Project	IEEE 802.16 Broadband Wireless Access Working Group <**http://ieee802.org/16**>
Title	LDPC coding for OFDMA PHY
Date Submitted	2004-11-12
Source(s)	Brian Classon brian.classon@motorola.com Yufei Blankenship yufei.blankenship@motorola.com Motorola Jerry Kim kimjy@samsung.com Panyuh Joo panyuh@samsung.com Gyubum Kyung gyubum.kyung@samsung.com Hongsil Jeong hongsil.jeong@samsung.com Sang-Hyo Kim sanghyo7.kim@samsung.com Hanju Kim hanju.kim@samsung.com DS Park dspark@samsung.com Jaeho Jeon jhjeon@samsung.com Samsung Eric Jacobsen eric.a.jacobsen@intel.com Bo Xia bo.xia@intel.com Intel Kyuhyuk Chung kyuhyuk@lge.com Min-seok Oh minoh@lge.com LG Electronics Brian Johnson brjohnso@nortelnetworks.com Alek Purkovic apurkovi@nortelnetworks.com Nina Burns nburns@nortelnetworks.com Sergey Sukobok ssukobok@nortelnetworks.com Nortel Networks Eli Sasha shasha@runcom.co.il Zion Hadad zionh@runcom.co.il Yigal Leiba yigall@runcom.co.il Itzik Kitroser itzikk@runcom.co.il Yossi Segal yossis@runcom.co.il Oren Elkayam Elkayam@runcom.co.il Runcom Technologies

Simon Litsyn Eran Sharon Tel-Aviv University.	litsyn@eng.tau.ac.il eransh@eng.tau.ac.il
Dale Hocevar Anuj Batra Texas Instruments	hocevar@ti.com batra@ti.com
Victor Stolpman Jianzhong (Charlie) Zhang Nico Van Waes Nokia	Victor.Stolpman@nokia.com Charlie.Zhang@nokia.com
Masoud Olfat Nextel Communications	masoud.olfat@nextel.com

Re:	IEEE P802.16-REVe/D5-2004, sponsor ballot
Abstract	This contribution contains additional text output from an informal LDPC group.
Purpose	Provide additional LDPC specification text.
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Overview

An informal LDPC group has been working on the goal of achieving consensus on a proposed LDPC code design as an optional advanced code for the OFDMA PHY. Many excellent code designs have been submitted. The codes have been qualitatively and quantitatively characterized, and it is clear that a LDPC code with excellent flexibility and performance, as well as low encoding and decoding complexity, can be defined for 802.16e. This contribution provides additional LDPC specification text.

Recommended Text Changes:

Add the following text to 802.16e_D5, adjusting the numbering as required:

<Add a new paragraph to section 8.4.9.2.5.1 Code Description, the new paragraph to precede the last paragraph of the section "$\mathbf{H}_b$ is partitioned into two sections…">
The permutations used are circular right shifts, and the set of permutation matrices contains the $z\times z$ identity matrix and circular right shifted versions of the identity matrix. Because each permutation matrix is specified by a single circular right shift, the binary base matrix information and permutation replacement information can be combined into a single compact model matrix $\mathbf{H}_{bm}$. The model matrix $\mathbf{H}_{bm}$ is the same size as the binary base matrix $\mathbf{H}_b$, with each binary entry (i,j) of the base matrix $\mathbf{H}_b$ replaced to create the model matrix $\mathbf{H}_{bm}$. Each 0 in $\mathbf{H}_b$ is replaced by a blank or negative value (e.g., by –1) to denote a $z\times z$ all-zero matrix, and each 1 in $\mathbf{H}_b$ is replaced by a circular shift size $p(i,j)\geq 0$. The model matrix $\mathbf{H}_{bm}$ can then be directly expanded to $\mathbf{H}$.

<Add the material below to the end of section 8.4.9.2.5.2 LDPC encoding, directly after the sentence "The LDPC codes are defined such that very low complexity encoding directly from $\mathbf{H}$ is possible." >
The following informative subsection shows two such methods.

Direct Encoding (Informative)

For the two methods, described below, section $\mathbf{H}_{b2}$ is further partitioned into two sections, where vector $\mathbf{h}_b$ has odd weight, and $\mathbf{H}'_{b2}$ has a dual-diagonal structure with matrix elements at row i, column j equal to 1 for $i=j$, 1 for $i=j+1$, and 0 elsewhere:

$$\mathbf{H}_{b2} = \left[\mathbf{h}_b \mid \mathbf{H}'_{b2}\right] = \left[\begin{array}{c|cccccc} h_b(0) & 1 & & & & & \\ h_b(1) & 1 & 1 & & \mathbf{0} & & \\ . & & 1 & \ddots & & & \\ . & & & \ddots & 1 & & \\ . & & \mathbf{0} & & 1 & 1 & \\ h_b(m_b-1) & & & & & 1 & \end{array}\right].$$

The base matrix has $h_b(0)=1$, $h_b(m_b-1)=1$, and a third value $h_b(j)$, $0<j<(m_b-1)$ equal to 1. The base matrix structure avoids having multiple weight-1 columns in the expanded matrix.

In particular, the non-zero submatrices are circularly right shifted by a particular circular shift value. Each 1 in $\mathbf{H}'_{b2}$ is assigned a shift size of 0, and is replaced by a $z\times z$ identity matrix when expanding to $\mathbf{H}$. The two 1s located at the top and the bottom of $\mathbf{h}_b$ are assigned equal shift sizes, and the third 1 in the middle of $\mathbf{h}_b$ is given an unpaired shift size. The unpaired shift size is 0.

Method 1

Encoding is the process of determining the parity sequence $\mathbf{p}$ given an information sequence $\mathbf{s}$. To encode, the information block $\mathbf{s}$ is divided into $k_b = n_b - m_b$ groups of z bits. Let this grouped $\mathbf{s}$ be denoted $\mathbf{u}$,

$$\mathbf{u} = [\mathbf{u}(0) \quad \mathbf{u}(1) \quad \cdots \quad \mathbf{u}(k_b - 1)],$$

where each element of $\mathbf{u}$ is a column vector as follows

$$\mathbf{u}(i) = [s_{iz} \quad s_{iz+1} \quad \cdots \quad s_{(i+1)z-1}]^T$$

Using the model matrix $\mathbf{H}_{bm}$, the parity sequence $\mathbf{p}$ is determined in groups of z. Let the grouped parity sequence $\mathbf{p}$ by denoted $\mathbf{v}$,

$$\mathbf{v} = [\mathbf{v}(0) \quad \mathbf{v}(1) \quad \cdots \quad \mathbf{v}(m_b - 1)],$$

where each element of $\mathbf{v}$ is a column vector as follows

$$\mathbf{v}(i) = [p_{iz} \quad p_{iz+1} \quad \cdots \quad p_{(i+1)z-1}]^T$$

Encoding proceeds in two steps, (a) initialization, which determines $\mathbf{v}(0)$, and (b) recursion, which determines $\mathbf{v}(i+1)$ from $\mathbf{v}(i)$, $0 \le i \le m_b-2$.

An expression for $\mathbf{v}(0)$ can be derived by summing over the rows of $\mathbf{H}_{bm}$ to obtain

$$\mathbf{P}_{p(x,k_b)}\mathbf{v}(0) = \sum_{j=0}^{k_b-1}\sum_{i=0}^{m_b-1}\mathbf{P}_{p(i,j)}\mathbf{u}(j) \tag{1}$$

where x , $1 \le x \le m_b-2$, is the row index of $\mathbf{h}_{bm}$ where the entry is nonnegative and unpaired, and $\mathbf{P}_i$ represents the $z\times z$ identity matrix circularly right shifted by size i. Equation (1) is solved for $\mathbf{v}(0)$ by multiplying by $\mathbf{P}^{-1}_{p(x,k_b)}$, and $\mathbf{P}^{-1}_{p(x,k_b)} = \mathbf{P}_{z-p(x,k_b)}$ since $p(x,k_b)$ represents a circular shift.

Considering the structure of $\mathbf{H}'_{b2}$, the recursion can be derived as follows,

$$\mathbf{v}(1) = \sum_{j=0}^{k_b-1}\mathbf{P}_{p(i,j)}\mathbf{u}(j) + \mathbf{P}_{p(i,k_b)}\mathbf{v}(0), \quad i = 0, \tag{2}$$

$$\mathbf{v}(i+1) = \mathbf{v}(i) + \sum_{j=0}^{k_b-1}\mathbf{P}_{p(i,j)}\mathbf{u}(j) + \mathbf{P}_{p(i,k_b)}\mathbf{v}(0), \quad i = 1,\ldots,m_b-2 \tag{3}$$

where

$$\mathbf{P}_{-1} \equiv \mathbf{0}_{z\times z}.$$

Thus all parity bits not in $\mathbf{v}(0)$ are determined by evaluating Equation (2) for $0 \le i \le m_b-2$.

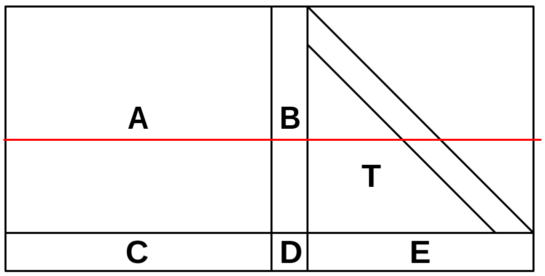
Equations (1) ~~and (2)~~to (3) completely describe the encoding algorithm. These equations also have a straightforward interpretation in terms of standard digital logic architectures. Since the non-zero elements $p(i,j)$ of $\mathbf{H}_{bm}$ represent circular shift sizes of a vector, all products of the form $\mathbf{P}_{p(i,j)}\mathbf{u}(j)$ can be implemented by a size-z barrel shifter.

Method 2

For efficient encoding of LDPC, **H** are divided into the form

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{T} \\ \boldsymbol{C} & \boldsymbol{D} & \boldsymbol{E} \end{pmatrix} \qquad (1)$$

where $\boldsymbol{A}$ is $(m-z)\times k$ $\cancel{(N_p-g)\times N_k}$, $\boldsymbol{B}$ is $(m-z)\times z$ $\cancel{(N_p-g)\times g}$, $\boldsymbol{T}$ is $(m-z)\times(m-z)$ $\cancel{(N_p-g)\times(N_p-g)}$, $\boldsymbol{C}$ is $z\times k$ $\cancel{g\times N_k}$, $\boldsymbol{D}$ is $z\times z$ $\cancel{g\times g}$, and finally, $\boldsymbol{E}$ is $z\times(m-z)$ $\cancel{g\times(N_p-g)}$. $\begin{pmatrix} B \\ D \end{pmatrix}$ and $\boldsymbol{D}$ correspond to the expanded $\mathbf{h}_b$ and $h_b(m_b-1)$, respectively. ~~The basic structure of the **H** matrix is~~



~~Further, all these matrices are sparse and *T* is lower triangular with ones along the diagonal. *B* and *D* part have the column degree 3 and *D* has shift value of a (a is an integer, 0<=a<=z-1). B is with the shift value a of the first entry and shift value 0 in the middle of the column. This other entry is non-zero.~~

Let $\boldsymbol{v}=(\boldsymbol{u}, \boldsymbol{p}_1, \boldsymbol{p}_2)$ ~~that~~ where $\boldsymbol{u}$ denotes the systematic part, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ combined denote the parity part, $\boldsymbol{p}_1$ has length $z\cancel{g}$, and $\boldsymbol{p}_2$ has length $(m-z)$ $\cancel{(N_p\text{-}g)}$. The definition equation $\boldsymbol{H}\cdot\boldsymbol{v}^t = 0$ splits into two equations, as in equations (2) and (3) ~~3 and 4 namely~~

$$\boldsymbol{A}\boldsymbol{u}^T + \boldsymbol{B}\boldsymbol{p}_1^T + \boldsymbol{T}\boldsymbol{p}_2^T = \mathbf{0} \qquad (2)$$

and

$$\left(\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{A}+\boldsymbol{C}\right)\boldsymbol{u}^T + \left(\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{B}+\boldsymbol{D}\right)\boldsymbol{p}_1^T = \mathbf{0}\ \cancel{\left(-\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{A}+\boldsymbol{C}\right)\boldsymbol{u}^T + \left(-\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{B}+\boldsymbol{D}\right)\boldsymbol{p}_1^T = \mathbf{0}} \qquad (3)$$

Define $f := \boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{B}+\boldsymbol{D}$ $\cancel{f := -\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{B}+\boldsymbol{D}}$ and ~~when we use~~with the parity check matrix as indicated ~~appendix we can get,~~ $f = I$. Then from (3~~4~~) we conclude that

$$\boldsymbol{p}_1^T = \left(\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{A}+\boldsymbol{C}\right)\boldsymbol{u}^T\ \cancel{\boldsymbol{p}_1^T = \left(-\boldsymbol{E}\boldsymbol{T}^{-1}\boldsymbol{A}+\boldsymbol{C}\right)\boldsymbol{u}^T} \qquad (4\cancel{5})$$

and

$$\boldsymbol{p}_2^T = \boldsymbol{T}^{-1}\left(\boldsymbol{A}\boldsymbol{u}^T + \boldsymbol{B}\boldsymbol{p}_1^T\right). \qquad (5\cancel{6})$$

As a result, the encoding procedures and the corresponding operations can be summarized below and illustrated in Fig. 1.

Encoding procedure

Step 1) Compute $\boldsymbol{A}\boldsymbol{u}^T$ and $\boldsymbol{C}\boldsymbol{u}^T$.

Step 2) Compute $\boldsymbol{E}\boldsymbol{T}^{-1}\left(\boldsymbol{A}\boldsymbol{u}^T\right)$.

Step 3) Compute $\boldsymbol{p}_1^T$ by $\boldsymbol{p}_1^T = \boldsymbol{E}\boldsymbol{T}^{-1}\left(\boldsymbol{A}\boldsymbol{u}^T\right)+\boldsymbol{C}\boldsymbol{u}^T$.

Step 4) Compute $\boldsymbol{p}_2^T$ by $\boldsymbol{T}\boldsymbol{p}_2^T = \boldsymbol{A}\boldsymbol{u}^T + \boldsymbol{B}\boldsymbol{p}_1^T$.

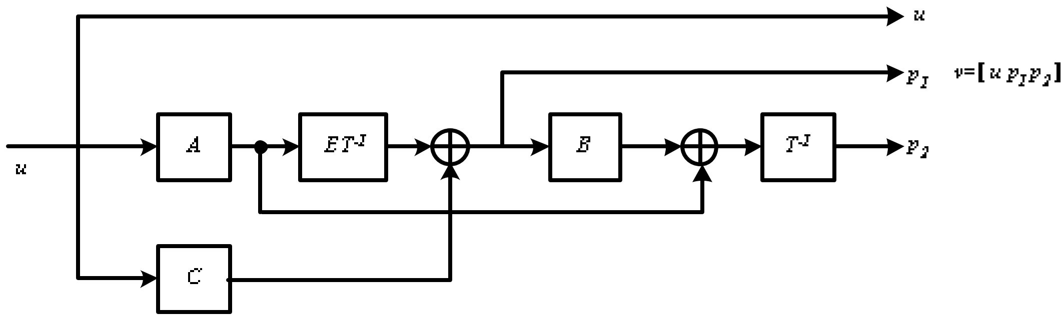


Fig. <u>1</u>~~2~~ Block diagram of the encoder architecture for the block LDPC code.