, or spatial signature vector seen at the array, and $s(t)$ is the transmitted complex information symbol. The model is based on the assumption that the antenna array channel response is flat over the bandwidth of the transmitted signal $s(t)$, which has been down-converted to complex baseband.

When there are multiple signals in the environment, the model is often written as,

$$\mathbf{x}(t) = \mathbf{a}s(t) + \mathbf{i}(t), \tag{1.2}$$

where $\mathbf{i}(t)$ contains both the background noise and the sum of all the other users in the environment, each with their own spatial signal vectors. It can be shown, [1], that the optimal linear receiver that extracts the signal of interest (SOI) $s(t)$, out of the interference, is given by

$$\mathbf{w} = \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{R}_{\mathbf{x}s}, \tag{1.3}$$

where $\mathbf{w}$ is an $M \times 1$ complex weight vector used to estimate the SOI via,

$$\hat{s}(t) = \mathbf{w}^H \mathbf{x}(t), \tag{1.4}$$

and where the time averaged auto-correlation and cross-correlation statistics are given by,

$$\mathbf{R}_{\mathbf{xx}} \equiv \left\langle \mathbf{x}(t)\mathbf{x}^H(t) \right\rangle_t \tag{1.5}$$

$$\mathbf{R}_{\mathbf{x}s} \equiv \left\langle \mathbf{x}(t)\mathbf{s}^*(t) \right\rangle_t . \tag{1.6}$$

Linear beamforming for an M dimensional array has the ability to separate M signals, provided that the array vectors $\mathbf{a}$ from each signal are sufficiently separated. The more wavefronts impinging on the array, the more loaded the array becomes, resulting in a degradation of performance. In general the array can become loaded through the introduction of more emitters as well as the presence of multipath. When there is multipath in the environment, the bandwidth of the emitters has a strong effect on array loading. A passage from [1] below, describes the effects of channel multipath on array loading.

Let B be the maximum bandwidth size of the SOI $s(t)$, and consider the effect of K reflectors in the environment as shown in Figure 1. Each reflector excites its own array vector yielding the baseband continuous time signal model,

$$\mathbf{x}(t) = \sum_{k=0}^{K} \mathbf{a}_k \xi_k d(t - \tau_k) + \boldsymbol{\varepsilon}(t), \tag{1.7}$$

where $\mathbf{a}_k$ is the spatial signature vector due to the k'th reflected path and ξ_k is the complex reflection coefficient. One also assumes here that the receiver has been synchronized to the direct path (path 0) so that τ_k is the differential multipath delay between the reflected path and the direct path. (This forces $\tau_0 = 0$.)

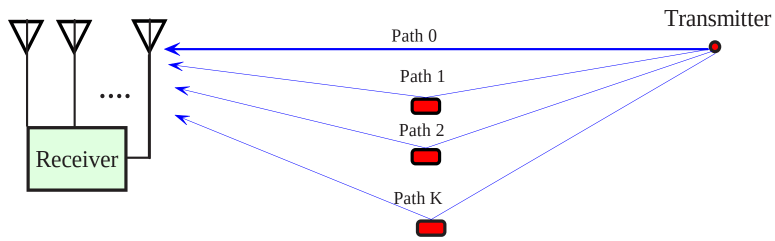

FIGURE 1. Transmitter with MultiPath

To make linear beamforming possible it is desirable to keep the array underloaded. That means that the number of significant emitters received by the array should be smaller than the number of sensors, M. If there is significant multipath, however, each emitter will load the array by a factor of K. Thus in a 5 element antenna array if there are 3 emitters, each with two significant multipaths, the array will be overloaded, seeing the equivalent of 6 emitters and preventing the application of conventional beamforming techniques.

To reduce this loading effect on the array, it is desirable that the signal be narrow-band enough so that the approximation,

$$d(t-\tau_k) \approx \alpha_k d(t) \tag{1.8}$$

is true. If it is, then (1.7) can be written as,

$$\begin{aligned} \mathbf{x}(t) &= \sum_{k=0}^{K} \mathbf{a}_k \xi_k \alpha_k d(t) + \boldsymbol{\varepsilon}(t) \\ &\equiv \tilde{\mathbf{a}} d(t) + \boldsymbol{\varepsilon}(t), \end{aligned} \tag{1.9}$$

which consolidates the effect of the multipath into a single spatial signature vector $\tilde{\mathbf{a}}$, preserving the basic structure of the narrow-band antenna assumption.

A simple analysis of the bandwidth required to validate this considers the frequency representation of the narrow-band signal $d(t-\tau_k)$.

$$d(t-\tau_k) = \int_0^B e^{2\pi jft} d(f) e^{-2\pi jf\tau_k} df. \tag{1.10}$$

It is apparent that (1.8) is true if $e^{-2\pi jf\tau_k} \approx \alpha_k$. A simple approximation of this type is,

$$e^{-2\pi jf\tau_k} \approx e^{-2\pi j\frac{B}{2}\tau_k}.$$

This approximation holds provided that,

$$\begin{aligned} \frac{B}{2}\tau_k &\ll 1 \\ B &\ll \frac{2}{\tau_k}. \end{aligned} \tag{1.11}$$

In [2] for a particular challenging suburban cellular environment, the RMS delay spread is measured to be on the order of $2\mu s$. Assuming a coherence bandwidth a factor of 20 smaller than the right hand side of (1.11) yields,

$$B \approx 50kHz. \tag{1.12}$$

Wideband signals, that exceed this design specification, can be channelized and processed over several sub-channels of bandwidth B. Of course the actual choice of B will depend on the application, the size of the array and the amount of multipath observed.

To get a qualitative feel for why the performance of a linear beamformer degrades as the bandwidth increases, consider the signal model of (1.2). We can write the signal to interference noise ratio (SINR) at the output of the beamformer as,

$$\gamma = \frac{E\left(|\mathbf{w}^H \mathbf{a} s(t)|^2\right)}{E\left(|\mathbf{w}^H \mathbf{i}(t)|^2\right)}. \tag{1.13}$$

The output SINR can be optimized over $\mathbf{w}$ and this optimum is asymptotically achieved by the Wiener spatial filter in (1.3) [1]. At optimality the output SINR becomes,

$$\gamma_* = \mathbf{a}^H \mathbf{R}_{\mathbf{ii}}^{-1} \mathbf{a}, \tag{1.14}$$

which represents the maximum obtainable linear beamformer performance.

Consider now the spectral decomposition of $\mathbf{R}_{\mathbf{ii}}$ of the form,

$$\mathbf{R}_{\mathbf{ii}} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H, \tag{1.15}$$

where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues of $\mathbf{R}_{\mathbf{ii}}$, ordered from smallest to largest. If we substitute $\mathbf{R}_{\mathbf{ii}}^{-1} = \mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^H$ into in (1.14), we see that the max SINR can be written as,

$$\gamma_* = \sum_{l=1}^{M} \frac{|\tilde{a}_l|^2}{\lambda_l}, \tag{1.16}$$

where $\tilde{a}_l$ is the l'th element of $\mathbf{U}^H\mathbf{a}$ and λ_l is the l'th smallest eigenvalue of $\mathbf{R}_{\mathbf{ii}}$. Note that the maximum SINR will be dominated by the smallest eigenvalues of $\mathbf{R}_{\mathbf{ii}}$, and these typically have values near the noise floor of σ_ε^2, provided that the array has not been overloaded.

We shall now see how this performance degrades as the array begins to load up as the signal bandwidth increases. Model the co-channel interference waveform as,

$$\mathbf{i}(t) = \sum_{q=1}^{Q} \mathbf{A}_q \mathbf{s}_q(t) + \boldsymbol{\varepsilon}(t), \tag{1.17}$$

where $\mathbf{A}_q$ is an $M \times K$ spatial signature vector matrix $\mathbf{A}_q$ whose columns contain the K steering vectors associated with each multipath ray of emitter q, $\mathbf{s}_q(t) \equiv [s_q(t)s_q(t-\tau_{1,q}), \cdots s_q(t-\tau_{K,q})]^T$ is the vector of transmitted symbols delayed by $\tau_{k,q}$ for multipath ray k, and $\boldsymbol{\varepsilon}(t)$ is the environment noise vector.

This model yields an interference autocorrelation matrix of the form,

$$\mathbf{R}_{\mathbf{ii}} = \sum_{q=1}^{Q} \mathbf{A}_q \mathbf{R}_{\mathbf{ss}} \mathbf{A}_q^H + \sigma_\varepsilon^2 \mathbf{I}, \tag{1.18}$$

where $\mathbf{R}_{\mathbf{ss}}$ is the emitter autocorrelation matrix, which we assume is approximately the same for all emitters. Also we assume that $s(t)$ is modeled by band-limited white noise whose spectral support is contained in $[0, B]$ Hz (complex baseband). Therefore it's idealized autocorrelation function is,

$$E(s(t+\tau)s^*(t)) = r_{ss}(\tau) = \sigma^2 e^{\pi j B \tau} \frac{sin(\pi B \tau)}{\pi \tau}. \tag{1.19}$$

A three tap multipath model might therefore yield an autocorrelation matrix of the form,

$$\mathbf{R_{ss}} = \begin{pmatrix} r_{ss}(0) & \eta r_{ss}(\tau) & \eta r_{ss}(2\tau) \\ \eta r^*_{ss}(\tau) & \eta^2 r_{ss}(0) & \eta^2 r_{ss}(\tau) \\ \eta r^*_{ss}(2\tau) & \eta^2 r^*_{ss}(\tau) & \eta^2 r_{ss}(0) \end{pmatrix}, \tag{1.20}$$

where τ is a typical multipath delay and η is a typical multipath gain relative to the main path. Assuming a unit power normalized signal, $\sigma^2 = 1/B$ ($r_{ss}(0) = 1$), and assuming the multipath gain η is -10 dB, and that $\tau = 2\mu s$ we plot the eigenvalues of $\mathbf{R_{ss}}$ as a function of the signal bandwidth in Figure 2 .

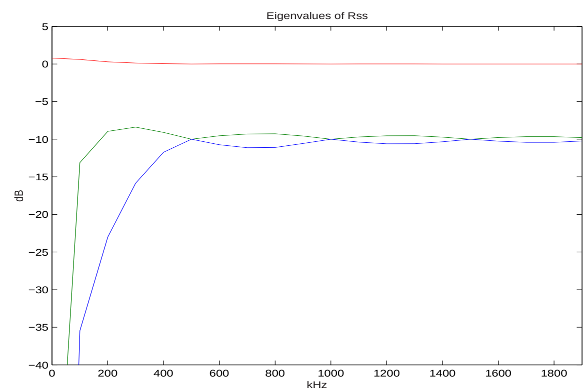

Figure 2. Eigenvalues of Signal Auto-Correlation Matrix

For narrow band signals ($B < 60$ kHZ), $\mathbf{R_{ss}}$ is nearly rank 1, having two zero eigenvalues. In this situation all the multipath steering vectors add coherently as described earlier and the total loading on the array is Q emitters. Assuming $Q < M$, from (1.16) the output SINR will be approximately,

$$\gamma_* \approx (M - Q)\gamma_0, \tag{1.21}$$

where γ_0 is the received signal to white noise power ratio for each emitter and assuming that the elements of the steering vector (after normalization) are approximately unity ($E(|\tilde{a}_l|^2) = 1$).

As the bandwidth increases **Rss** becomes increasingly a diagonal matrix, representing the fact that the multipath signals become increasingly uncorrelated. The rank of **Rss** increases, with the second and third eigenmodes approaching the expected multipath relative gain of -10 dB. If we assume that $3Q > M$, then $\mathbf{R_{ii}}$ will become full rank. The eigenvalues of $\mathbf{R_{ii}}$, will be roughly proportional to the eigenvalues of $\mathbf{R_{ss}}$. This would be exact in fact if the $\mathbf{A}_q$ all formed a set of orthonormal basis vectors. We might expect the smallest eigenvalues of $\mathbf{R_{ii}}$ to fill in roughly proportional to $(3Q - M)\sigma_s^2\eta^2 + \sigma_\varepsilon^2$, where $3Q - M$ are the number of emitters (including multipath) that overload the array and σ_s^2 is the received signal power per antenna element. This makes the approximate output SINR for the high-bandwidth signal,

$$\gamma_H \approx \frac{(M - Q)\gamma_0}{\gamma_0\eta^2(3Q - M)_+ + 1}, \tag{1.22}$$

where $(3Q-M)_+ = 0$ when $3Q < M$ and is $3Q - M$ otherwise. For the general case of K multipath rays we might expect,

$$\gamma_H \approx \frac{(M-Q)\gamma_0}{\gamma_0\eta^2(KQ-M)_+ + 1}. \tag{1.23}$$

The output SINR will tend to zero in this case as the number of significant multipath rays increase.

For a fully loaded array, wherein Q is near $M-1$, $(3Q-M)$ can approach $2M-3$. If for example $\gamma_0\eta^2$ is about 0 dB, then a performance degradation of 15 dB is possible. In the following numerical experiments we have $Q = 12$, $M = 16$ and $\gamma_0\eta^2$ anywhere from 0 to 5 dB. The formula in (1) predicts SINR degradations as much as $12.5dB$ in this case. Several of the experiments which follow, exhibit output SINR reductions of approximately this magnitude.

2. Numerical Experiments

A series of numerical experiments are performed to verify the qualitative discussion of the previous section. The simulations rely on on the Stanford University Interim (SUI) models proposed in [3]. For these experiments an 8 element antenna array is assumed, with 5 co-channel emitters transmitting for each trial. The emitters are assumed to be power managed so that their signal power is received at 15 dB above the noise floor at the multi-antennae receiver.

The simulated channels adhere closely to the SUI-1 to SUI-6 models, with some notable exceptions. The first exception addresses the fact that the standard's specified antenna cross-correlation matrix in [3] can not be consistent with the physics of the complex baseband noise envelope assumed for each channel.

For example if x_1 is the complex received signal from sensor 1, and x_2 is the signal from sensor 2, [3] specifies that the real random vector, $\mathbf{z}_r \equiv [\Re(x_1), \Im(x_1), \Re(x_2), \Im(x_2)]^T$ has the autocorrelation matrix,

$$\mathbf{R}_{\mathbf{z}_r\mathbf{z}_r} = \begin{pmatrix} 1 & 0 & \rho & \rho \\ 0 & 1 & \rho & \rho \\ \rho & \rho & 1 & 0 \\ \rho & \rho & 0 & 1 \end{pmatrix}. \tag{2.1}$$

Unfortunately this specification does not have the required symmetry for complex baseband noise [4] (see also http://www.ece.wpi.edu/~mattb/spring02courses/ComplexBase.pdf .) In fact this matrix is not even positive definite, as it clearly exhibits negative eigenvalues as $\rho \to 1$. To restore the correct symmetry for complex baseband noise, we define the complex autocorrelation between two sensors directly as,

$$E(\mathbf{z}\mathbf{z}^H) = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{2.2}$$

where $\mathbf{z} \equiv [x_1, x_2]^T$.

The second change accommodates the reality that the received spatial signature vector from different emitters are expected to be substantially different. Proper cell/frequency assignment should assure that in a given cell, co-channel emitters are spatially separated. This is simulated here by applying a random unit modulus scalar to each element of the array. The scalars represent the assumption of a random phase, and are independent over the sensor index and over the emitter number, but are constant for each multipath delay.

The performance is measured assuming a training sequence of 1500 symbols and ideal linear Wiener spatial filtering. This might simulate an initial short preamble followed by information symbols totaling 1500 symbols, since either blind processing or decision direction will usually have nearly identical performance to a non-blind processor. The time bandwidth product is held constant at 1500 symbols, whilst the bandwidth is varied from 10 kHz to 3.5 MHz.

The signal to interference noise measured at the output of the beamformer (output SINR) is sorted and the 50% and 90% SINR values in dB are plotted as a function of bandwidth. Most data points are found from 200 trials each containing 12 emitters for a total of 2400 measured output SINRs. Some plots are generated using 50 data points, to speed data compilation. The results of the experiments for the omni antennae and 30^o sectorized antennae are shown for all six SUI models below in Figures 3 to 14.

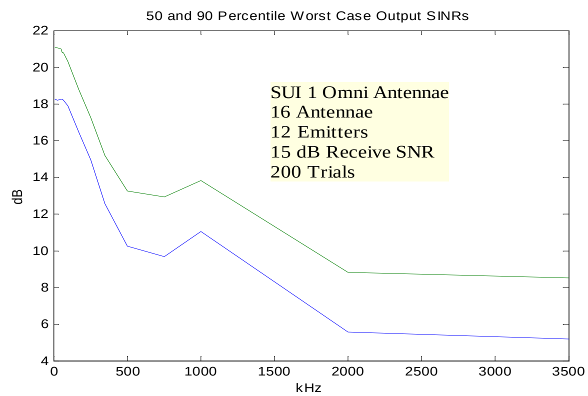

Figure 3. SUI 1 Omni Antenna Model

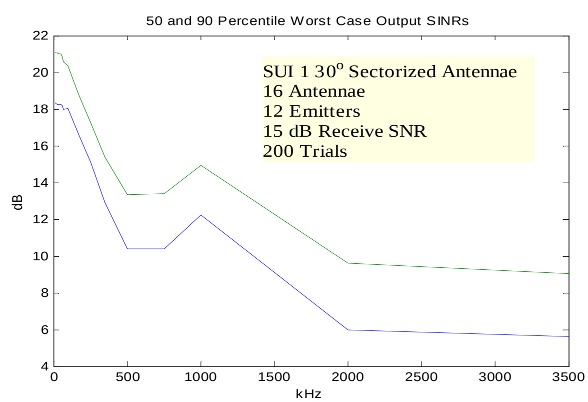

Figure 4. SUI 1 Sectorized Antenna Model

The Figures show a marked deterioration with increased bandwidth. For narrow bandwidths, the 16 element array can easily handle the 12 emitters in most of the SUI environments. All of the channels, except those with the most benign

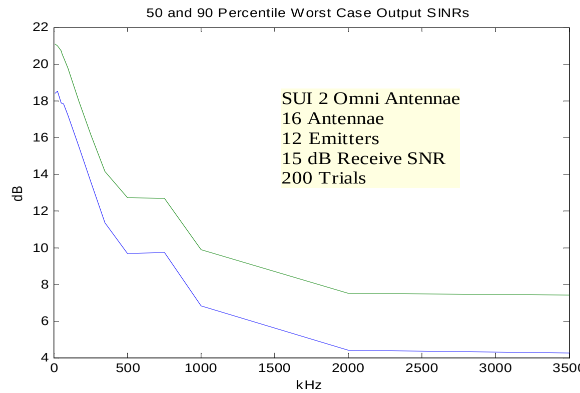

FIGURE 5. SUI 2 Omni Antenna Model

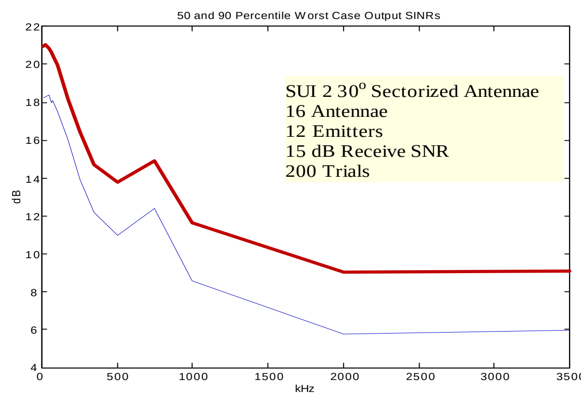

FIGURE 6. SUI 2 Sectorized Antenna Model

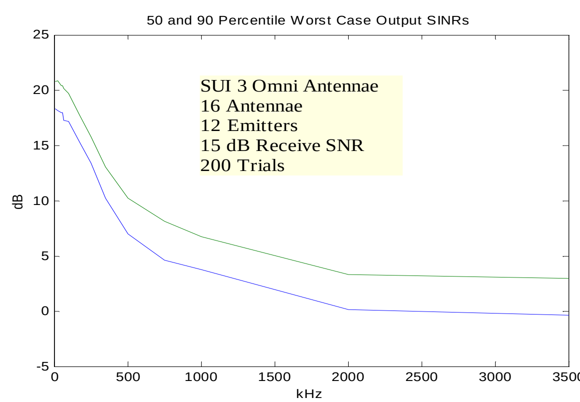

FIGURE 7. SUI 3 Omni Antenna Model

multipath, experience severe degradation as the bandwidth increases, overloading the array. Note that the degradation due to signal bandwidth, for an environment

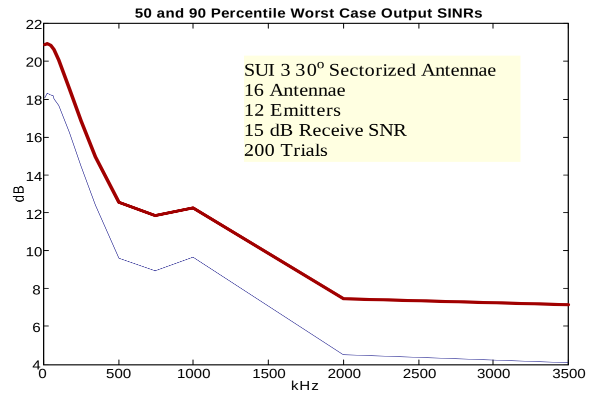

FIGURE 8. SUI 3 Sectorized Antenna Model

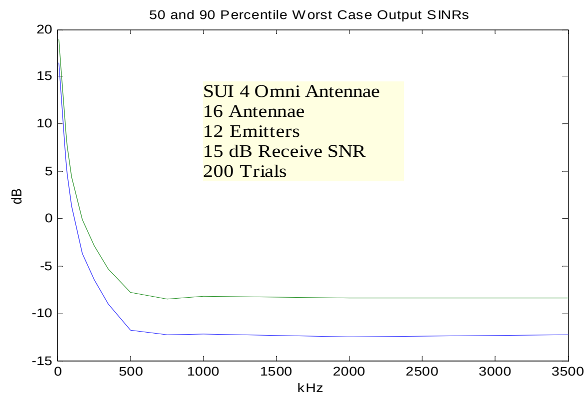

FIGURE 9. SUI 4 Omni Antenna Model

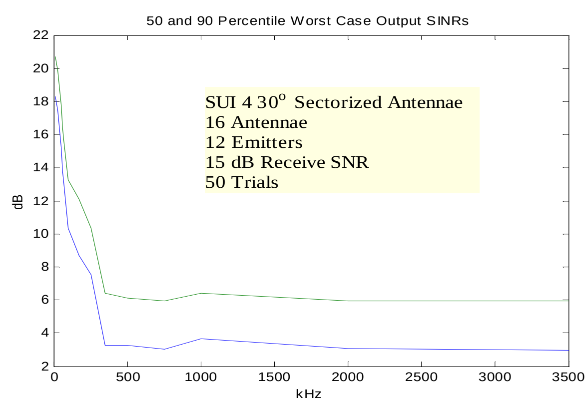

FIGURE 10. SUI 4 Sectorized Antenna Model

with many users, far outweighs the negative effects of flat fading, even for the worst channels. It is also interesting to note that for very small bandwidths, the

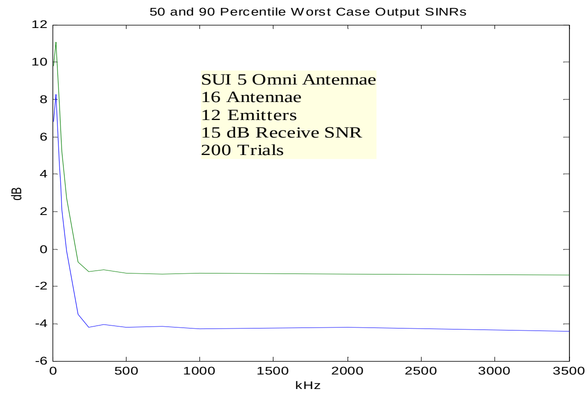

FIGURE 11. SUI 5 Omni Antenna Model

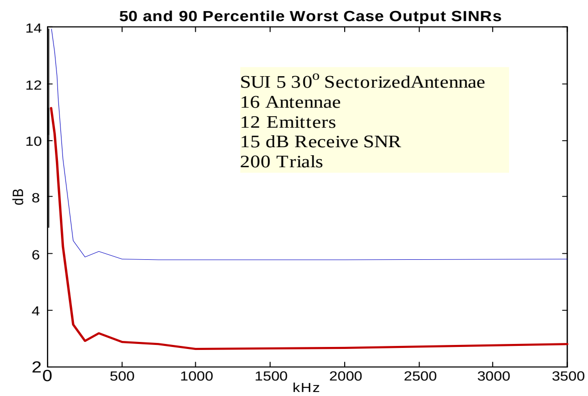

FIGURE 12. SUI 5 Sectorized Antenna Model

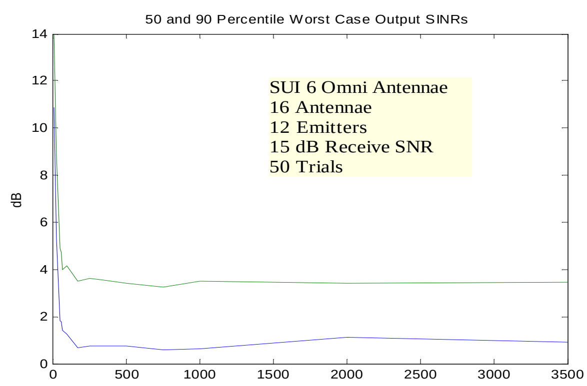

FIGURE 13. SUI 6 Omni Antenna Model

performance also degrades due to channel time variation though this is a far smaller

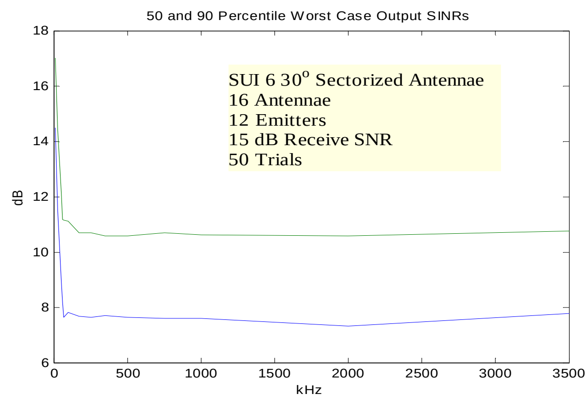

FIGURE 14. SUI 6 Sectorized Antenna Model

effect for fixed wireless channels. This effect can be seen clearly for example in the omni SUI 5 channel, and the sectorized SUI 3 experiments in Figures 8 and 11.

Channel dispersion due to time variation causes the spatial signature vector to change over a single adapt frame. This results in a degradation of the output SINR as the length of the adapt frame increases beyond the minimum required to obtain good auto-correlation and cross-correlation statistics. Smaller channel bandwidths forces a longer adapt frame. As noted, however, this effect is not nearly as significant in fixed wireless channels as the frequency dispersion due to multipath for wider bandwidth channels.

Flat fading degradation for the SUI3 Omni channel can be characterized by observing the CDF of the output SINRs for a 50 kHz channel shown in Figure 15. The CDF represents 200 trials each containing 5 emitters for a receiver with 8 antenna elements.

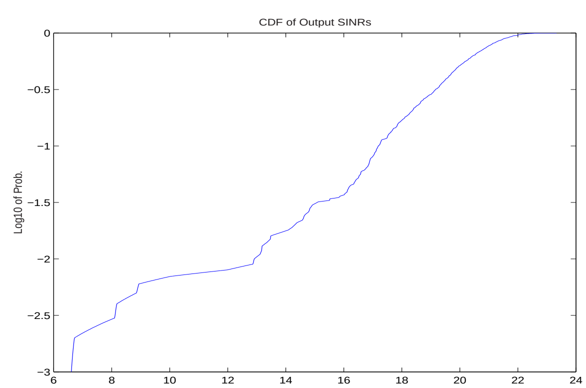

FIGURE 15. Output SINR Cumulative Density Function

Flat fading is infrequent in this case, due to the antenna diversity of the 8 element array at the receiver. Almost 99% of the time, the output SINR is at 14 dB or better.

3. Conclusions

This report demonstrates that smaller bandwidths achieve significantly better performance for classical Wiener spatial filtering. A smaller bandwidth overcomes the problem of additional loading due to multipath wavefronts from multiple emitters, which is evidently a far more significant issue than flat fading. For a multiple access, fixed wireless communication system, channelizing the available bandwidth into channels of approximately 50 kHz is recommended if it is intended to use spatial beamforming to increase the user density of fixed wireless cellular networks.

References

[1] M. Bromberg and D. Brown. *The Application of Programmable DSPs in Mobile Communications*, chapter The Use of Programmable DSPs in Antenna Array Processing. Wiley, November 2001.

[2] T. Rappaport. *Wireless Communications*. Prentice Hall, 1996.

[3] V. E. Erceg, K.V.S. Hari, and M.S. Smith Et. Al. Channel models for fixed wireless applications. *IEEE 802.16 Broadband Wireless Access Working Group*, 2001.

[4] N. R. Goodman. Statistical analysis based on a certain multivariate complex gaussian distribution (an introduction). *Annals of Mathematical Statistics*, 34(1):152–177, March 1963.