Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Link Performance Abstraction based on Mean Mutual Information per Bit (MMIB) of the LLR Channel**
Date Submitted	~~**2007-05-02**~~
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Re:	Call for comments on Draft IEEE802.16m Evaluation Methodology Document
Abstract	Link abstraction based on MMIB without the need of adjustment factors and extendable to MIMO ML receivers
Purpose	For discussion and approval by TGm
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Link Performance Abstraction based on Mean Mutual Information per Bit (MMIB) of the LLR Channel

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1.0 Purpose

This contribution provides detailed description of a link abstraction technique. The level of details herein was absent in the Draft IEEE 802.16m Evaluation Methodology Document, even though the concept was alluded on page 40 (line 19) to 42 (line 6). Hence, the details are provided and explained to allow simulation study to be conducted to verify/improve the proposed method.

In summary, with the proposed modeling technique, accurate link abstraction can be obtained based on a mean mutual information per coded bit (MMIB) metric which is the mean mutual information between coded bits and their LLR values. The known Mutual Information/Capacity ESM method for link abstraction has a closed-form expression for BPSK/QPSK. But for 16QAM/64QAM, some empirical compensation factor must be introduced, just like in the well-known EESM method. On the other hand, the MMIB metric itself, once computed for QPSK, 16QAM, and 64QAM, can be used to model the decoded performance for any MCS and coding rate, without the need of defining any MCS-dependent adjustment factors. The method is also extended to model HARQ (both chase and IR) and MIMO ML or quasi-ML receiver.

2.0 Introduction

Methods of block error rate (BLER) prediction conditioned on measurable physical parameters such as signal-noise ratio and multi-path channel state are required for modeling a link performance in system level simulation. A well-known approach to link performance prediction is the Effective Exponential SINR Metric (EESM) method. This approach has been widely applied to OFDM link layers [2][3][6], but this approach is only one of many possible methods of computing an 'effective SINR' metric.

One of the disadvantages of the EESM approach is that a normalization parameter (usually represented by a scalar, β) must be computed for each modulation and coding (MCS) scheme. In particular, for broader link-system mapping applications, it can be inconvenient to use EESM when combining codewords mapped onto different modulation types, where the EESM method can require the use of so-called symbol de-mapping penalties. Seeking a means to overcome some of the shortcomings of EESM, we focus here on the Mutual Information based approach to link performance prediction.

The first part of the contribution focuses on the single-stream transmissions where a single equivalent channel can be readily constructed (e.g.., Single Input Single Output (SISO), MISO with 802.16e Matrix A encoding, or simply a SIMO channel). In this case, the proposed approach links the SINR of each subcarrier (or group of sub-carriers) to the *mutual information between each encoded bit comprising the received QAM symbols and the corresponding log-likelihood ratio (LLR)*. This yields a mean mutual information per bit (MMIB) measure that may be attributed to "quality" of the entire codeword if sent on that particular channel. This measure can then be used to predict the block error rate for a hypothesized MCS transmission, or space-time coding scheme, or spatial multiplexing scheme. In this document, we derive an appropriate mutual information measure for each component bit comprising the QAM symbol by using Monte Carlo integration and then show the adjustment parameter β for each MCS can be avoided.

In the second part of this contribution, we extend the MMIB method to open loop MIMO links (e.g., 802.16e Matrix B mode). Link performance prediction methods such as EESM may be extended to 2x2 MIMO systems, by reducing the MIMO channel to two equivalent SISO channels and associating an SNR metric to each such channel. However, with a quasi maximum likelihood (QML) receiver, such as a receiver constructed from the general class of sphere decoding methods, this type of separation is not justified. Approximations can be made assuming a) a perfect successive cancellation receiver or b) eigenmode transmission (i.e. assuming a known channel) at the transmitter, but we will show they cannot accurately model the true QML/ML receiver performance. MMIB-based approach parameterized by three variables is shown to have very good prediction.

3.0 Link Performance Prediction using MMIB – Overview

For communication systems like OFDM where multiple channel states may be obtained on a transmitted codeword, link performance prediction, in general, is based on determining a function $I(SINR_1, SINR_2, \cdots)$ which maps multiple physical SINR observations (or more generally of the channel states itself for MIMO channels as we will show later) into a single "effective SINR" metric $SINR_{eff}$ (or equivalent) which can then be input to a second mapping function $B(SINR_{eff})$ to generate a block error rate (BLER) estimate for a hypothesized codeword transmission. We assume the access to a set Ω of N SINR measures, denoted $SINR_n$, $0 \le n < N$. Note that the precise definition of these observations will depend on the SISO/MIMO transmission mode and a receive type, but for the simple SIOS case, the SINR measures may be assumed to correspond to SINR observations of individual data sub-carriers (and therefore of associated QAM symbols) transporting the hypothesized codeword of interest.

The first mapping function I, and effective SINR metric $SINR_{eff}$, may be generally defined as

$$\Gamma \triangleq I\left(\frac{SINR_{eff}}{\alpha_1}\right) = \frac{1}{N}\sum_{n=1}^{N} I\left(\frac{SINR_n}{\alpha_2}\right) \quad (0.1)$$

where α_1 and α_2 are constants (and maybe constrained to be equal), which may be MCS-specific, and Γ may correspond to a defined statistical measure. $I(.)$ is a reference function usually selected to represent a performance model. Exponential ESM is derived by using an exponential function, which is based on using Chernoff approximation to the union bounds on the code performance. Similarly other performance measures like capacity or mutual information can be used. The accuracy of the model to some extent is dependent on how closely the reference model represents the code performance (with sufficient parameterization a given model can yield a reasonably good accuracy as in EESM).

In the method proposed here, Γ is the ***mean mutual information per coded bit*** (**MMIB**), or simply denoted as M, and α_1 and α_2 are discarded (i.e. set to unity). That is, equation (0.1) becomes

$$M = I\left(SINR_{eff}\right) = \frac{1}{N}\sum_{n=1}^{N} I_m\left(SINR_n\right)$$
$$\Rightarrow SINR_{eff} = I^{-1}(M) = I^{-1}\left(\frac{1}{N}\sum_{n=1}^{N} I_m\left(SINR_n\right)\right) \quad (0.2)$$

where $I_m(.)$ is a function that depends on the modulation type identified by m and the associated bit labeling in the constellation[1], where $m \in \{2,4,6\}$ corresponding to QPSK, 16-QAM, 64-QAM respectively. $I_m(.)$ maps the sub-carrier SINR to the mean mutual information between the log-likelihood ratio and the binary codeword bits comprising the QAM symbol.

Due to the asymmetry of bit-to-symbol mapping in the constellation, each bit in the m-tuple labeling of each QAM symbol perceives a different equivalent channel (commonly referred to as unequal error protection). An equivalent bit channel is defined and appropriate bit-wise measures are derived in the following sections. In fact, the average of the mutual information of these bit-wise mutual information measures is derived. More precisely, for an m-tuple input word there exist m mutual information functions[2] $I_{m,i}$, where

$$M = I_m(SINR) = \frac{1}{m}\sum_{i=1}^{m} I_{m,i}(SINR) \quad (0.3)$$

Note that I_m may be approximated using numerical methods, and then stored for later use in link performance prediction. *We will refer to the above quantity as Mutual Information per coded Bit or MIB, with the understanding that it is derived by averaging over the "m" bit channels. Furthermore, mean mutual information per bit (MMIB) is used to refer to the mean obtained over different channel states or SNR measures.* We will show how I_m can be accurately computed without the need of defining any adjustment factor and only three I_m functions (i.e., for m=2,4,6) need to be specified that do not depend on the coding rate. The three MMIBs are adequate for predicting performance for any modulation and coding scheme.

The second functional relationship necessary to estimate the BLER – that is, the BLER function $B(M)$ – may be derived simply from the performance of a specific coding type and decoder under AWGN conditions. Typically, a distinct function $B(M)$ is required for each possible MCS type supported for system simulation. In later sections, means of generating and storing $B(M)$, and simplifications to reduce the number of distinct functions required for storage are discussed.

The basic SISO MMIB method described above can be readily extended to SIMO (i.e. 1x2, or MS diversity) and MISO (the 802.16e Matrix A space-time code) channels by the application of the appropriate MS combining operations. These modes result in a single SINR value per QAM symbol and equation (0.2) can then be simply re-applied.

The extension of MMIB method in the case of linear MIMO receiver is straightforward. However, an SINR measure cannot be obtained per QAM symbol in the case of Maximum Likelihood (ML) or Sphere Decoding receivers. This presents a challenging problem, and we will show in later sections that

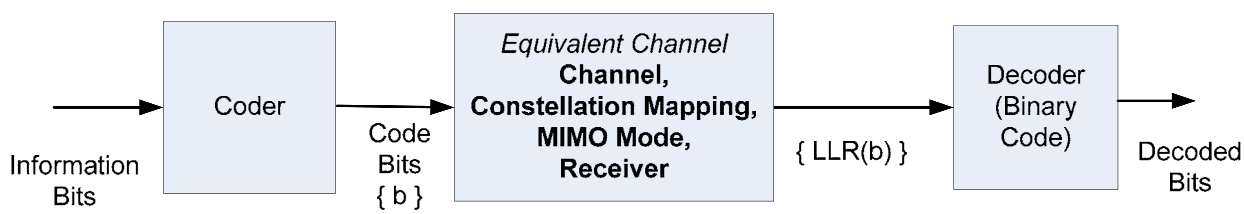
[1] The constellation mappings in this document are specified in Section 8.4.9.4 ,[1].
[2] Only $m/2$ of these measures are distinct, due to the quadrature symmetry of 802.16e constellations.

with the definitions and models we introduce here, a very accurate abstraction can be obtained for these receivers without reducing the MIMO channel to two parallel SISO channels. This is another advantage of MMIB-based link abstraction.

4.0 Concept of Bit LLR Channel

In general, the accuracy of a mutual information based metric depends to a large extent on the equivalent channel over which this metric is defined. For example, a modulation constrained capacity metric is the mutual information of a “symbol channel” (i.e., constrained by the symbol constellation). It is possible to obtain a mutual information per bit metric from the symbol channel by simply normalizing this constrained capacity (i.e, by dividing by the modulation order [1]).

However, given that our goal is to abstract the performance of the underlying binary code, the closest approximation to the actual performance is obtained by defining an information channel at the coder-decoder level, i.e., defining the mutual information between bit input (into the QAM mapping) and LLR output (out of the LLR computing engine at the reciever), as shown below. The concept of bit channel encompasses MIMO channel and receiver. We will demonstrate that this definition will greatly simplify PHY abstraction by moving away from an empirically adjusted model and introducing instead MIB functions of equivalent bit channels.



In the bit channel above, the task now is to define efficient functions that capture the mutual information per bit. The following sections further develop an efficient approach for MIB computation by approximating the LLR PDF with a mixture Gaussian PDFs. We will begin with the development of explicit functions for MIBs in SISO and later extend to MIMO.

5.0 MIB Mapping for Single Input Single Output Systems (SISO)

This section describes MIB defintion for SISO systems, focusing on the theoretical concepts and notations. The numerical expressions/approximations for the actual MIB mapping functions for implementation purposes are elaborated in the next section.

After the encoding step using the CTC or CC encoders to generate a binary codeword bit stream c_k, the QAM modulation step can be represented as a labeling map $\mu : \mathrm{A} \rightarrow X$, where A is the set of m-tuples – $m \in \{2,4,6\}$ – of binary bits and X is the constellation. Given the observation y_n corresponding to the n^{th} QAM symbol, the demodulator computes the log-likelihood ratio (LLR) $LLR(b_{i,n})$ of the i^{th} bit comprising the symbol via the following expression (where the symbol index n is dropped for convenience)

$$LLR(b_i) = \ln\left(\frac{P(b_i = 1 \mid y)}{P(b_i = 0 \mid y)}\right) = \ln\left(\frac{P(y \mid b_i = 1)}{P(y \mid b_i = 0)}\right) \quad (0.4)$$

The computed LLR's may then be input to a BCJR (or similar) decoder. When the coded block sizes are very large in a bit-interleaved coded system (BICM), the bit interleaver effectively breaks up the memory of the modulator, and the system can be represented as a set of parallel independent bit-channels [4]. Conceptually, the entire encoding process can be represented as follows:

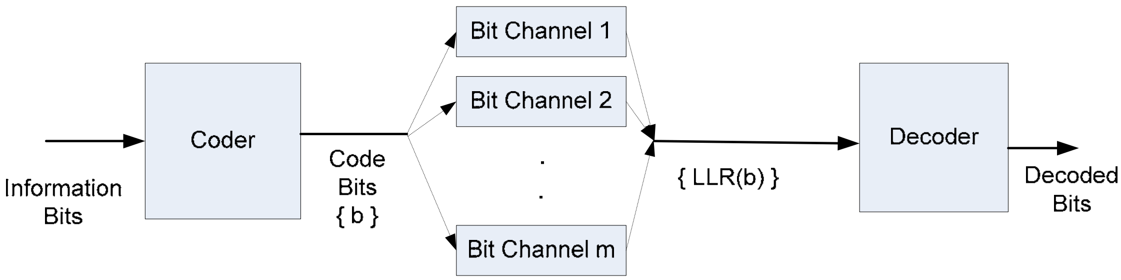


Figure 1 – Conceptual .16e encoding & decoding process with a BICM model.

Due to the asymmetry of the modulation map, each bit location in the modulated symbol experiences a different 'equivalent' bit-channel. In the model, each coded bit is randomly mapped (with probability $1/m$) to one of the m bit-channels. The mutual information of the equivalent channel can be expressed as:

$$I(b, LLR) = \frac{1}{m}\sum_{i=1}^{m} I(b_i, LLR(b_i)) \quad (0.5)$$

where $I(b_i, LLR(b_i))$ is the mutual information between input bit to the QAM mapper and output LLR for i^{th} bit in the modulation map.

More generally, however, the mean mutual information – computed by considering the symbol observations at all N sub-carriers over the codeword – may be computed as

$$M = \frac{1}{mN}\sum_{n=1}^{N}\sum_{i=1}^{m} I(b_i, LLR(b_i)) \quad (0.6)$$

The mutual information function $I(b_i, LLR(b_i))$ is, of course, a function of the QAM symbol SINR, and so the mean mutual information M may be alternatively written

$$M = \frac{1}{mN}\sum_{n=1}^{N}\sum_{i=1}^{m} I_{m,b_i}(SINR_n) \triangleq \frac{1}{N}\sum_{n=1}^{N} I_m\left(SINR_n\right) \quad (0.7)$$

The mutual information function is in turn dependent on the SINR (itself a function of the sub-carrier index n) and the code bit index i, and varies with the constellation order m. Accordingly, the relationship $I_{m,b_i}(SINR)$ is required for each modulation type and component bit index in order to construct $I_m(SINR)$.[3]

5.1 Mutual Information Computation – BPSK/QPSK

Generally, if $H(X)$ is the entropy of X, then

$$I(b,LLR)=H(b)-H(b\,|\,LLR) \quad (0.8)$$

That is, the mutual information between the coded bit value b and the LLR is equal to the uncertainty concerning b (which is assumed to be unity) minus the uncertainty concerning b given that LLR is available. But, clearly $H(b)=1$ and so

$$H(LLR\,|\,b)=\frac{1}{2}\int_{-\infty}^{\infty} p_{LLR}(z\,|\,b=1)\log_2[p_{LLR}(z\,|\,b=1)]dz + \frac{1}{2}\int_{-\infty}^{\infty} p_{LLR}(z\,|\,b=0)\log_2[p_{LLR}(z\,|\,b=0)]dz \quad (0.9)$$

However, the required mutual information function can also be expressed in a more convenient form for numerical evaluation, specifically

$$I(b,LLR)=\frac{1}{2}\sum_{b=0,1}\int_{-\infty}^{+\infty} p_{LLR}(z\,|\,b)\log_2\left(\frac{2p_{LLR}(z\,|\,b)}{p_{LLR}(z\,|\,b=0)+p_{LLR}(z\,|\,b=1)}\right)dz \quad (0.10)$$

where z is a dummy variable equal to LLR.

The received signal can then be represented as

$$y=x+n \quad (0.11)$$

where $E[|x|^2]=1$- bit '0' is transmitted as '+1' and '1' as '-1'- and $E[|n|^2]=\sigma_n^2=1/(2E_s/N_o)$, $N_o/2$ being the noise variance per complex dimension. Substituting in equation(0.11), it can be easily shown that the LLR simplifies to

$$LLR=\frac{2}{\sigma_n^2}(x+n) \quad (0.12)$$

i.e., it is a scaled value of the received signal and is thus Gaussian, conditioned on a specific value of x, where $\mu=2/\sigma_n^2$ (conditioned on $x=1$) and $\sigma^2=4/\sigma_n^2=8E_s/N_o$ are the mean and the variance of the LLR respectively. Since the LLR satisfies $\mu=\sigma^2/2$, the above expression simplifies to

$$I(b,LLR)=1-\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-\sigma^2/2)^2}{2\sigma^2}}\log_2(1+e^{-z})dz = J(\sigma)=J\left(\sqrt{8E_s/N_o}\right)=J\left(\sqrt{8SINR}\right) \quad (0.13)$$

The above expression[4] can be computed numerically (see [5]), and appears in Figure 2.

[3] Note that in the 802.16e specification, bit indexing typically proceeds from 0.

[4] Note that $J(.)$ is *not* related to the well-known Bessel function of the first kind conventionally designated $J_n(.)$, where n denotes the function order.

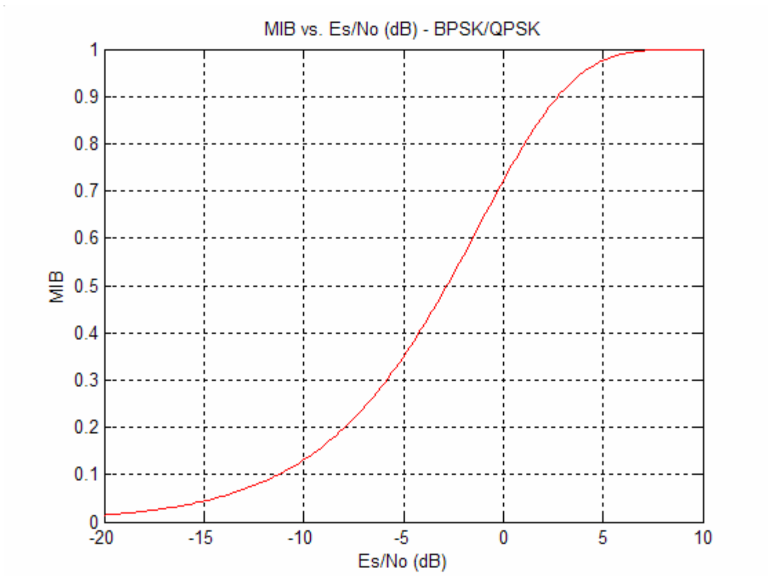


Figure 2 – MIB vs. Es/No (dB), BPSK/QPSK[5]

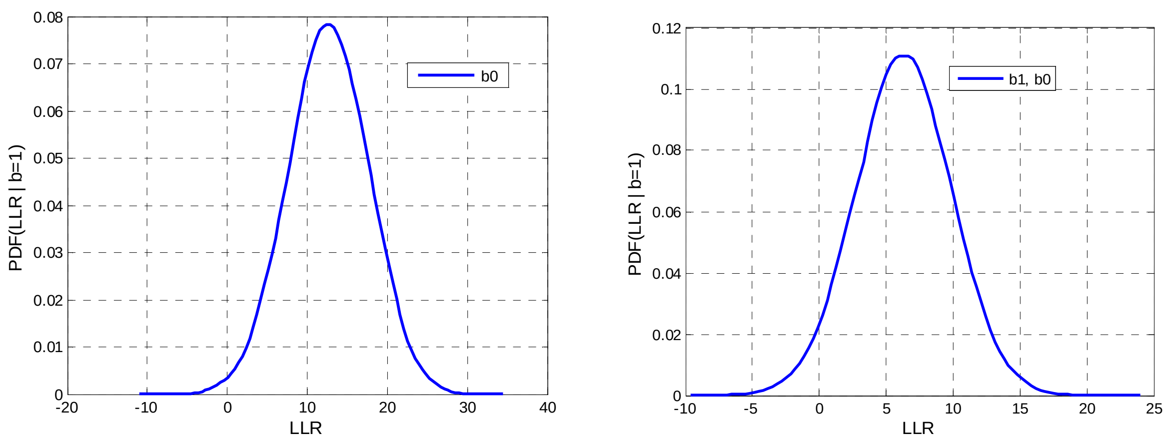


Figure 3 – BPSK and QPSK bit-wise conditional LLR distributions

We can note that, as expected, for BPSK, the LLR distribution is Gaussian with mean $2/\sigma_n^2 = 4E_s/N_o = 12.65$ (SNR = 5 dB). Predictably for QPSK, the distribution is also Gaussian with a mean which is one half of the BPSK mean.

5.2 Mutual Information Computation – M-QAM

BPSK/QPSK MIB is obtained by a known closed-form expression. It is clear that a corresponding non-linear function exists for higher order QAM. Before proceeding with determining these with the proposed LLR channel model in the next section, we briefly discuss possible approaches that can be considered to obtain similar functions.

- ESM with BPSK MIB (MIESM): Simply use the BPSK MIB function in place of exponential in EESM. This approach would require "beta" parameterization adjustment similar to EESM for higher order modulation.

[5] This does not mean it is the same function for both BPSK, QPSK. The difference is only a fixed scaling of SNR by 3 dB.

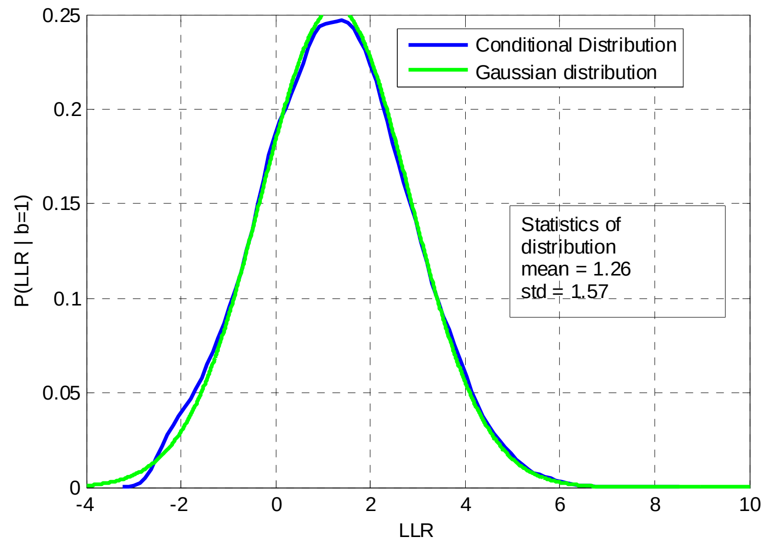
- Constrained Capacity with a Modulation (Ungerboeck). Also referred to as CM (Coded Modulation Capacity). This model does not accurately reflect performance in a fading channel with coding and interleaving. Further, it does not take into account the constellation mapping.
- Bit-Interleaved Coded Modulation (BICM) Capacity (Caire). This model captures the capacity of each bit channel in an interleaved coded modulation, and reduces to the general LLR channel model developed here for SISO. Non-linear functions must be considered for bit channel capacities/MI with a given modulation constellation
- Proposed Approach (see detail in the next section): With the LLR model, the framework is general and it is *applicable to all cases including MIMO* since it uses the baseline receiver models to approximate the MIB with actual decoding. Numerical characterization of functions is required, but such characterization is based on a simple transmission and the receiver models and does not require any exhaustive link simulations (the end result is verified).

6.0 Generalized LLR PDF Model - Mixture of Gaussians

It is shown that there exists a known closed form expression for BPSK. We now derive functions of similar complexity for higher order modulations. The LLR PDF of 16QAM is shown in the figure below for SNR = 5 dB for all the four (m=4) component bits. It is shown in the figure, that it can be approximated as a mixture Gaussian distribution with two component Gaussian distributions defined by individual means, standard deviations and the associated marginal probability. If similar PDF is plotted for higher SNRs, it can be seen that the component distributions do not overlap. Conceptually, it can be easily proved using minimum distance arguments, that at asymptotically high SNRs, we will have a mixture distribution composed of Gaussians in the conditional LLR PDFs (we will skip the proof for brevity here).

The implications of this observation are profound, and it indicates that some kind of structure exists in the non-linear MIB functions. In other words, *if the LLR distribution can be approximated by a mixture of Gaussian distributions (which are non overlapping), then it follows that the corresponding MIB can be expressed as a sum of J(.) functions, which corresponds to the MIB for a Gaussian conditional LLR PDF distribution.*

$$\text{Mixture of Gaussians } \rightarrow I(x) = \sum_{i=1}^{K} a_i J(c_i x) \qquad \sum_{\mathrm{i}} a_i = 1$$



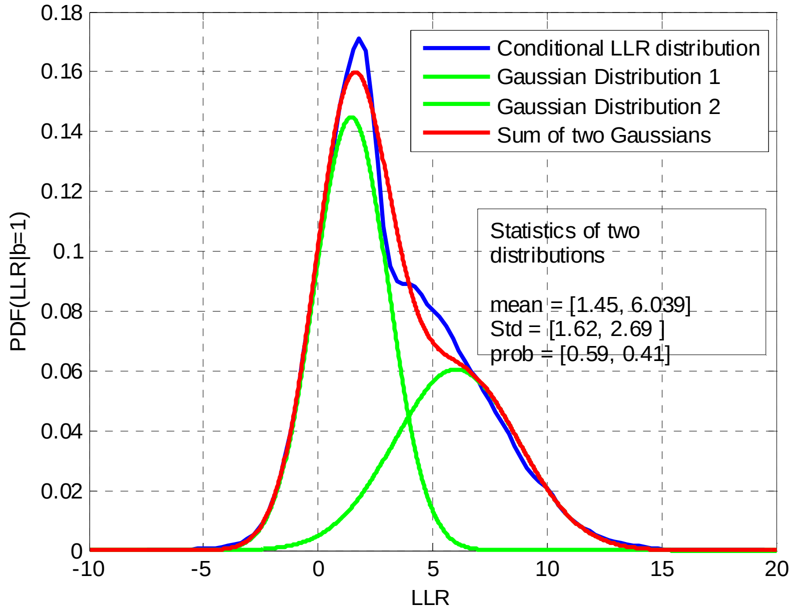


Figure 4 - 16QAM : Conditional LLR distributions modeled as a mixture of Gaussian distributions at SNR = 5dB a) b2,b0 b) b3,b1

The MIB is defined by considering a conditional hypothesis on all the individual bits. In the above figure, LLR PDFs corresponding to two of the bits is a Gaussian distribution. The PDFs for other two can be expressed as mixture of two Gaussian distributions. It is clear that that the MIB of 16QAM can be represented by a sum of three J(.) functions. Similarly, other modulations orders can be expressed as a mixture of Gaussian distributions and as the modulation order is increased could typically be composed of greater than 3 Gaussians.

However, limiting the maximum to 3 dominant Gaussians is found to yield very good approximation to the actual non-linear MIB function for typical cases of interest. The approximations are obtained by numerical optimization and are summarized in Table 1. The accuracy is verified in Figure 5 and the deviation from the actual curve is less than 5e-3.

MI Function	Numerical Approximation
$I_2(\gamma)$ (QPSK)	$M = J(2\sqrt{\gamma})$ $(Exact)$
$I_4(\gamma)$ (16 QAM)	$M = \frac{1}{2}J(0.8\sqrt{\gamma}) + \frac{1}{4}J(2.17\sqrt{\gamma}) + \frac{1}{4}J(0.965\sqrt{\gamma})$
$I_6(\gamma)$ (64 QAM)	$M = \frac{1}{3}J(1.47\sqrt{\gamma}) + \frac{1}{3}J(0.529\sqrt{\gamma}) + \frac{1}{3}J(0.366\sqrt{\gamma})$

Table 1 – Numerical approximations for MMIB mappings.

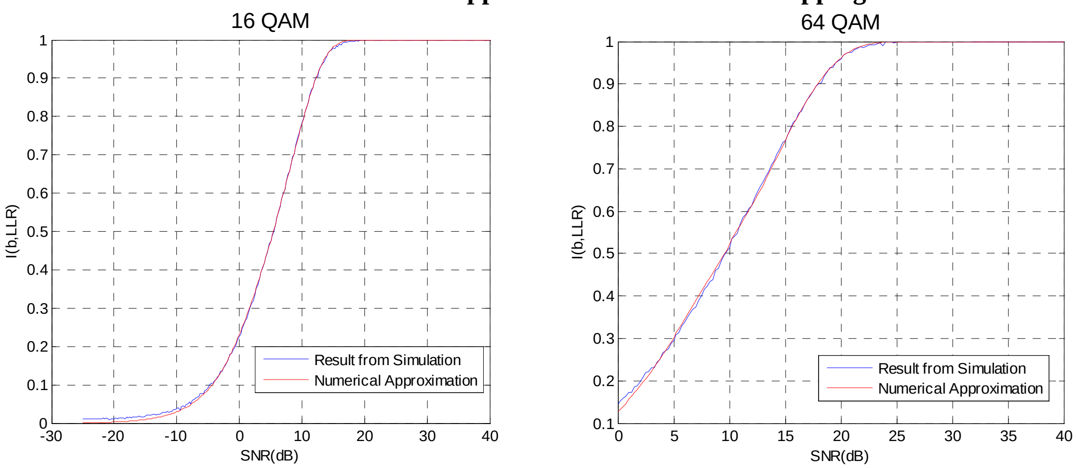


Figure 5 - Comparison of Numerical Approximations with simulated results for a) 16 QAM b) 64 QAM.

6.1 Numerical Simulation to Obtain LLR PDFs and MIBs

For reference, the following steps can be used for obtaining the above approximations

Step 1 LLR conditional PDF's of all the bits for each specific modulation are obtained by numerical simulation at each SNR for a scalar channel
Step 2 MIB is then obtained by numerical integration
Step 3 Approximation using sum of basis functions J(.) by curve fitting considering all SNRs

7.0 MMIB Link Abstraction for SISO/SIMO – Detailed Description of the Simulation Step

The current 802.16e ECINR reporting requirement (802.16e Table 298a, Section 8.4.5.4.10.4) lists the MCS levels specified in Table 2 (to be updated based on simulation methodology support for different MCSs, packet sizes etc., specific to 16m).

MCS Label	Modulation	Code Rate	Repetition Factor	Max. Inf. Word Length	Max. Code Word Length (bits)

				(bits)	
1c	QPSK	1/2	6	See MCS 1	
1b	QPSK	1/2	4		
1a	QPSK	1/2	2		
1	QPSK	1/2	1	480	960
2	QPSK	3/4	1	432	576
3	16-QAM	1/2	1	480	960
4	16-QAM	3/4	1	432	576
5	64-QAM	1/2	1	432	864
6	64-QAM	2/3	1	384	576
7	64-QAM	3/4	1	432	576
8	64-QAM	5/6	1	480	576

Table 2 –Example MCS Set for Simulation.

The associated codeword lengths are adopted to be the maximum codeword lengths possible corresponding to each modulation and code rate combination (for illustrative purposes). Accordingly, for the purpose of MMIB based link abstraction, we require

1) Mutual information mapping functions $I_m(x)$ for all modulation types {QPSK, 16-QAM, 64-QAM} to obtain MMIB from a given channel realization, and
2) A block error rate (BLER) mapping function $B_\varphi(M)$ for mapping MMIB to a predicted BLER, where φ is the index identifying the codeword length and code rate.

The second requirement is based on the observation that MMIB to block error rate mapping is found to be independent of the modulation itself to a good approximation (see below).

8.0 BLER Mapping Function – Detailed Description and Numerical Approximations

The BS can store the AWGN reference curves for different MCS levels in order to map the MMIB to BLER. Another alternative is to approximate the reference curve with a parametric function. For example, we consider a Gaussian cumulative model with 3 parameters which provides a close fit to the AWGN performance curve, parameterized as

$$y = \frac{a}{2}\left[1 - erf\left(\frac{x-b}{\sqrt{2}c}\right)\right], \quad c \neq 0 \qquad (0.14)$$

where a is the “transition height” of the error rate curve, b is the “transition center” and c is related to the “transition width” (transition width = 1.349c) of the Gaussian cumulative distribution. In the linear BLER domain, the parameter a can be set to 1, and the mapping requires only two parameters, which are given for each MCS index in the table below.

The accuracy of the curve fit with this model is verified in below with MCS modes in 802.16e.

Modulation	**Code Rate**	*b*	*c*
QPSK	1/2	0.5512	0.0307
16QAM	3/4	0.7863	0.03375
64QAM	5/6	0.8565	0.02622

Table 3 - Parameters for Gaussian cumulative approximation to BLER mapping. (Block Size = 480 bits)

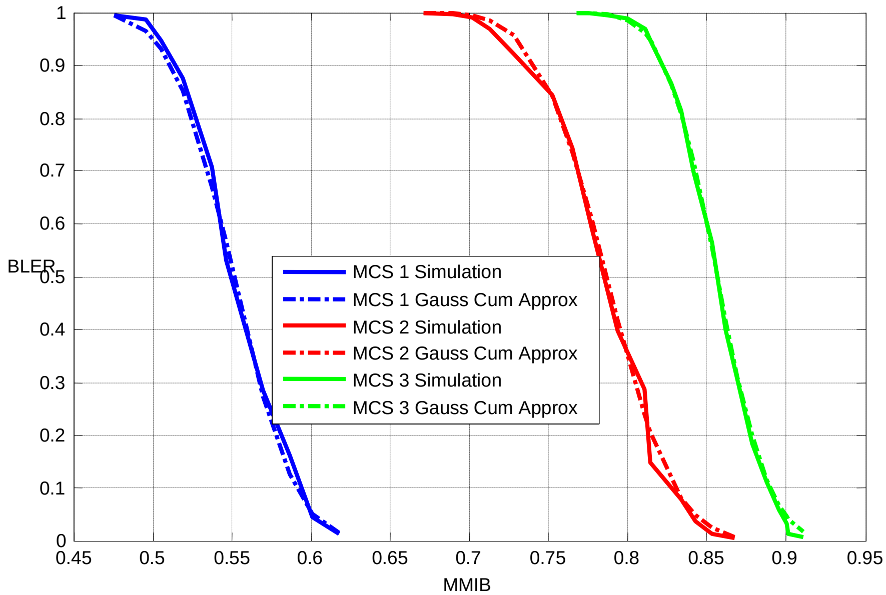


Figure 6 - Curve fit for BLER mapping.

So for each MCS the BLER is obtained as

$$BLER_{MCS} = \frac{1}{2}\left[1 - erf\left(\frac{x - b_{MCS}}{\sqrt{2}c_{MCS}}\right)\right], \quad c \neq 0 \qquad (0.15)$$

Further, we can achieve an additional simplification. The following figure plots MMIB vs BLER (i.e. $B_{\varphi}(M)$) for numerical results obtained in 802.16e simulations using 6 different MCS's with rates 1/2 and 3/4 on an AWGN channel.

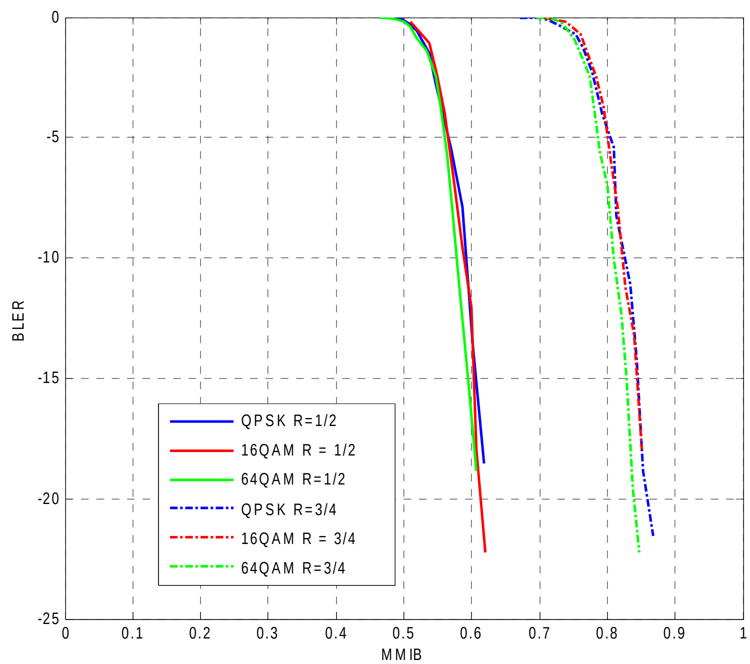


Figure 7 - BLER mappings for MMIB from AWGN performance results.

It can be seen from the figure that – to a first-order approximation – the mapping from MMIB to BLER can be assumed independent of the QAM modulation type. However, since code performance is strongly dependent on code sizes and code rates, $B_{\varphi}(M)$ will not be independent of these parameters.

With the above result, we generalize the AWGN reference curves to be a function of the block size and coding rate (BCR)

$$BLER_{BCR} = \frac{1}{2}\left[1 - erf\left(\frac{x - b_{BCR}}{\sqrt{2}c_{BCR}}\right)\right], \quad c \neq 0 \qquad (0.16)$$

With this simplification, a base station needs to store two parameters for each supported BCR mode.

Note: The choice of this particular MMIB to BLER mapping is due to the underlying physical interpretation. The parameter b is closely related to the binary code rate and will be equal to the code rate for an ideally designed code. Similarly, parameter c represented the rate of fall of the curve and is also related to the block size.

Note 2: . It is also possible to express these parameters as simple 2-dimensional parameterized functions of block size and code rate as follows, which could further reduce storage requirements and streamline simulation methodology.

$$b = f(R,L) = R + f^{'}(R,L)$$
$$c = g(R,L)$$

9.0 Performance Prediction for HARQ

It is clear that once compted, MMIB metric relates to the underlying coder-decoder performance and is independent of the modulation order transmission modes etc., Due to this property, link prediction for HARQ can be accomplished easily when the multiple (re)transmissions corresponding to an information packet do not correspond to the same modulation. This is illustrated in the above section, where we have shown that the MMIB to BLER mappings are independent of the modulation order – to a good approximation.

This allows the transmitter to predict performance with hybrid ARQ, even when the retransmissions support modulation and transmission modes different from the first transmission. We outline the general approach here for performance prediction with HARQ

9.1 Chase Combining

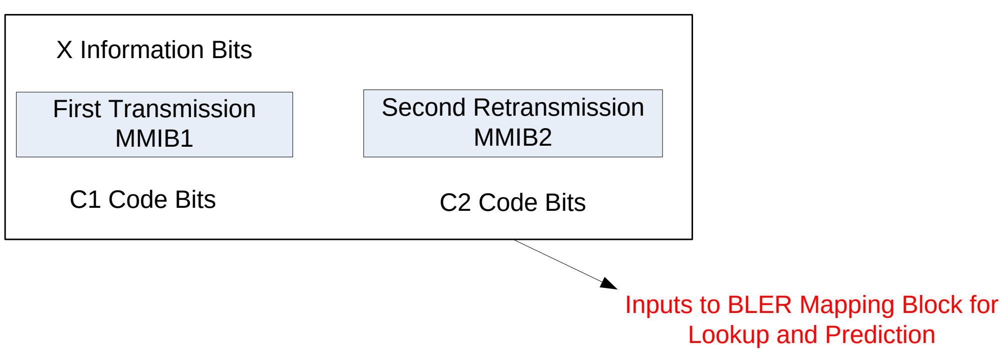
This is a straight-forward extension since the post-processing SNRs can be obtained as simple sum of the SNRs on the first transmission and subsequent retransmissions.

$$MMIB = \sum_{i=1}^{N} I_m \left(\sum_{j=1}^{q} \gamma_{ij}\right)$$

where γ_j is the i th symbol SNR during j th retransmission.

9.2 Incremental Redundancy

With IR, typically a transmitter transmits packets which are components of a mother code. Given a packer is received in error, the BS tries to transmit independent code information as much as possible to maximize coding gains at the receiver. Typically, only when these combinations are exhausted, a retransmission of previous packet transmissions is performed. In this context, the performance of the decoder at each stage is that corresponding to a binary code with the modified equivalent code rate and code size (as shown below), and *neglecting any partial repetition* of previously sent packets for modelling purposes.



$$\text{Effective Code Rate} = \frac{x}{c_1 + c_2}$$

$$\text{Effective Code Size} = c_1 + c_2$$

$$MMIB_{HARQ,2} = \frac{c_1 MMIB_1 + c_2 MMIB_2}{c_1 + c_2}$$

Figure 8 – MMIB Update after a Retransmission and the Required Parameters for BLER Lookup

The performance prediction can be performed by combining the MMIBs on the transmissions as shown in the above figure, and *looking up the BLER relationship corresponding to the modified effective code rate and code size.*

Note: A code rate-code size parameterized relationship for b,c parameters in the AWGN reference (see note at the end of previous section), is clearly very helpful to cover the new possible combinations with IR.

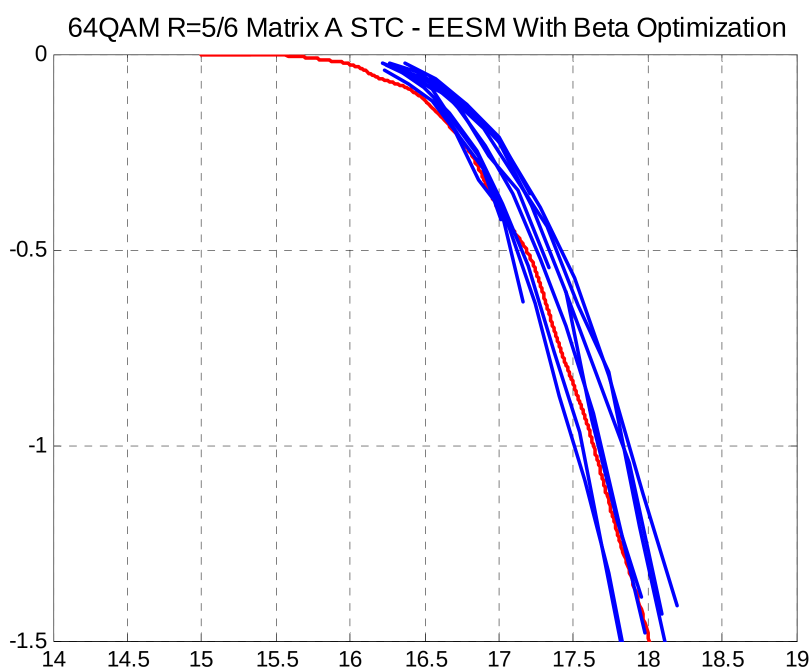
[To be inserted in future. The supported IR modes and the corresponding parameters in 16m modeling.]

[To be inserted by another contribution: The partial IR modeling]

9.3 Numerical Results with SISO

The following results show the performance prediction accuracy of EESM and MMIB approaches. Optimal beta is used for EESM obtained by link simulations. The proposed 'sum of Gaussians' mappings are used for MMIB and no further fudge factors are used. It is clear that performance prediction is close to (slightly better) EESM. Further, MMIB approach is more robust to channel models etc., variation compared to EESM.

Note that the "Effective SNR" in the plot is the SNR of the reference (AWGN) curve which results in the same FER as the given fading channel realization. The curves are plotted in an effective SNR domain for comparison purposes only. For MMIB mapping, we can operate in MMIB domain directly.



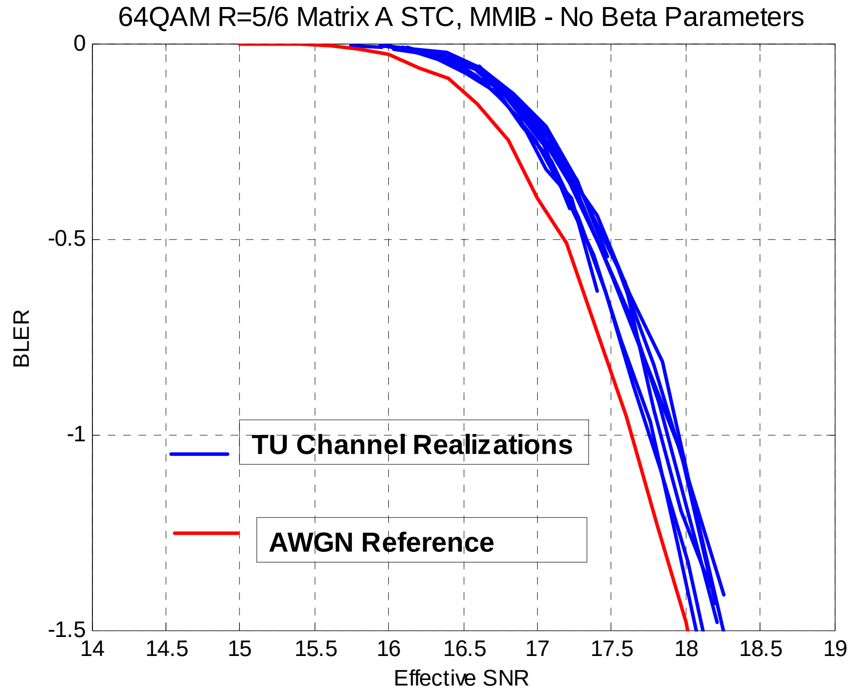


Figure 9 – Performance Prediction for SISO with a) EESM b) MMIB

10.0 MIMO Mapping based on SISO MMIB Mapping Functions

We briefly discuss the approaches that may be used to derive the mappings for Matrix B mode. Note that the mapping for matrix A is fairly straightforward once the post processing SNR with STC is derived.

10.1 Linear Receivers

With linear receivers like MMSE, each MIMO channel is treated as two equivalent SISO channels with SNRs given by post combining SNRs of the linear receiver. The MIB can be obtained as

$$M = \frac{1}{NN_t}\sum_{i=1}^{N}\sum_{j=1}^{N_t} I_m(\gamma_{ij})$$

$$BER = B_{\varphi}(M) \qquad (0.17)$$

where γ_{ij} is the post combining SNR of layer j on subcarrier i, N_t is the number of transmit antennas, N is the total number of coded subcarriers, and the mapping functions $I_m(.)$ and $B_{\varphi}(.)$ are defined in sections on SISO for each MCS.

10.2 Successive Cancellation for Non-Linear Receivers

Successive cancellation approaches can be considered for decoding of SM schemes (e.g., matrix-B) and give improved performance compared to linear receivers. ZF and MMSE based approaches can be considered. Here, we summarize the algorithm with QR decomposition (equivalent to ZF approach).

The QR decomposition of the channel matrix is given by

$$\mathbf{H} = \mathbf{QR} \tag{0.18}$$

where $\mathbf{Q}$ is a 2x2 unitary matrix and $\mathbf{R}$ is a 2x2 upper triangular matrix. By pre-multiplying the received vector with $\mathbf{Q}^H$, we obtain

$$\begin{aligned}\mathbf{Q}^H\mathbf{y} &= \mathbf{Rs} + \mathbf{Q}^H\mathbf{n} \\ \mathbf{y'} &= \mathbf{Rs} + \mathbf{n'}\end{aligned} \tag{0.19}$$

where $E[\mathbf{n}^H\mathbf{n}] = E[\mathbf{n}'^H\mathbf{n}'] = \sigma^2 I$, where σ^2 is the variance per receive antenna and I is the 2x2 identity matrix. With this transformation, the second symbol has no interference from the first symbol and the LLRs can be computed from the following equation

$$y_2' = R_{22}s_2 + n_2' \tag{0.20}$$

A hard or soft estimate of the second symbol can be used to cancel the interference and decode the first symbol.

$$y_1' = R_{11}s_1 + R_{12}s_2 - R_{12}\hat{s}_2 + n_2' \tag{0.21}$$

Assuming perfect cancellation, the SNRs of the two layers are given by

$$\gamma_1 = \frac{|R_{11}|^2}{\sigma^2}, \qquad \gamma_2 = \frac{|R_{22}|^2}{\sigma^2} \tag{0.22}$$

and the MIB mapping can be obtained using the SISO MIB mappings as follows

$$M = \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{2} I_m(\gamma_{ij}) \tag{0.23}$$

The optimal ordering for cancellation requires maximizing the SNR of the first detected layer. This can be done by permuting the columns of $\mathbf{H}$, and choosing the best possible QR decomposition.[6]

10.3 Eigen Decomposition with Channel Knowledge for Non-Linear Receivers

Define $\mathbf{W} = \mathbf{H}^H\mathbf{H}$ (or $\mathbf{HH}^H$). $\mathbf{W}$ is a 2x2 random non-negative matrix and has real non-negative eigen values. The capacity can then be written in terms of eigen values λ_1, λ_2 of $\mathbf{W}$,

$$\begin{aligned}C &= \log_2 \det(I + \mathbf{H}^H\mathbf{H} * SNR) \\ &= \sum_{i=1}^{2} \log(1 + \lambda_i * SNR)\end{aligned} \tag{0.24}$$

The capacity of an instantaneous channel matrix remains the same, regardless of whether the data is transmitted on eigenmodes or not, assuming that no water-filling is allowed on the eigenmodes. However, with linear receivers, pre-coding/beamforming on eigen modes allows us to achieve channel capacity without added implementation complexity of a non-linear receiver.

[6] For higher number of receive and transmit antennas, reduced complexity ordering algorithms are proposed, but they are not required for a 2x2 system.

With this observation, we now assume that data is transmitted along the eigen modes, and treat each of the modes as a separate layer, which allows us to employ the MMIB models developed for SISO channels. The MMIB mapping is given by

$$M = \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{2} I_m(\lambda_{ij}) \qquad (0.25)$$

where $I_m(.)$ are the MMIB mappings for a SISO system. Numerical approximations for these functions are provided as before. Note that this is an approximate model, since the arguments are based on capacity, and does not exactly capture the performance of an ML receiver with non-Gaussian constellations which are used in practice.

11.0 Non-Linear Receiver Modelling

The most accurate way to model the performance of non-linear receivers is to operate in the MIB domain itself, without requiring an SNR interpretation. In other words, we will deal with the LLR channel directly with the hypothesis of an ML receiver. In general, we can think of MIB as now being defined for a hyper-constellation induced by the instantaneous channel matrix.

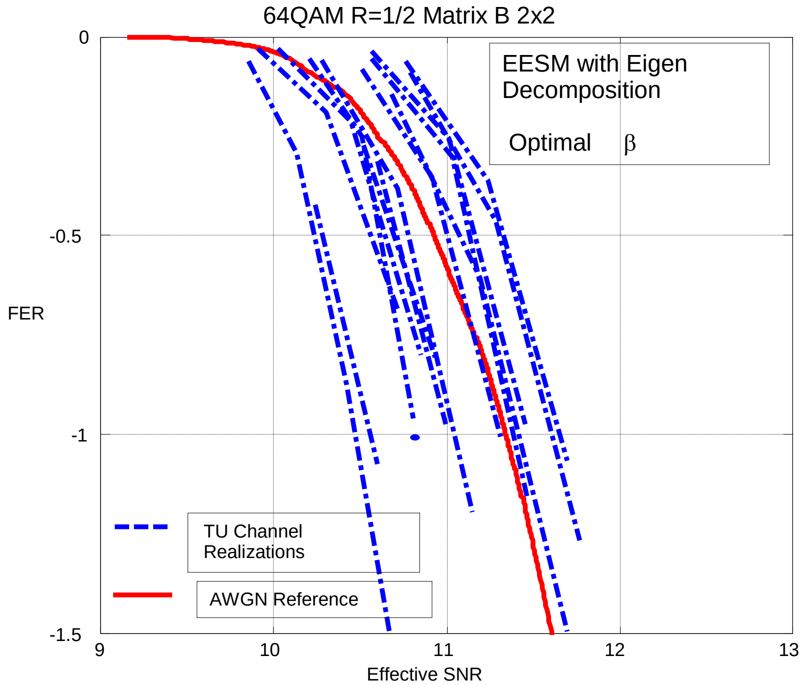
In addition by imposing the structure of mixture Gaussian distributions, we identify three dominant Gaussians corresponding to a channel matrix

$$\mathbf{H} \to [\gamma_1, \gamma_2, \gamma_3]$$
$$I(\mathbf{H}) = \sum_{i=1}^{3} c_i(a_i\gamma_i), \qquad a_1 + a_2 + a_3 = 1$$

Step 1: Determine the dominant Gaussian means by simple algebraic mappings from the channel matrix
Step 2: Determine the parameterized sum of Gaussians with these means by numerical simulation and curve fit.

The effort required for Step 2 can be intensive, but once determined these functions are fixed and do not have any runtime impact on simulation modelling.

The plots below compare EESM with Eigen decomposition and MMIB with the above approach showing 15 different TU channel realizations. The spread of the blue curves represents the accuracy of the performance prediction.



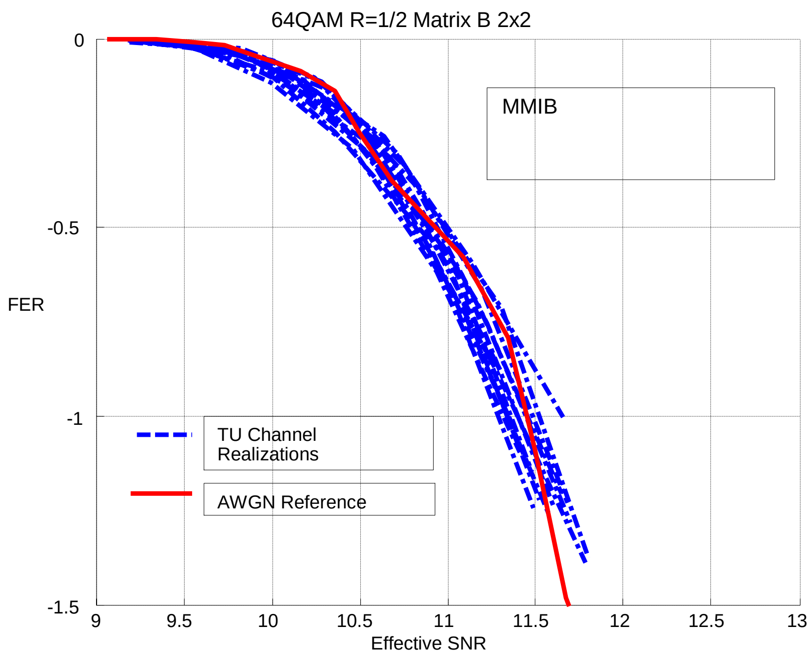


Figure 10 – Performance prediction for a MIMO ML receiver with a) EESM with Eigen decomposition b) MMIB for ML receivers

In this case, with EESM, the error in effective SNR evaluation is -1/+0.5 dB at 10% frame error rate. It is -0.2/+0.1 dB with the MMIB mappings. It is further noted that similar result is obtained with EESM when other mappings based on MMSE or SIC are used. It is clear using MMIB based mapping targeted at non-linear receiver operation results in significant improvement compared to EESM. Using other SISO based mappings, i.e. mutual information mapping itself would result in similar degradation. But the proposed approach is shown to have prediction accuracy similar to SISO, and with no additional beta parameters specific to MCSs (the functions once defined for each modulation, are common for all MCSs)

12.0 Conclusions

This contribution proposes an LLR based bit-channel model to define mutual information measures applicable to both SISO and MIMO. Further, we have shown that MIB in all cases can be expressed as a sum of Gaussian approximation, which allows us to implement MIB evaluation with simple functions obtained and approximated numerically. MMIB approach permits accurate prediction of code performance independent of modulation order and the channel (Validated with TU, PA, PB etc.)

Similar MMIB vs. BLER relationship is observed for TU channel and AWGN reference channels, which avoids optimization of parameters required for EESM mapping. Further, a 2 parameter Gaussian Cumulative curve fit is recommended for MMIB to BLER/FER mappings, due to its accuracy and physical interpretation.

MMIB allows performance prediction when code words from different modulation orders are combined for decoding in a HARQ systems. Additional parameters are not required for HARQ. Accurate mappings are also developed for an ML receiver, which obtain MIB as a function of channel matrix itself, without the need to generate parallel channel approximations. Further, the beta parameters of these approximate models are typically sensitive to MIMO channel models used in link simulations.

In conclusion, MMIB is a highly accurate tool to study and compare system performance of advanced MIMO receivers and transmission modes in 16m systems.

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