Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Unified MIMO Pre-Coding based on Givens Rotation**
Date Submitted	**2004-11-14**
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Re:	Response to Recirculation Sponsor Ballot
Abstract	To unify the closed loop MIMO pre-coding based Givens rotation. The modification is in green.
Purpose	To incorporate the changes here proposed into the 802.16e D6 draft.
Notice	
Release	
Patent Policy and Procedures	

Unified MIMO Pre-coding Based on Givens Rotation

1 Introduction

For the SVD based MIMO pre-coding technique, the MSS is required to send beam-forming V matrix to the BS, due to the unitary structure of matrix V, a number of research work have done to quantize the matrix V in order to reduce the feedback overhead in the UL. In this contribution, we show that by using Givens decomposition of matrix V, the Givens parameter can be further quantized by using simple 1-bit scalar delta modulation to allow the further reduction of the redundancy in time/frequency, the differential-Givens (D-Givens) provide an straightforward scalability to arbitrary antenna configurations while achieve the much less computational complexity, lower quantization noise and requires less feedback resource. The D-Givens method discussed in this contribution is compared with the Householder method introduced in [1].

2 Background

2.1 Givens Rotation

In the following, we assume that the number of BS transmit antennas is *M* and the number of MSS receive antennas is *N* and the vector representation of the received signal is: *Y=HX+n*. In the beam-forming MIMO pre-coding method, the BS transmitter needs to know the right-singular matrix *V*, when the channel matrix is singular-value decomposed as $H=USV^H$. The number of non-zero singular values is at most *min (M,N)*. The matrix *V* contains M^2 complex elements but based on the fact that it is a unitary matrix, the number of independent variables is *M(M-1)* real values. By using the Givens decomposition, the matrix *V* is decomposed to a set of *M(M-1)/2* unitary matrices. Each matrix is an identity matrix except for four of its elements and can be represented by two real values. Besides their ability to decompose the unitary matrix to the minimum number of parameters, the resulting parameters are statistically independent. The independence property facilitates the quantization procedure. In this contribution, we chose a Givens representation as:

$$G(c,\theta)=\begin{bmatrix} \hat{c} & |\hat{s}|e^{j\hat{\theta}} \\ -|\hat{s}|e^{-j\hat{\theta}} & \hat{c} \end{bmatrix}$$

where the distribution of $\hat{\theta}$ and $\hat{c}$ are independent and $s=\sqrt{1-|c|^2}$, in particular, $\hat{\theta}$ is uniformly distributed and $\hat{c}$ is non-uniformly distributed. Based on the statistical distribution of Givens, the optimum quantizer can be designed to achieve maximum compression ratio.

2.2 Delta Modulation

The Givens parameters $\hat{\theta}$ and $\hat{c}$ are further compressed by using the simplest delta modulator to exploit the channel correlation in time or frequency domain, the delta modulator is shown in Figure 1.

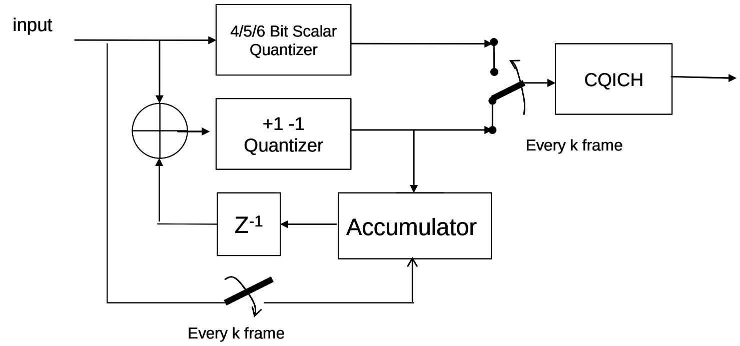

Figure 1 Delta Modulator

3 Proposed Solution

For the unitary pre-coding feedback, the MSS is required to perform the Givens decomposition of unitary matrix, the Givens expansions can be truncated (variable number of Givens rotations) to allow feedback partial or full the unitary pre-coding vectors. The parameters of each Givens rotations can be quantized and compressed by scalar quantizer such as delta modulation and feedback to BS. The BS reconstructs the unitary pre-coding vectors/matrix.

4 Advantages

- Scalability:
  - Scalable to MIMO pre-coding with large number of transmit antennas to allow standard future proof.
  - Scalable to MIMO pre-coding stream selection. The Givens expansion can be shortened to scale the feedback of partial or full set of the Givens rotations to allow BS to perform sub-space or full-space pre-coding
  - Givens decomposition will significantly reduce the complexity since the code books search complexity, since for Householder based method increases ***exponentially*** with respect of the number of transmit antennas, the complexity of Givens rotation based method increases ***polynomial*** with respect of the number of transmit antennas.
  - Lower quantization noise
  - Lower feedback resource required.
- Reuse:
  - The Givens rotation engine can be implemented with very efficient ORDIC computing. It can be used to compute:
    - The decomposition of unitary matrix, such as V for the compression of the feedback of V [1],[2],[3]
    - The Gentleman-Kung systolic based matrix inversion the receiver based schemes [4],[5]

5 Simulation Results

The simulation conditions and set-up is listed in Table 1

Table 1 Simulation Set Up

Configurations	**Parameters**	**Comments**
Optional BAND AMC sub-channel		The band allocation in time-direction shall be fixed at center band
Coding Modulation Set	CC coding , K=7, TB	Coded Symbol Puncture for MIMO Pilot
	QPSK ½, QPSK, ¾, 16QAM ½, 16QAM R=¾, 64QAM R=1/2, 64QAM R= 3/4	
Code Modulation Mapping	Single encoder block with uniform bit-loading	
MIMO Receiver	MMSE-one-shot for SVD	

	MLD receiver for OL and CL SM	
FFT parameters	Carrier 2.6GHz, 10MHz, 1024-FFT Guard tone 79 left, 80 right CP=11.2ms, Sampling rate = 8/7, Sub-carrier spacing = 11.2kHz	
Frame Length	5ms frame, DL:UL=2:1	
Feedback delay	2 frames	
MIMO Configurations	4x2	
Channel Model	ITU-PA, 3km/h, Antenna Correlation: 20% Perfect Channel Estimation	
Feedback	SVD: perfect pre-coding matrix V without quantization D-Givens: per this contribution Householder: Ref[1]	

5.1.1 Performance

The partial simulation results based on 0 frame delay are shown in Figure 2. and Figure 3.

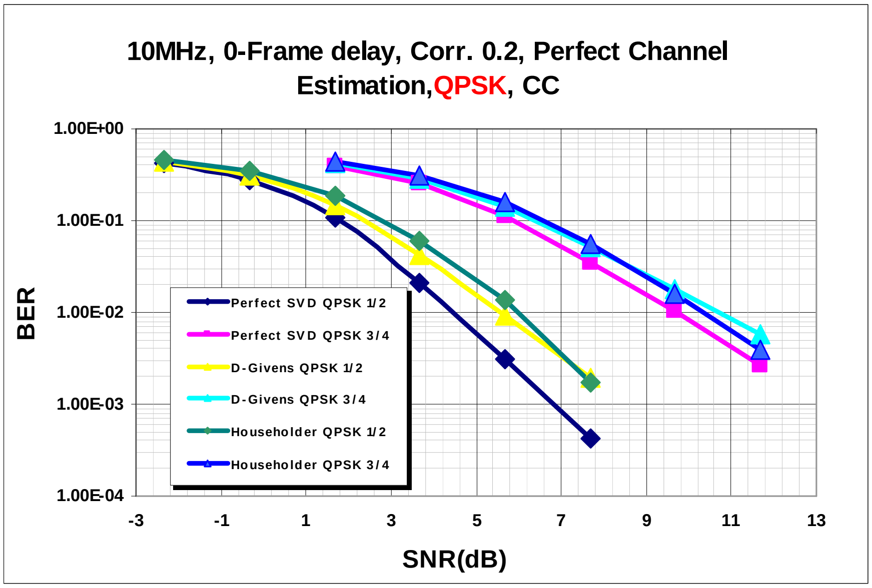

Figure 2 Performance Comparison of D-Givens/Householder based pre-coding

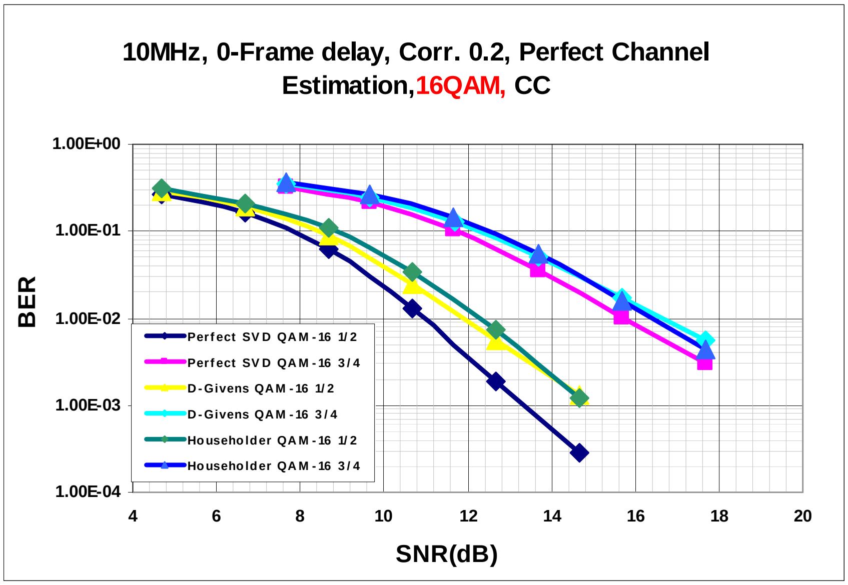

Figure 3 Performance Comparison of Givens/Householder based pre-coding

It can be seen that the D-Givens based method achieve the better performance than the Householder method. Figure 4 shows the throughout curve for comparing perfect SVD, D-Givens and Householder based methods with 2-frame delay.

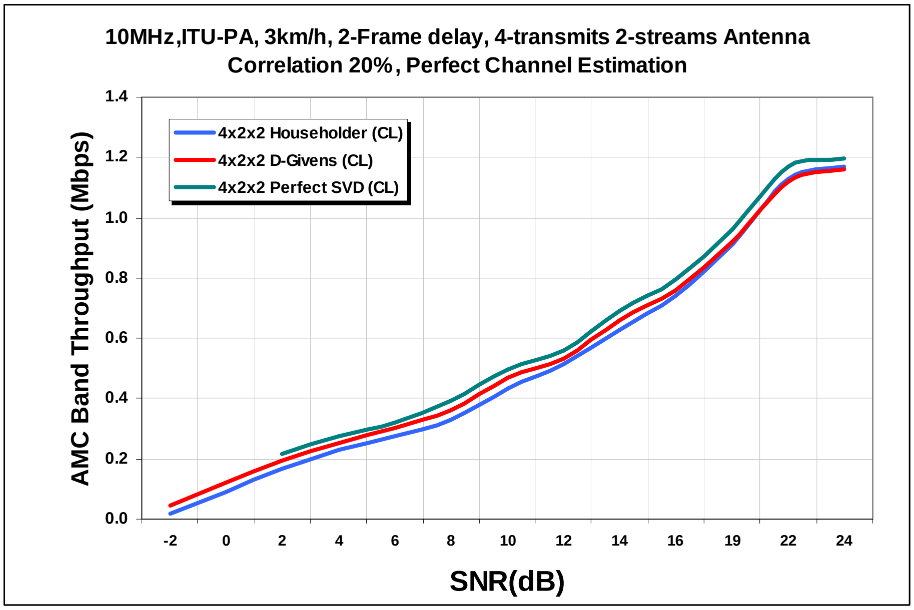

Figure 4 Comparison of perfect SVD, D-Givens and Householder

5.1.2 Feedback Resource Requirement

The feedback resources requirement and comparison is shown in Figure 5. As we can see the D-Givens requires less feedback resources. See Appendix-A for details.

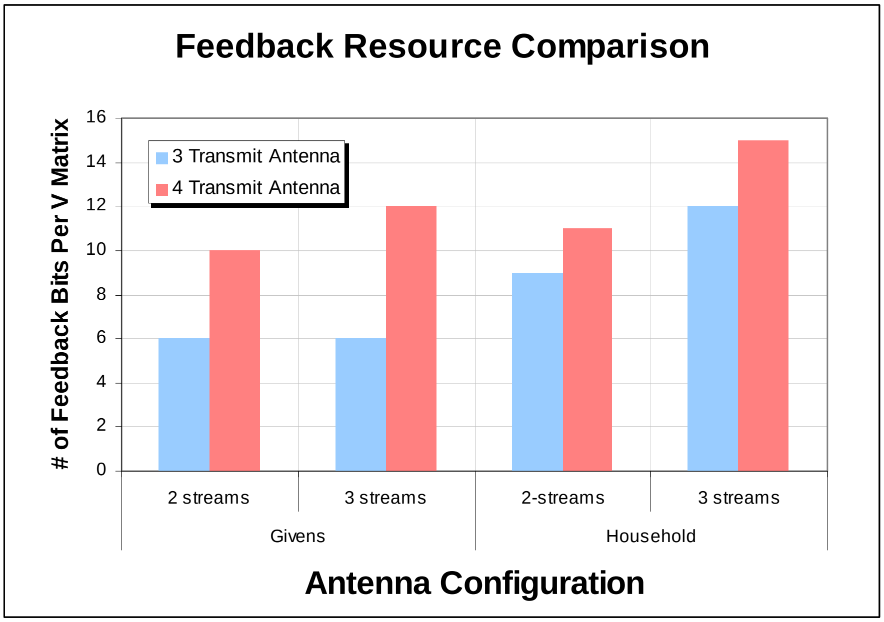

Figure 5 Feedback Resources Comparison

5.1.3 Quantization Error

In Figure 6, we can see the D-Givens based method has less quantization error than the Householder based method, see Appendix-B for details. This reduces the inter-stream interface significantly.

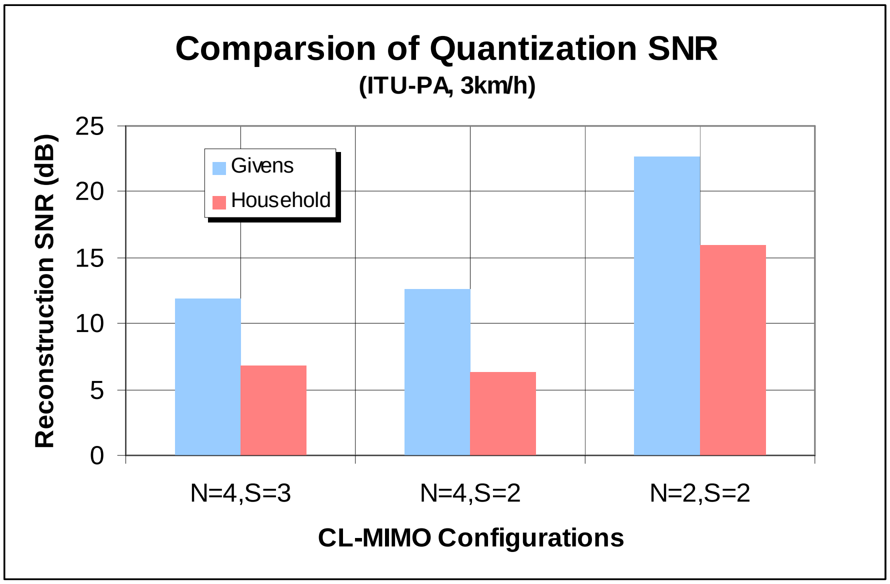

Figure 6 Comparison of Quantization SNR

5.1.4 Compression Complexity

Figure 7 shows the computational complexity comparison of Givens based method and Householder based method, the major advantages of the Givens based method is the low complexity at MSS side. In this Figure, we show the complexity for the direct Givens computation approach, indeed, by using CORDIC technique fast Givens rotations can be achieved with even less computing complexity.

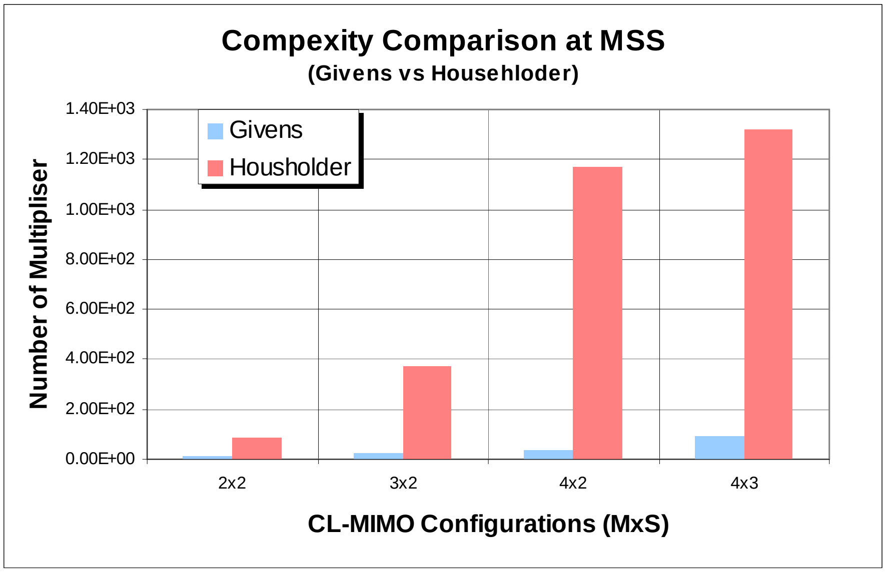

Figure 7 Comparison of computational complexity

6 Summary

In summary, the Differential Givens based unitary pre-coding method has the following advantages.

- Scalability to allow flexible extension to transmit antenna and data streams
- D-Givens based unitary pre-coding has significant lower complexity at MSS side
- D-Givens based unitary pre-coding requires less feedback resource
- D-Givens based unitary pre-coding generate lower quantization noise
- D-Givens based unitary pre-coding has better performance

7 Text Proposal

Start text proposal

[Add a new section 8.4.8.3.6.2 as follows]

8.4.8.3.6.1 Unitary Matrix Pre-coding for 3 and 4 Transmit Antennas

A unitary matrix V can be applied at BS actual transmit antennas to perform the closed loop MIMO pre-coding with s transmit streams. The V matrix is expanded by using Givens decomposition as: $V = \prod_{i=1}^{S} \prod_{j=i+1}^{P} G(i,j,a,b)$ where

$$G(i,j,a,b) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & a & \cdots & \sqrt{1-|a|^2}\,e^{jb} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sqrt{1-|a|^2}\,e^{-jb} & \cdots & a & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix},$$ the delta modulation is further applied to the

parameters $a(k)$ and $b(k)$. For $a(k)$, delta $d(k) = a(k) - \hat{a}(k-1)$ is quantized by a 1-bit quantizer which outputs $\tilde{a}(k) = Q[d(k)]$. $\hat{a}(k-1) = \sum_{i=1}^{k-1} \tilde{a}(i)$ is the reconstruction of $a(k-1)$. The 1-bit quantization index for $\tilde{a}(k)$ and $\tilde{b}(k)$ is mapped onto CQICH and fed back to BS to re-generate the unitary matrix V.

End text proposal

8 Reference

[1] Intel: Improved MIMO Feedback and Per-Stream ABL for OFDMA/OFDM Systems
[2] Beceem: Closed Loop MIMO Pre-coding
[3] Motorola: Closed-Loop MIMO for IEEE 802.16e
[4] TI: An enhanced closed-loop MIMO design for OFDM/OFDMA-PHY
[5] Nokia: Closed-Loop MIMO Pre-coding with Limited Feedback
[6] IEEE P802.16-REVd/D5-2004 Draft IEEE Standards for local and metropolitan area networks part 16: Air interface for fixed broadband wireless access systems

9 APPENDIX-A

9.1 Feedback Resources

The feedback resource requirement for Givens based method vs. the Householder based method is listed in Table 2.

	D-Givens			Householder		
	2 streams	3 streams	4 streams	2-streams	3 streams	4 streams
3	6	6		9	12	
4	10	12	12	11	15	21
6	18	24	20	15	21	29
8	26	36	44	19	27	37

Table 2 Feedback resource

In this case, with n transmit antennas, we assume that the Householder method requires $\sum_{i=1}^{S} n+3-i$ bits for S streams, while the D-Givens method requires $(n^2-n)-((n-s)^2-(n-s))$ bit for S streams.

10 APPENDIX-B

10.1 Compression Quantization SNR

The performance of each schemes are evaluated based on the following metric: $SIR = mean[10 * \log_{10}(\gamma_{k,l})]$ where $\gamma_{k,l}$ is signal-to-interference ratio for the l^{th} sub-carrier of the k^{th} frame due to quantization. It is defined by $\gamma_{k,l} = \dfrac{|h(k,l)|^2}{|h(k,l) - h(\hat{k},l)|^2}$. The $h(k,l)$ is the ideal channel coefficient, and $h(\hat{k},l)$ is reconstructed channel coefficient for the l^{th} sub-carrier of the k^{th} frame, respectively.

Case Number	CL-MIMO	Givens	Household
ITU-PA, 3 km/hr	N = 4, S = 3	11.8884dB	6.7908dB
	N = 4, S = 2	12.6734dB	6.3509dB
	N = 2, S = 2	22.6832dB	15.9478dB

Table 3 Comparison of Quantization SNR

11 APPENDIX-C

11.1 Complexity Comparison

The following notations are used in this appendix:

N		Number of transmitter antenna
M		Number of receiver antenna
S		Number of streams
C_N	C_4 = 64	Size of codebook of unit N-vector
	C_3 = 32	
	C_2 = 16	

Table 4: Definition of common parameters

11.1.1 Complexity of Householder Method

Step 1: Quantization. In this step, vector quantization of a column vector should be done by searching a codebook according to the following criterion

$$\hat{v} = \arg\max \| u^H v \|$$

where v is the vector to be quantized, u is the codeword vector in a pre-determined codebook. $\hat{v}$ is the output of the quantizer.

As Householder scheme is an iterative approach, the first quantized vector is the first column of matrix V and in next iteration, first quantized vector is the first column of a reduced size matrix within FV, where F is the Householder reflection matrix from first iteration. This process could continue until reaching number of streams.

To search the codebook, inner product of the vector to be quantized and each codeword in the codebook should be calculated and compared, and the codeword with the largest norm of inner product with vector to be quantized is chosen as the quantized vector. The complexity of this process is summarized in Table 5, which takes into account that the first element of the vector to be quantized and the first element of each codeword are all real.

Iterations	Real Multiplications	Complex Multiplications
1	C_N	$(N-1)*C_N$
2	C_{N-1}	$(N-2)*C_{N-1}$
3	C_{N-2}	$(N-3)*C_{N-2}$

Table 5: Code book search complexity for Householder method

Step 2: Computing Householder reflection matrix F . The Householder transform matrix can be calculated as follows

$$F = I - \frac{2}{\|w\|^2} ww^H$$

I is an identity matrix. $w = \hat{v} - e$ where $\hat{v}$ is the output of quantization process and $e = [1 \quad 0 \quad \ldots \quad 0]^T$. Table 6 shows the complexity of calculating the Householder reflector matrix, in which the fact such as F is a Hermitian matrix is considered.

Iterations	Complex Multiplications	Real Divisions
1	$N(N-1)/2$	$N(N-1)$
2	$(N-1)(N-2)/2$	$(N-1)(N-2)$
3	$(N-2)(N-3)/2$	$(N-2)(N-3)$

Table 6: Complexity of calculation of Householder reflection matrix

If the codebook is not large, the Householder reflection matrix F can be pre-calculated for each codeword and stored. If this is the case, then this part of complexity can be saved.

Step 3: In the 3rd step, the obtained Householder reflection matrix F calculated in step 2 is used to convert V in an iterative way. The complexity of calculation of FV is summarized in Table 7. In complexity estimation, such factors are taken into account that first row and column of the matrix don't have to be calculated and only those columns (specified by number of streams) to be feedback are updated.

Iterations	Complex Multiplications
1	$N(N-1)(R-1)$
2	$(N-1)(N-2)(R-2)$
3	$(N-2)(N-3)(R-3)$

Table 7: Complexity of Householderreflection operation

Summary: The complexity of Householder scheme is summarized in Table 8. As each complex multiplication equals to four real multiplications, total complexity given is expressed in terms of real multiplication and divisions. It should be noted that number of stream R should be smaller than or equal to N

Steps	Complex Multiplications	Complex Divisions

1	$\sum_{i=1}^{S}\left[4(N-i)C_{N-(i-1)}+C_{N-(i-1)}\right]$	
2	$\sum_{i=1}^{S}\left[2(N-i+1)(N-i)\right]$	$\sum_{i=1}^{S}(N-i+1)(N-i)$
3	$\sum_{i=1}^{S}4(N-i+1)(N-i)(R-i)$	
Total	$\sum_{i=1}^{S}\left[(N-i)\left(4C_{N-(i-1)}+(N-i+1)(2+4R-4i)\right)+C_{N-(i-1)}\right]$	$\sum_{i=1}^{S}(N-i+1)(N-i)$
Total (without step 2)	$\sum_{i=1}^{S}\left[4(N-i)\left(C_{N-(i-1)}+(N-i+1)(R-i)\right)+C_{N-(i-1)}\right]$	0

Table 8: Summary of Householdercomplexity in MSS

The complexity analysis shown in Table 8 is terminal side. For the base station, the feedback information from the terminal is used to reconstruct the right singular matrix V and complexity required for that are in fact the combined complexity of steps 2 and 3 in Table 8 as in the base station, there is no need for codebook search. If Householder matrix is pre-calculated, then the complexity for base station is only the complexity of step 3 in terminal. For convenience, Table 9 lists the complexity at the base station for Householder method.

	Complex Multiplications	Complex Divisions
Complexity in BS	$\sum_{i=1}^{S}4(N-i+1)(N-i)(R-i)$	

Table 9: Householder complexity for BS

11.1.2 Complexity of Givens Rotation

In Givens method, the right side singular matrix V is first decompose into a set of Givens matrices. Each Given matrix contains one real element and one complex element, which is quantized using non-uniform scalar quantizer. The quantized bits are fed back.

The Givens method can be implemented into two major steps:

Step 1: In this step, the right side singular matrix V is decomposed into product of a set of Givens matrices. The decomposition is done in an iterative way. The first Givens matrix denoted by $G_{2,1}$ is calculated, where $G_{2,1}$ is the same as an identity matrix except that it has non-trivial elements at $G(1,1)$, $G(1,2)$, $G(2,1)$, $G(2,2)$. It is then multiplied with V to obtain

$$V' = G_{2,1}V$$

where $V'(2,1)$ is zeroed. If the whole matrix V needs to be feedback, the procedure continues iteratively until an identity matrix is obtained

$$I = G_{N,N-1} \ldots G_{3,1}G_{2,1}V$$

Normally, the elements of the first column is zeroed out one by one first which is then followed zeroing of elements along the 2nd column and so on. As V is unitary, only lower triangle of the matrix needs to be zeroed in order to get the identity matrix.

Table 10 shows the complexity of Givens decomposition. Some considerations are taken into account when getting the numbers, which include the fact that as Givens matrix contains only four non-trivial elements, therefore only certain rows needs to be calculated when doing each step of Givens decomposition like $V' = G_{i,j}V$. Also if only certain streams are needed to feedback, the columns corresponding to rest streams do not need to be updated in Givens decomposition.

Givens Decomp	Real Multiplications
Zero 1st column	$12(N-1)(R-1)$
Zero 2nd column	$12(N-2)(R-2)$
….	
Total	$\sum_{i=1}^{R} 12(N-i)(R-i)$

Table 10: Complexity of Givens decompostion

Step 2: After obtaining the set of Givens matrices, the non-trivial elements in them are quantized using scalar non-uniform quantization. In fact there are only two independent numbers in each Given matrix which needs quantization. The quantization process is trivial and needs some table look-up. Its complexity can be ignored.
At BS, the feedback information are used to reconstruct the right singular matrix V. In Givens scheme, the reconstruction process is similar as the decomposition process in the MSS, namely,

$$V_r = G_{2,1}^H G_{3,1}^H \dots G_{N,N-1}^H I$$

So the complexity is similar as shown in Table 10.

11.1.3 Complexity Comparison

The complexity of Householder and givens are calculated from above analysis for different scenarios. Table 11 list the results at terminal side. As can be seen from the table, complexity of Householder scheme is in general much higher than the Givens scheme at the MSS side. In the scenarios presented here, the Householder scheme requires 7 to 32 times of complexity required by the Givens scheme. This is particularly intolerable for the MSS where the size the power consumption budget is normally very limited.

	N=2 S=2	N=3 S=2	N=4 S=2	N=4 S=3
1. Givens	12	24	36	96
2. Householderwithout step 2	88	372	1168	1320
Ratio: 2 over 1	7.3	16.3	32.4	13.8

Table 11: Complexity comparison at MSS

However, at the BS side, the story is quite different. As shown in Table 12, the Givens and Householder schemes require similar complexity at BS. The reason for this is that the dominate factor in Householder complexity at BS side is due to the codebook search. In base station, there is no codebook search for Householder and therefore, complexity of these two schemes is similar.

	N=2 S=2	N=3 S=2	N=4 S=2	N=4 S=3
1. Givens	12	24	36	96
2. Householder	8	24	48	120
Ratio: 2 over 1	0.6	1	1.3	1.3

Table 12: Complexity comparison at base station