Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Compact codebooks for transmit beamforming in closed-loop MIMO**
Date Submitted	**2005-1-22**
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Re:	
Abstract	
Purpose	Adoption of proposed changes into P802.16e ~~Crossed-out indicates deleted text,~~ underlined blue indicates new text change to the Standard
Notice	
Release	
Patent Policy and Procedures	

Compact Codebooks for Transmit Beamforming in Closed-loop MIMO

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1 Introduction

The channel feedback mechanism in 16e D5a is not efficient for MIMO Precoding. The channel feedback defined 8.4.5.4.10.6 requires a large feedback bandwidth, which can't be fitted into two CQICH channels with 6 bits payload each for even small MIMO systems such as 2x2, 3x3, and 4x4. Therefore, this scheme can not be used for high mobility feedback using CQICH. A compact feedback format is proposed, which can be fit into either 3, or 6, or 9 bit CQICH payloads. Simulations demonstrate that 6 bit feedback per AMC band provides more than 3 dB gain over STC with the same rate for 4x1 and 4x2. More than 1 dB gain over STC is achieved for 2x2 with 1 stream using 3 bit feedback per AMC band.

The proposed feedback employs a codebook for the beamforming matrixes for each combination of number of BS antennas and number of data streams. The whole set of codebooks require less than 200 byte memory. Codebooks for various numbers of BS antenna and various numbers of data streams can be dynamically generated with low complexity.

2 Compact Codebooks

2.1 Usage mode

The codebook is employed in the feedback from SS to BS. The SS learns the channel state information from downlink and selects a transmit beamforming matrix for the codebook. The index of the matrix in the codebook is then fed back to the BS. Each codebook corresponds to a combination of N_t, N_s, and L, where N_t, N_s, and L are the numbers of BS transmit antennas, available data streams, and bits for the feedback index respectively. Once N_t, N_s, and L are determined in the SS, the SS will feed back the codebook indexes each of L bits. After receiving a L bit index, the BS will look up the corresponding codebook and select the matrix (or vector) according to the index. The selected matrix will be used as the beamforming matrix in MIMO precoding as in 8.4.8.3.6.

In order to make use of the existing CQICH feedback mechanism, the SS may feed back one index per AMC band. The CQICH codebook feedback mechanism and payload sizes for codebook index are specified in contribution 552, which are 3, 6, and other multiples of 3.

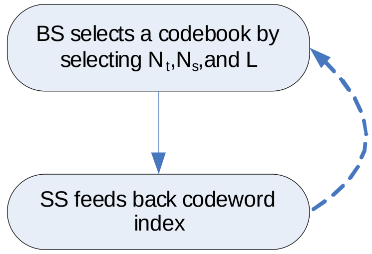

Figure 1 Illustration of the usage model.

2.2 Codebook construction

Since there are many combinations of N_t, N_s, and L and each of them requires a corresponding codebook, the storage of all codebooks are burdensome. A set of codebooks is proposed, which can be dynamically generated with low complexity. It is shown in [2] and the references therein that random codebook has better performance than that of structured codebook. Unfortunately, random codebook needs to be stored completely while structured codebook can be generated dynamically using parameters. The proposed set of codebooks is combination of both.

For small size codebooks, i.e. 2x1, 3x1, and 4x1 with 3 bit index, three optimized, random codebooks are stored. For 3x1 and 4x1 with 6 bit index, two structured codebooks are proposed, which can be dynamically generated using an improved Hochwald method. For all the other matrix codebooks such as 3x2 and 4x2, structured codebooks are proposed, which can also be dynamically generated with low complexity.

2.2.1 Vector codebooks

The stored vector codebooks for 2x1, 3x1, and 4x1 with 3 bit index are listed in Table 1, Table 2, and Table 3. The notation $V(N_t, L)$ denotes the vector codebook (i.e. the set of complex unit vectors), which consists of 2^L unit vectors of a dimension N_t. The number L is the number of bits required for the feedback index that can indicate any vector in the codebook.

Table 1 $V(2,3)$

Vector index	1	2	3	4	5	6	7	8
v_1	1	0.794	0.794	0.794	0.794	0.329	0.511	0.329
v_2	0	-0.580 + 0.182i	0.058 + 0.605i	-0.298 - 0.530i	0.604 + 0.069i	0.661 + 0.674i	0.475 - 0.716i	-0.878 - 0.348i

Table 2 $V(3,3)$

Vector index	1	2	3	4	5	6	7	8
v_1	1	0.500	0.500	0.500	0.500	0.495	0.500	0.500
v_2	0	-0.720 - 0.313i	-0.066 + 0.137i	-0.006 + 0.653i	0.717 + 0.320i	0.482 - 0.452i	0.069 - 0.139i	-0.005 - 0.654i
v_3	0	0.248 - 0.268i	-0.628 - 0.576i	0.462 - 0.332i	-0.253 + 0.263i	0.296 - 0.480i	0.620 + 0.585i	-0.457 + 0.337i

Table 3 $V(4,3)$

Vector index	1	2	3	4	5	6	7	8

v_1	1	0.378	0.378	0.378	0.378	0.378	0.378	0.378
v_2	0	-0.270 - 0.567i	-0.710 + 0.133i	0.283 - 0.094i	-0.084 + 0.648i	0.525 + 0.353i	0.206 - 0.137i	0.062 - 0.333i
v_3	0	0.596 + 0.158i	-0.235 - 0.147i	0.070 - 0.826i	0.018 + 0.049i	0.412 + 0.183i	-0.521 + 0.083i	-0.346 + 0.503i
v_4	0	0.159 - 0.241i	0.137 + 0.489i	-0.280 + 0.049i	-0.327 - 0.566i	0.264 + 0.430i	0.614 - 0.375i	-0.570 + 0.211i

For the $V(3,6)$ and $V(4,6)$ codebooks, we use the improved Hochwald structured construction method. We first note that a Householder transformation is defined here as a transformation that generates a unitary N by N matrix $\mathrm{H}(\mathbf{v})$ using a N vector $\mathbf{v}$ as

$$\mathrm{H}(\mathbf{v}) = \begin{cases} \mathbf{I}, & \mathbf{v} = \mathbf{e}_1 \\ \mathbf{I} - p\,\mathbf{w}\mathbf{w}^H, & \text{otherwise} \end{cases}, \qquad (1)$$

where $\mathbf{w} = \mathbf{v} - \mathbf{e}_1$ and $\mathbf{e}_1 = [1 \quad 0 \quad \cdots \quad 0]^T$; $p = \dfrac{2}{\left\|\mathbf{w}^H\mathbf{w}\right\|}$ and it is a real number that can be pre-computed and stored for each vector in the tables; $\mathbf{I}$ is the N by N identity matrix; ^{H} denotes the conjugate transpose operation. It should be noticed that $\mathrm{H}(\mathbf{v})$ is Hermitian and only N^2 real multiplications are needed to generate $\mathrm{H}(\mathbf{v})$. In the improved Hochwald construction all the vector codewords $\mathbf{v}_i$, $i = 2,\cdots,2^L$, are derived from the first codeword $\mathbf{v}_1$ by:

$$\mathbf{v}_i = \mathbf{H}(\mathbf{s})Q^i(\mathbf{u})\mathbf{H}^H(\mathbf{s})\mathbf{v}_1, \text{ for } i = 2,\cdots,2^L \qquad (2)$$

where $Q^i(\mathbf{u}) = \mathrm{diag}\left(\mathrm{e}^{j\frac{2\pi}{L}u_1 i}, \cdots, \mathrm{e}^{j\frac{2\pi}{L}u_{N_t} i}\right)$ is a diagonal matrix that is fully parameterized by an integer vector $\mathbf{u} = [u_1 \quad \cdots \quad u_{N_t}]$; $\mathbf{v}_1 = \dfrac{1}{\sqrt{N_t}}\left[1 \quad \mathrm{e}^{j\frac{2\pi}{N_t}} \quad \cdots \quad \mathrm{e}^{j\frac{2\pi}{N_t}(N_t-1)}\right]^T$. The codebook $V(3,6)$ and $V(4,6)$ are completely parameterized by the vectors $\mathbf{s}$ and $\mathbf{u}$ as shown in Table 4.

Table 4 Parameters for $V(3,6)$ and $V(4,6)$

N_t	L	$\mathbf{u}$ in $Q^i(\mathbf{u})$	$\mathbf{s}$ in $\mathrm{H}(\mathbf{s})$
3	6	$[1 \quad 26 \quad 57]$	$[1.2518 - j0.6409,\ -0.4570 - j0.4974,\ 0.1177 + j0.2360]^T$
4	6	$[1 \quad 45 \quad 22 \quad 49]$	$[1.3954 - j0.0738,\ 0.0206 + j0.4326,\ -0.1658 - j0.5445,\ 0.5487 - j0.1599]^T$

2.2.2 Matrix codebooks

The matrix codebooks for multiple stream transmission are constructed from the vector codebooks in the previous section using three operations depicted next. We assume that all unit vectors in the section are complex with unit norm and the first entry of each vector is real. The first operation is called Householder reflection transformation mentioned above in (1). The other two operations are built on Householder transformation. One of them is called H-concatenation, and the other is called H-expansion, where the "H" stands for Householder. The H-concatenation (HC) generates a N by $M+1$ unitary matrix from a unit N vector and a unitary $N-1$ by M matrix using Householder transformation as

$$\mathrm{HC}\left(\mathbf{v}_N, \mathbf{A}_{(N-1)\times M}\right) = \mathrm{H}\left(\mathbf{v}_N\right)\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \mathbf{A}_{(N-1)\times M} & \\ 0 & & & \end{bmatrix}, \quad (3)$$

where $N-1 \geq M$; the $N-1$ by M matrix unitary matrix has property $\mathbf{A}^H\mathbf{A} = \mathbf{I}$. Since both terms on the left are unitary the output of HC is a unitary matrix. The H-expansion (HE) generates a N by l matrix from a unit N vector, $\mathbf{v}_N$, by taking the last l columns of $\mathrm{H}(\mathbf{v}_N)$ as

$$\mathrm{HE}(\mathbf{v}_N, l) = \mathrm{H}(\mathbf{v}_N)_{:,N-l+1:N}. \quad (4)$$

Three operations defined in (1), (3), and (4) jointly generate matrix codebooks as follows. In Table 55, by L bit codebook we mean the codebook has 2^L matrixes, which requires a L bit feedback index.

Table 5 Construction operations for N_t by N_s beamforming matrix codebooks with 3, 6, and 9 bit indexes.

N_s / N_t	2	3	4
2 ant., 3 bit codebook	$\mathrm{H}(V(2,3))$		
3 ant., 3 bit codebook	$\mathrm{HE}(V(3,3),2)$	$\mathrm{H}(V(3,3))$	
4 ant., 3 bit codebook	$\mathrm{HE}(V(4,3),2)$	$\mathrm{HE}(V(4,3),3)$	$\mathrm{H}(V(4,3))$
3 ant., 6 bit codebook	$\mathrm{HC}(V(3,3),V(2,3))$	$\mathrm{HC}(V(3,3),\mathrm{H}(V(2,3)))$	
4 ant., 6 bit codebook	$\mathrm{HC}(V(4,3),V(3,3))$	$\mathrm{HE}(V(4,6),3)$	$\mathrm{H}(V(4,6))$
3 ant., 9 bit codebook	$\mathrm{HC}(V(3,6),V(2,3))$	$\mathrm{HC}(V(3,6),\mathrm{H}(V(2,3)))$	
4 ant., 9 bit codebook	$\mathrm{HC}(V(4,6),V(3,3))$	$\mathrm{HC}(V(4,3),\mathrm{HC}(V(3,3),V(2,3)))$	$\mathrm{HC}(V(4,3),\mathrm{HC}(V(3,3),\mathrm{H}(V(2,3))))$

The set notation $V(N_t, L)$ in the input parameter of the operations (i.e. H, HC, and HE) denotes that each vector in the codebook $V(N_t, L)$ is sequentially taken as an input parameter to the operations. The output of the operation (i.e., any one of H, HC, and HE) with a codebook as an input is also a codebook. For example, in $\mathrm{HC}(V(3,6),\mathrm{H}(V(2,3)))$, HC has two codebooks as input. The first one is $V(3,6)$ with 64 vectors and the second one is $\mathrm{H}(V(2,3))$ with 8 2 by 2 matrixes, which are computed from $V(2,3)$. The feedback index is constructed by concatenating all the indexes of the input argument vector codebooks in binary format. For example, the feedback index of $\mathrm{HC}(V(4,6),V(3,3))$ is constructed as $i_2 j_2$, where i_2 and j_2 are the indexes of the vectors in codebooks $V(4,6)$ and $V(3,3)$ in binary format respectively; $_2$ denotes binary format for the indexes.

3 Simulation results

The set of codebooks are evaluated by simulations. The channel model is ITU downlink, pedestrian A and B with 3 km/h. Transmit antenna correlation is 0.2 and receive antenna correlation is 0. The feedback delay is 2 frames, i.e. 10 ms. System bandwidth is 10 MHz with 5 ms per frame. Packet size is 64 byte. One index is fed back per AMC band. Both codebook SVD and STC are simulated. The scheme using the proposed codebooks outperforms STC significantly as shown in the following figures. MMSE receiver is employed.

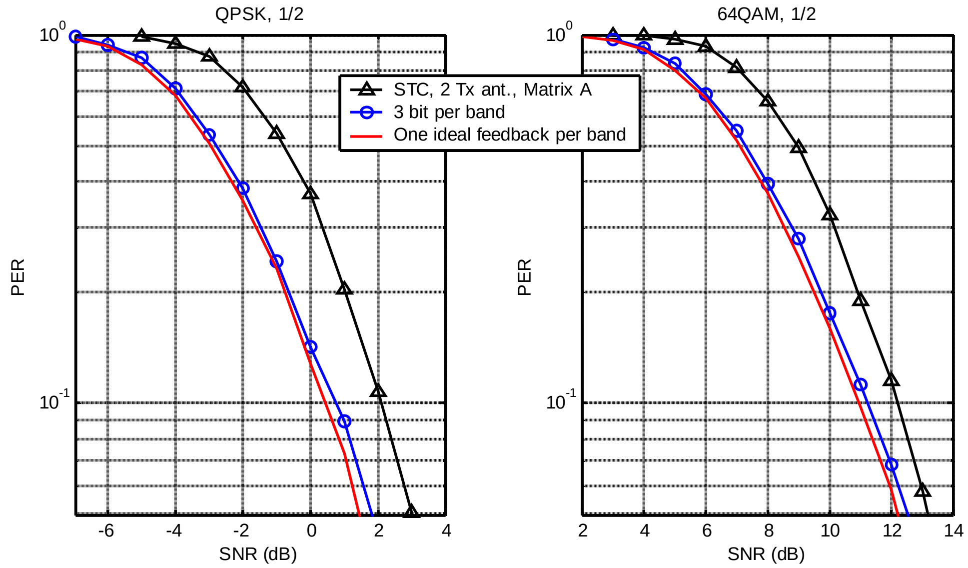

Figure 2 PER performance, 2x2 with 1 data stream, ITU pedestrian B.

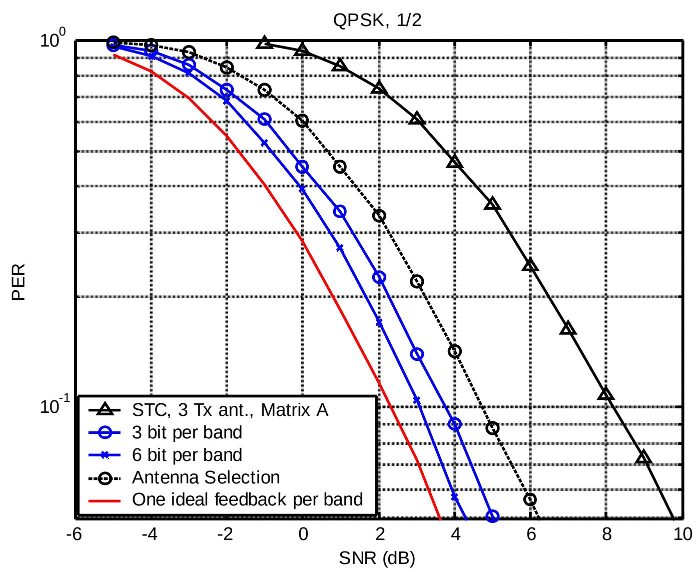

Figure 3 PER performance, 3x1 with 1 data stream, ITU pedestrian A.

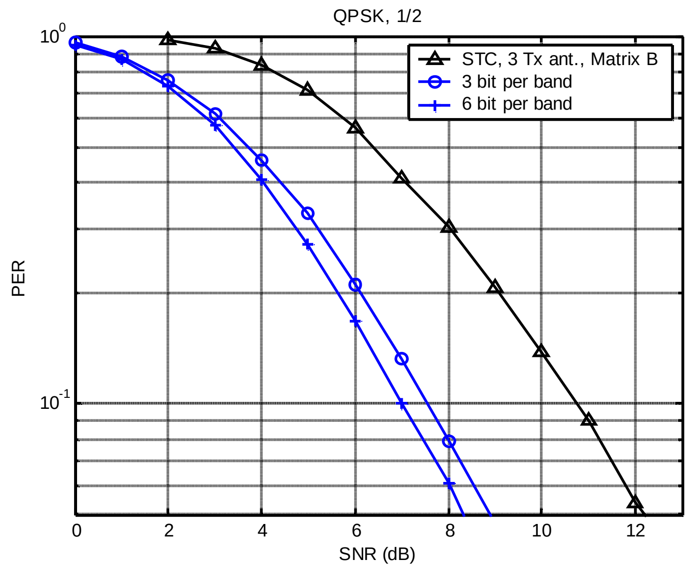

Figure 4 PER performance, 3x2 with 2 data streams, ITU pedestrian A.

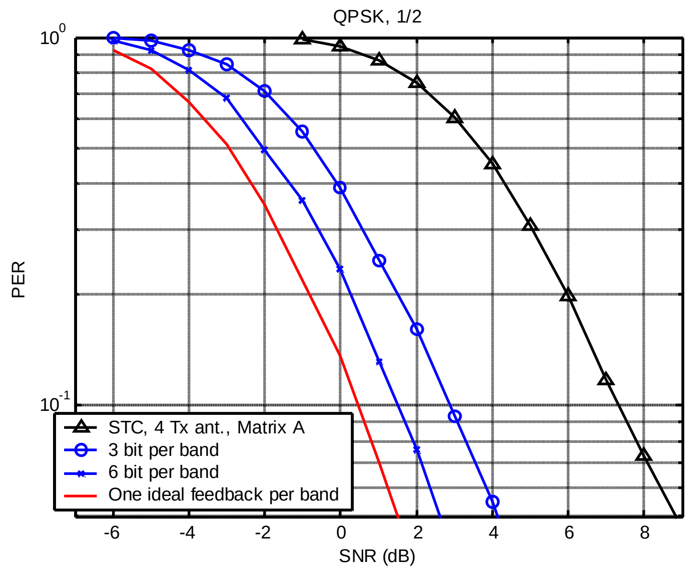

Figure 5 PER performance, 4x1 with 1 data stream, ITU pedestrian A.

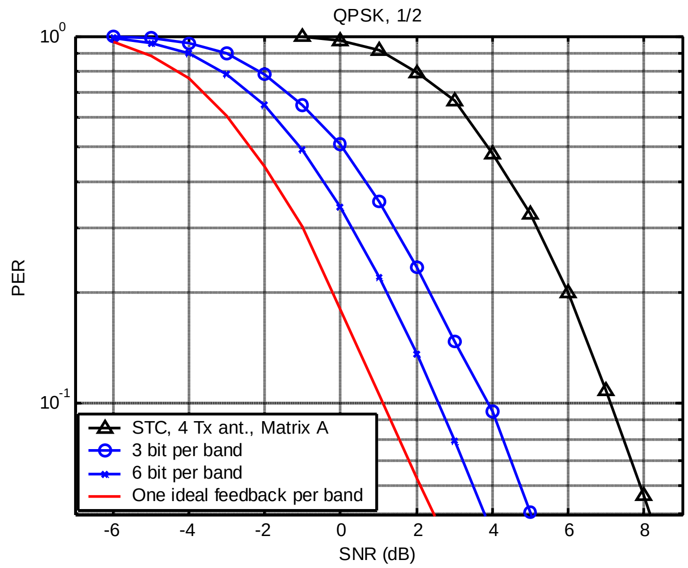

Figure 6 PER performance, 4x1 with 1 data stream, ITU pedestrian B.

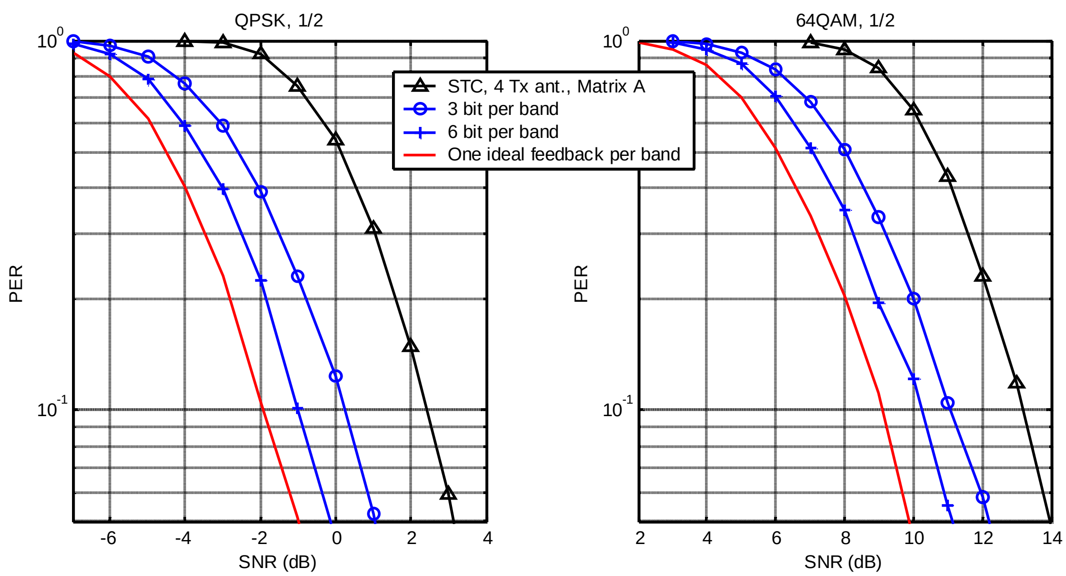

Figure 7 PER performance, 4x2 with 1 data stream, ITU pedestrian B.

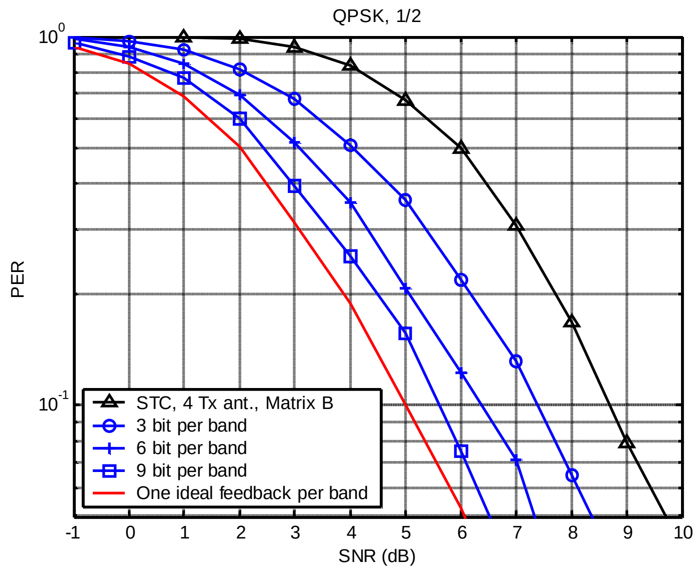

Figure 8 PER performance, 4x2 with 2 data streams, ITU pedestrian B.

4 Specific Text Changes

Added at the end (i.e., line 49) in section 8.4.5.4.10.12 on page 270 of [1] as follows

8.4.5.4.10.12 MIMO feedback for transmit beamforming

Codebooks are defined for the feedback of MIMO transmit beamforming, whose codeword may be employed as the beamforming matrix in MIMO precoding in 8.4.8.3.6. The vector codebooks for 2x1, 3x1, and 4x1 with 3 bit feedback index are listed in Table 1, Table 2, and Table 3. The notation $V(N_t, L)$ denotes the vector codebook, which consists of 2^L complex, unit vectors of a dimension N_t. The integer L is the number of bits required for the index that can indicate any vector in the codebook.

Table 5 $V(2,3)$

Vector index	1	2	3	4	5	6	7	8
v_1	1	0.7940	0.7940	0.7941	0.7941	0.3289	0.5112	0.3289
v_2	0	-0.5801 + j0.1818	0.0576 + j0.6051	-0.2978 - j0.5298	0.6038 + j0.0689	0.6614 + j0.6740	0.4754 - j0.7160	-0.8779 - j0.3481

Table 6 $V(3,3)$

Vector index	1	2	3	4	5	6	7	8
v_1	1	0.500	0.500	0.500	0.500	0.4954	0.500	0.500
v_2	0	-0.7201 - j0.3126	-0.0659 + j0.1371	-0.0063 + j0.6527	0.7171 + j0.3202	0.4819 - j0.4517	0.0686 - j0.1386	-0.0054 - j0.6540
v_3	0	0.2483 - j0.2684	-0.6283 - j0.5763	0.4621 - j0.3321	-0.2533 + j0.2626	0.2963 - j0.4801	0.6200 + j0.5845	-0.4566 + j0.3374

Table 7 $V(4,3)$

Vector index	1	2	3	4	5	6	7	8
v_1	1	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780
v_2	0	-0.2698 - j0.5668	-0.7103 + j0.1326	0.2830 – j0.0940	-0.0841 + j0.6478	0.5247 + j0.3532	0.2058 - j0.1369	0.0618 – j0.3332
v_3	0	0.5957 + j0.1578	-0.2350 – j0.1467	0.0702 – j0.8261	0.0184 + j0.0490	0.4115 + j0.1825	-0.5211 + j0.0833	-0.3456 + j0.5029
v_4	0	0.1587 - j0.2411	0.1371 + j0.4893	-0.2801 + j0.0491	-0.3272 - j0.5662	0.2639 + j0.4299	0.6136 - j0.3755	-0.5704 + j0.2113

An operation, $\mathrm{H}(\mathbf{v})$, is defined. It generates a unitary N by N matrix $\mathrm{H}(\mathbf{v})$ using a N vector $\mathbf{v}$ as

$$\mathrm{H}(\mathbf{v}) = \begin{cases} \mathbf{I}, & \mathbf{v} = \mathbf{e}_1 \\ \mathbf{I} - p\,\mathbf{w}\mathbf{w}^H, & \text{otherwise} \end{cases},$$

where $\mathbf{w} = \mathbf{v} - \mathbf{e}_1$ and $\mathbf{e}_1 = [1 \quad 0 \quad \cdots \quad 0]^T$; $p = \frac{2}{\left\|\mathbf{w}^H \mathbf{w}\right\|}$; $\mathbf{I}$ is the N by N identity matrix; ^{H} denotes the conjugate transpose operation. Vector codebooks $V(3,6)$ and $V(4,6)$ are generated as follows. All the vector codewords $\mathbf{v}_i$, $i = 2, \cdots, 2^L$, are derived from the first codeword $\mathbf{v}_1$ as

$$\tilde{\mathbf{v}}_i = \mathrm{H}(\mathbf{s}) Q^i(\mathbf{u}) \mathrm{H}^H(\mathbf{s}) \mathbf{v}_1, \text{ for } i = 2, \cdots, 2^L,$$

$$\mathbf{v}_i = \tilde{\mathbf{v}}_i e^{-j\phi_i}, \text{ for } i = 2, \cdots, 2^L,$$

where $Q^i(\mathbf{u}) = \mathrm{diag}\left(\mathrm{e}^{j\frac{2\pi}{L}u_1 i}, \cdots, \mathrm{e}^{j\frac{2\pi}{L}u_{N_t} i} \right)$ is a diagonal matrix; $\mathbf{u} = \left[u_1 \quad \cdots \quad u_{N_t}\right]$ is an integer vector;

$$\mathbf{v}_1 = \frac{1}{\sqrt{N_t}} \left[1 \quad \mathrm{e}^{j\frac{2\pi}{N_t}} \quad \cdots \quad \mathrm{e}^{j\frac{2\pi}{N_t}(N_t - 1)} \right]^T$$

; ϕ_i is the phase of the first entry of $\tilde{\mathbf{v}}_i$. The parameters for the generation of $V(3,6)$ and $V(4,6)$ are listed in Table 4.

Table 8 Generating parameters for $V(3,6)$ and $V(4,6)$

N_t	L	$\mathbf{u}$ in $Q^i(\mathbf{u})$	$\mathbf{s}$ in $\mathrm{H}(\mathbf{s})$
3	6	$[1 \quad 26 \quad 57]$	$[1.2518 - j0.6409,\ -0.4570 - j0.4974,\ 0.1177 + j0.2360]^T$
4	6	$[1 \quad 45 \quad 22 \quad 49]$	$[1.3954 - j0.0738,\ 0.0206 + j0.4326,\ -0.1658 - j0.5445,\ 0.5487 - j0.1599]^T$

The matrix codebooks for multiple stream transmission are constructed from the vector codebooks using three operations. The first operation is $\mathrm{H}(\mathbf{v})$. The second denoted as $\mathrm{HC}(\mathbf{v}_N, \mathbf{A}_{(N-1)\times M})$ generates a N by $M+1$ unitary matrix from a unit N vector and a unitary $N-1$ by M matrix as

$$\mathrm{HC}\left(\mathbf{v}_N, \mathbf{A}_{(N-1)\times M}\right) = \mathrm{H}(\mathbf{v}_N)\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \mathbf{A}_{(N-1)\times M} & \\ 0 & & & \end{bmatrix},$$

where $N-1 \geq M$; the $N-1$ by M matrix unitary matrix has property $\mathbf{A}^H\mathbf{A} = \mathbf{I}$. The third operation denoted as $\mathrm{HE}(\mathbf{v}_N, M)$ generates a N by M matrix from a unit N vector, $\mathbf{v}_N$, by taking the last M columns of $\mathrm{H}(\mathbf{v}_N)$ as

$$\mathrm{HE}(\mathbf{v}_N, M) = \mathrm{H}(\mathbf{v}_N)_{:,N-M+1:N}.$$

The three operations jointly generate matrix codebooks as listed in Table 55.

Table 5 Generating operations for N_t by N_s codebooks with 3, 6, and 9 bit indexes.

N_t, L \ N_s	2	3	4
2 antennas, 3 bit	$\mathrm{H}(V(2,3))$		
3 antennas, 3 bit	$\mathrm{HE}(V(3,3),2)$	$\mathrm{H}(V(3,3))$	
4 antennas, 3 bit	$\mathrm{HE}(V(4,3),2)$	$\mathrm{HE}(V(4,3),3)$	$\mathrm{H}(V(4,3))$
3 antennas, 6 bit	$\mathrm{HC}(V(3,3),V(2,3))$	$\mathrm{HC}(V(3,3),\mathrm{H}(V(2,3)))$	
4 antennas, 6 bit	$\mathrm{HC}(V(4,3),V(3,3))$	$\mathrm{HE}(V(4,6),3)$	$\mathrm{H}(V(4,6))$
3 antennas, 9 bit	$\mathrm{HC}(V(3,6),V(2,3))$	$\mathrm{HC}(V(3,6),\mathrm{H}(V(2,3)))$	
4 antennas, 9 bit	$\mathrm{HC}(V(4,6),V(3,3))$	$\mathrm{HC}(V(4,3),\mathrm{HC}(V(3,3),V(2,3)))$	$\mathrm{HC}(V(4,3),\mathrm{HC}(V(3,3),\mathrm{H}(V(2,3))))$

The set notation $V(N_t, L)$ in the input arguments of the operations (i.e. H, HC, and HE) denotes that each vector in the codebook $V(N_t, L)$ is sequentially taken as an input to the operations. The output of the operation with one or more codebooks as input arguments is a codebook. For example, in $\mathrm{HC}(V(3,6),\mathrm{H}(V(2,3)))$, HC has two codebooks as input. The first is $V(3,6)$ with 64 vectors and the second is $\mathrm{H}(V(2,3))$ with 8 2 by 2 matrixes, which are computed from $V(2,3)$. The feedback index is constructed by sequentially concatenating all the indexes of the input argument vector codebooks in binary format. For example, the feedback index of $\mathrm{HC}(V(3,6),\mathrm{H}(V(2,3)))$ is constructed as $i_2 j_2$, where i_2 and j_2 are the indexes of the vectors in codebooks $V(3,6)$ and $V(2,3)$ in binary format respectively; $_2$ denotes binary format for the indexes.

References:

[1] IEEE P802.16e/D5a Air Interface for Fixed and Mobile Broadband Wireless Access Systems – Amendment for Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands, 2004.
[2] Q. Li, *et al.*, "Improved feedback for MIMO precoding," IEEE C80216e-04/527r4, 2004.