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Re:	Call for comments on Draft IEEE 802.16m Evaluation Methodology Document, IEEE 802.16m-07/023
Abstract	This contribution proposes a link performance abstraction method which is a comprehensive extension of EESM method stated in section 4.3.3 of EMD
Purpose	For discussion and approval by 802.16 TGm
Notice	
Release	
Patent Policy	

Link Performance Abstraction based on Bit-wise Exponential Effective SINR Metric (BEESM)

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1. Introduction

The dynamic PHY abstraction methodology based on Exponential Effective SINR Metric (EESM) has been adopted as part of the draft IEEE 802.16m Evaluation Method Document (C802.16m-07/080r3 P67). The original EESM method stated in this document computes the effective SINR based on symbol level SINR, and a *linear* scaling factor (usually represented by β) is used to account the effect of different modulation and coding scheme (MCS). In practice, an optimal β should be derived for each MCS by using numerical method. However, this method may not necessarily be optimized and has been shown that it can not achieve a comparable performance with mutual information (MI) based metric. This contribution proposes a comprehensive extension to this method called Bit-wise Exponential Effective SINR Metric (BEESM) with following features:

a. non-linear mapping function from symbol level SINR to bit level SINR instead of linear scaling factor β
b. support different modulation constellations within one coded block

It is further shown that BEESM and MMIB-ESM are quite similar to each other in terms of the BE/MMIB vs. SINR curves for different modulation constellations.

2. Bit-wise Exponential ESM (BEESM)

As stated in relevant literatures, the exponential average operation used by EESM is actually derived based on the Chernoff bound for the probability of error. However, it should be pointed out that this exponential average operation should be imposed on the effective SINR of each individual bit rather than the SINR of modulated symbol as used in original EESM. It is believed that the *linear* scaling factor used in original EESM actually intends to map the symbol level SINR to bit level SINR. However, the mapping relationship between symbol-level SINR and bit-level SINR is in fact not *linear* such that EESM can not achieve good performance even though β is optimized numerically. To solve this problem, we must derive the accurate bit-level SINR from symbol-level SINR.

2.1 Mapping from symbol-level SINR to bit-level SINR

In every common receiver for digital communication, there is a demapper block which demapping the received modulated symbol to soft information of each individual bit (soft demapping is assumed here). The key issue is how to evaluate the *equivalent effective SINR* contained in the soft information of each individual bit. For BPSK and QPSK, the soft information of each individual bit can actually be treated as noised BPSK symbol whose *equivalent effective SINR* can be easily calculated exactly. However, for 16QAM and 64QAM, the soft information of each individual bit can no longer be treated as a noised BPSK symbol whose *equivalent effective SINR* can only be approximated in other smart way. In this contribution, we approximate the *equivalent effective SINR* by using a equivalent AWGN BPSK channel concept, i.e.,

Assume the SINRs of all symbols in a coded block are $SINR_n$ ($n = 1,\cdots,N$, where N is the number of symbols in a coded block), the *equivalent effective SINR* for each individual bit $SINR_{n,b}$ ($b = 1,\cdots,m_n$, where m_n is the number of bits carried by the nth symbol) can be approximated with following equation

$$P_{e,1}^{1}(SINR_{n,b}) = P_{e,b}^{m_n}(SINR_n), \quad (b = 1,\cdots,m_n) \quad , \tag{1}$$

Such that

$$SINR_{n,b} = P_{e,1}^{1\ -1}\left[P_{e,b}^{m_n}(SINR_n)\right]$$

where $P_{e,1}^{1}(\lambda)$ is the bit error probability function of BPSK, and $P_{e,b}^{m_n}(\lambda)$ is the bit error probability function of the b-th bit in the 2^{m_n}-QAM symbol with symbol SNR γ. $P_{e,1}^{1}(\lambda)$ and $P_{e,b}^{m_n}(\lambda)$ can be easily derived theoretically (Gray bit mapping is assumed):

Modulation	m_n	b	$P_{e,b}^{m_n}(\lambda)$
BPSK	1	1	$P_{e,1}^{1}(\lambda)=Q(\sqrt{2\lambda})$
QPSK	2	1, 2	$P_{e,1}^{2}(\lambda)=P_{e,1}^{2}(\lambda)=Q(\sqrt{\lambda})$
16QAM	4	1, 3	$P_{e,1}^{4}(\lambda)=P_{e,3}^{4}(\lambda)=\frac{1}{2}\sum_{i=1}^{2}Q(\sqrt{\frac{(2i-1)^2}{5}\lambda})$
		2, 4	$P_{e,2}^{4}(\lambda)=P_{e,4}^{4}(\lambda)=Q(\sqrt{\frac{1}{5}\lambda})+\frac{1}{2}\sum_{i=2}^{3}(-1)^{i}Q(\sqrt{\frac{(2i-1)^2}{5}\lambda})$
64QAM	6	1, 4	$P_{e,1}^{6}(\lambda)=P_{e,4}^{6}(\lambda)=\frac{1}{4}\sum_{i=1}^{4}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})$
		2, 5	$P_{e,2}^{6}(\lambda)=P_{e,5}^{6}(\lambda)=\frac{1}{2}\sum_{i=1}^{2}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})+\frac{1}{4}\sum_{i=3}^{4}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})$ $-\frac{1}{4}\sum_{i=5}^{6}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})$
		3, 6	$P_{e,3}^{6}(\lambda)=P_{e,6}^{6}(\lambda)=Q(\sqrt{\frac{1}{21}\lambda})+\frac{3}{4}\sum_{i=2}^{3}(-1)^{i}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})$ $-\frac{2}{4}\sum_{i=4}^{5}(-1)^{i}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})+\frac{1}{4}\sum_{i=6}^{7}(-1)^{i}Q(\sqrt{\frac{(2i-1)^2}{21}\lambda})$

Table 1. BER of each individual bit for 2^{m_n}-QAM with Gray mapping

By this operation, each QAM modulated channel can be decoupled into parallel BPSK modulated channels with corresponding bit-level *equivalent effective SINR*, as illustrated in **Figure** . It is obviously that the modulation schemes for different symbols are not necessary to be the same by using the proposed approximation method, i.e., different modulation schemes for different symbols are supported.

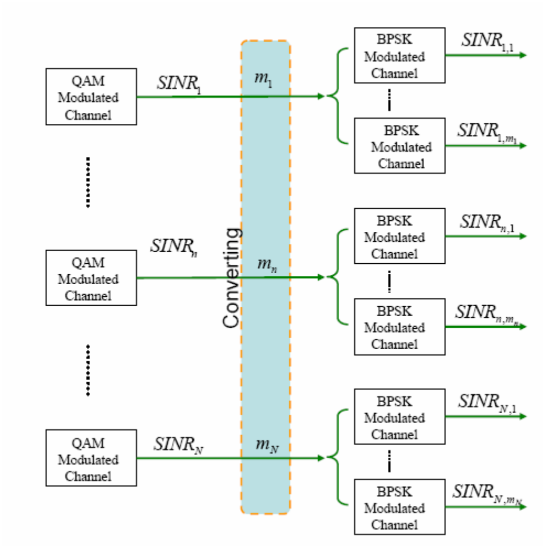

Figure 1 Converting from QAM domain to BPSK domain

To illustrate the mapping process from symbol-level SINR to bit-level SINR, figure 2 shows an example for a 64 QAM symbol. Assume that the SINR of the 64 QAM symbol is 26 dB. The SINRs of the 6 bits of the 64 QAM symbol are almost 10.336, 10.061, and 9.768 dB respectively. In practice, lookup table generated from these BER curves can be used instead of real time computation.

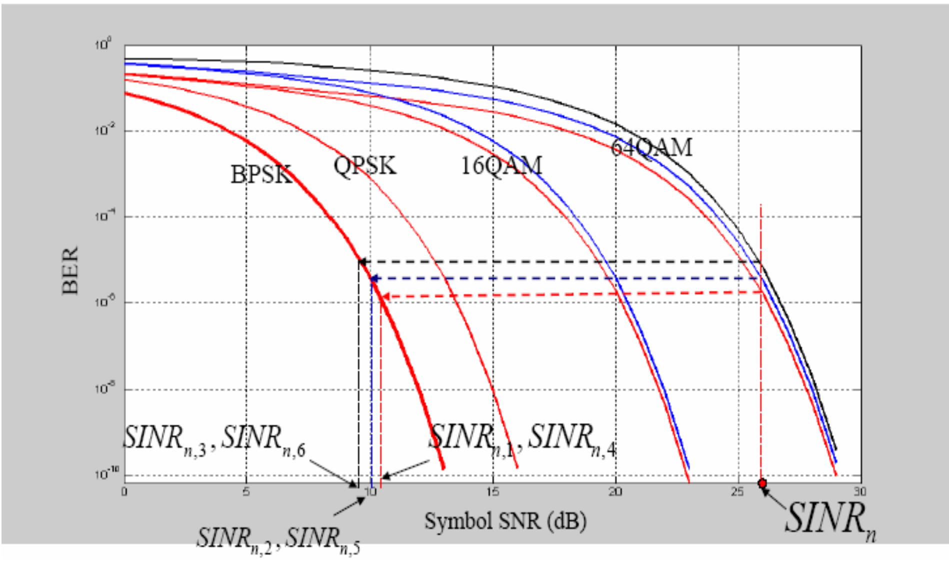

Figure 2 Mapping process for 64QAM as an example

2.2 Effective SINR

After the decoupling process, the exponential average operation is imposed on the *equivalent effective SINR* of each individual bit to get the Bit-wise Exponential ESM (BEESM) as

$$SINR_{eff} = -\ln\left\{ \frac{1}{\sum_{n=1}^{N} m_n} \sum_{n=1}^{N} \sum_{b=1}^{m_n} e^{-SINR_{n,b}} \right\}, \quad (2)$$

The effective SINR computation can be straightly extended to the MIMO system where per-tone post processing SINR is available by increasing the number of symbols in a codeword, N , in equation (2).

2.3 Mapping from Effective SINR to BLER

The mapping function between BLER and BEESM, $BLER = f_2(SINR_{eff})$, can be obtained by computer simulation. Figure 3 presents one example where the x-axis is the BEESM calculated based on the instant channel state, and y-axis is the simulated BLER. The simulated system is an OFDM system with 16QAM, 1/2 convlutional coding and 1000 bytes block length. 20 channel realizations are simulated and corresponding BLER vs. BEESM curves are drawn in this figure. It is shown that these curves are quite close to each other which indicates the goodness of the derived BEESM. The red cycle curve is obtained through curve fitting over these simulated curves for physical layer abstraction purpose.

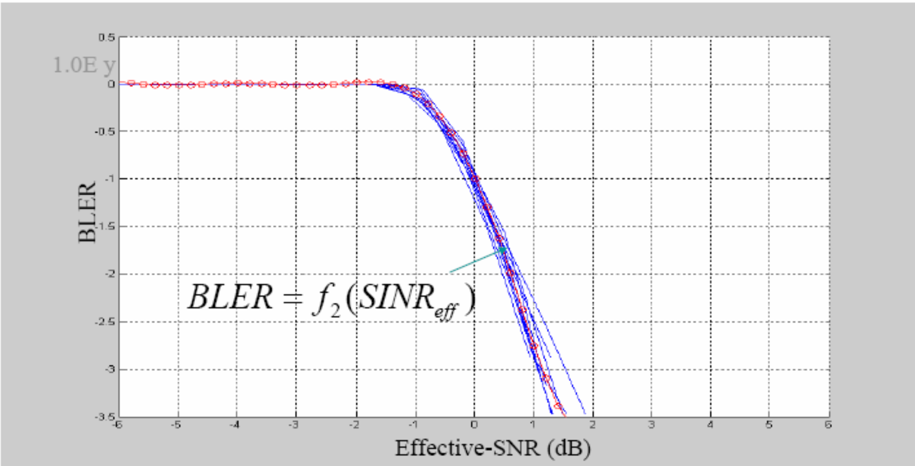

Figure 3. one example for generation of BLER vs. BEESM curve

Because BEESM is calculated based on the bit-level SINR, the BLER vs. BEESM curves are actually independent of the modulation scheme at all. To further elaborate the performance of the derived BEESM, a comprehensive computer simulation has been carried out for various system configurations:

A MIMO-OFDM system with 2 by 2 antennas. MCSs with one spatial stream and two spatial streams are considered. For all one-stream MCSs, STBC is used, while for MCSs with two-spatial streams, MMSE is used for detection. Convolutional coding with coding rate 1/2, 2/3, 3/4 and 5/6 are considered.

MCS Label	Code rate	Number of spatial streams	Modulation
0	1/2	1	BPSK
1	1/2	1	QPSK
2	3/4	1	QPSK
3	1/2	1	16QAM

4	3/4	1	16QAM
5	2/3	1	64QAM
6	3/4	1	64QAM
7	5/6	1	64QAM
8	1/2	2	Substream 1: BPSK, Substream 1: BPSK
9	1/2	2	Substream 1: QPSK, Substream 1: QPSK
10	3/4	2	Substream 1: QPSK, Substream 1: QPSK
11	1/2	2	Substream 1: 16QAM, Substream 1: 16QAM
12	3/4	2	Substream 1: 16QAM, Substream 1: 16QAM
13	2/3	2	Substream 1: 64QAM, Substream 1: 64QAM
14	3/4	2	Substream 1: 64QAM, Substream 1: 64QAM
15	5/6	2	Substream 1: 64QAM, Substream 1: 64QAM
16	1/2	2	Substream 1: 16QAM, Substream 1: QPSK
17	1/2	2	Substream 1: 64QAM, Substream 1: QPSK
18	3/4	2	Substream 1: 16QAM, Substream 1: QPSK
19	1/2	2	Substream 1: 64QAM, Substream 1: 16QAM
20	3/4	2	Substream 1: 64QAM, Substream 1: QPSK
21	3/4	2	Substream 1: 64QAM, Substream 1: 16QAM

Table 2 MCS for simulation

Figure 4 shows the BLER versus BEESM for all MCSs listed in above table.

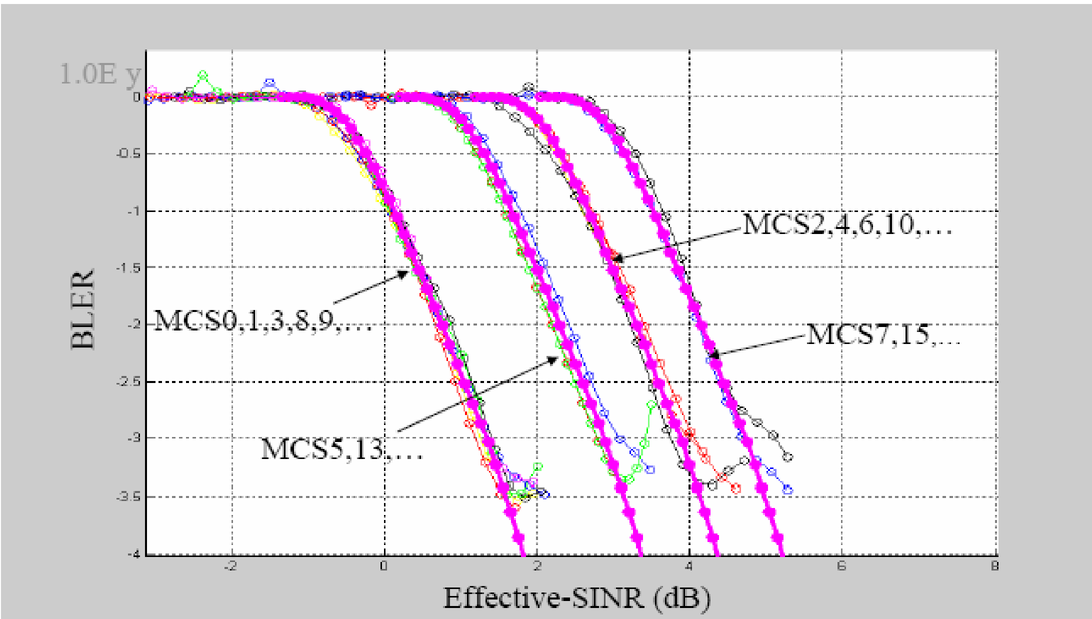

Figure 4 BLER versus SINR for different MCSs

It is shown that BLER versus BEESM curves are independent of modulation scheme but dependent on coding rate. In addition, it is also independent of the number of spatial stream.

3 Proposed Text Changes

Replace or enhance the content in section 4.3.3 Exponential ESM (EESM) by embedding following context:

The BEESM based on bit-wise SINR exponential averaging method is given by

$$SINR_{eff} = -\frac{1}{\alpha}\ln\left\{\frac{1}{\sum_{n=1}^{N} m_n}\sum_{n=1}^{N}\sum_{b=1}^{m_n} e^{-\Psi_b^{m_n}(SINR_n)*\alpha}\right\}$$

Where $SINR_n \ (n=1,\ldots,N)$ is the experienced SINR values of the coded block. m_n dontes the number of bits carried by the modulated symbol on tone n. $\Psi_b^{m_n}(\cdot)$ denotes the bit-level SINR mapping function for b-th bit of 2^{m_n}-QAM symbol. α is a factor which will be explained in Appendix. The utilized mapping functions $\Psi_b^{m_n}$ are listed in following table.

Modulation	m_n	b	$\Psi_b^{m_n}(x)$
BPSK	1	1	$\Psi_1^1(x) = x$
QPSK	2	1, 2	$\Psi_1^2(x) = \Psi_2^2(x) = \frac{1}{2}x$
16QAM	4	1, 3	$\Psi_1^4(x) = \Psi_3^4(x) = \frac{1}{2}\left\{Q^{-1}\left[\frac{1}{2}\sum_{i=1}^{2} Q(\sqrt{\frac{(2i-1)^2}{5}x})\right]\right\}^2$
		2, 4	$\Psi_2^4(x) = \Psi_4^4(x) = \frac{1}{2}\left\{Q^{-1}\left[Q(\sqrt{\frac{1}{5}x}) + \frac{1}{2}\sum_{i=2}^{3}(-1)^i Q(\sqrt{\frac{(2i-1)^2}{5}x})\right]\right\}^2$
64QAM	6	1, 4	$\Psi_1^6(x) = \Psi_4^6(x) = \frac{1}{2}\left\{Q^{-1}\left[\frac{1}{4}\sum_{i=1}^{4} Q(\sqrt{\frac{(2i-1)^2}{21}x})\right]\right\}^2$
		2, 5	$\Psi_2^6(x) = \Psi_5^6(x) = \frac{1}{2}\left\{Q^{-1}\left[\frac{1}{2}\sum_{i=1}^{2} Q(\sqrt{\frac{(2i-1)^2}{21}x}) + \frac{1}{4}\sum_{i=3}^{4} Q(\sqrt{\frac{(2i-1)^2}{21}x}) - \frac{1}{4}\sum_{i=5}^{6} Q(\sqrt{\frac{(2i-1)^2}{21}x})\right]\right\}^2$

		3, 6	$\Psi_3^6(x)=\Psi_6^6(x)=\frac{1}{2}\left\{Q^{-1}\left[\begin{array}{l}Q(\sqrt{\frac{1}{21}x})+\frac{3}{4}\sum_{i=2}^{3}(-1)^i Q(\sqrt{\frac{(2i-1)^2}{21}x})\\ -\frac{2}{4}\sum_{i=4}^{5}(-1)^i Q(\sqrt{\frac{(2i-1)^2}{21}x})+\frac{1}{4}\sum_{i=6}^{7}(-1)^i Q(\sqrt{\frac{(2i-1)^2}{21}x})\end{array}\right]\right\}^2$

4 Reference

[1] IEEE C802.16m-07_080 r3:Draft IEEE 802.16m Evaluation Methodology Document.

[2] IEEE C802.16m-07_097 : Link Performance Abstraction based on Mean Mutual Information per Bit (MMIB) of the LLR Channel

Appendix

This appendix shows similarity between MMIB ESM and BEESM.

Assume that equal modulation is applied, i.e., $m = m_1 = ... = m_N$, then

$$SINR_{eff} = -\ln\left\{\frac{1}{\sum_{n=1}^{N} m_n}\sum_{n=1}^{N}\sum_{b=1}^{m_n} e^{-\Psi_b^{m_n}(SINR_n)}\right\} = -\ln\left\{\frac{1}{mN}\sum_{n=1}^{N}\sum_{b=1}^{m} e^{-\Psi_b^{m}(SINR_n)}\right\}$$

It can be further written as

$$SINR_{eff} = -\ln\left\{1-\frac{1}{mN}\sum_{n=1}^{N}\sum_{b=1}^{m}(1-e^{-\Psi_b^{m}(SINR_n)})\right\}$$

$$= -\ln\left\{1-\frac{1}{N}\sum_{n=1}^{N}\left[\frac{1}{m}\sum_{b=1}^{m}(1-e^{-\Psi_b^{m}(SINR_n)})\right]\right\}$$

Let's define a new function

$$B_m(\gamma)=\frac{1}{m}\sum_{b=1}^{m}(1-e^{-\Psi_b^{m}(\gamma)})$$

It is not difficult to figure out that function $B_m(\gamma)$ corresponds to the mean mutual information function $I_m(\gamma)$ used in MMIB based method. In order to show the similarity between these two function, we draw the function curves as presented in *figure a* for different modulation schemes (different m). It is shown that they are almost same to each other except that there is a SINR offset between each pair of them.

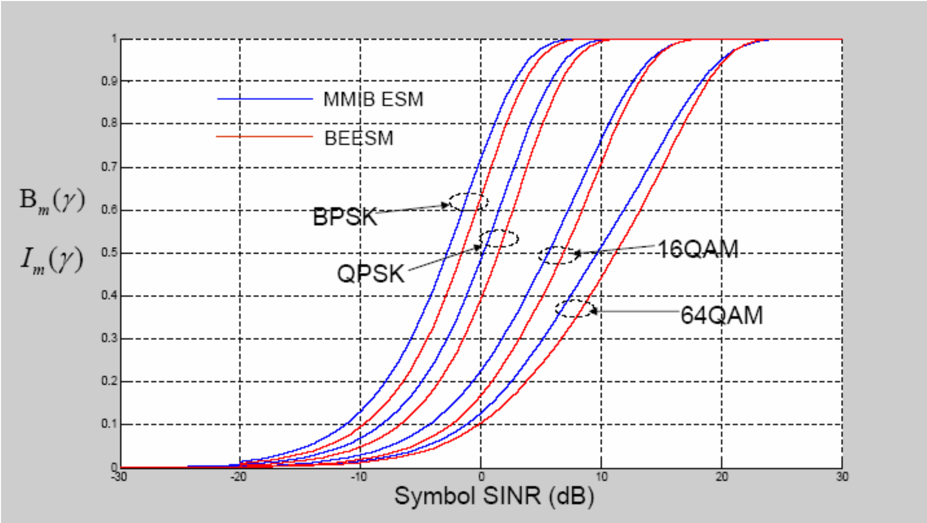

Figure a Comparison between MMIB ESM and BEESM

This offset can easily be removed by introducing an exact scaling factor α as follows

$$\widetilde{B}_m(\gamma) = \frac{1}{m}\sum_{b=1}^{m}(1 - e^{-\Psi_b^m(\gamma)*\alpha})$$

When $\alpha = 1.36$, we can draw another set of curves as follows.

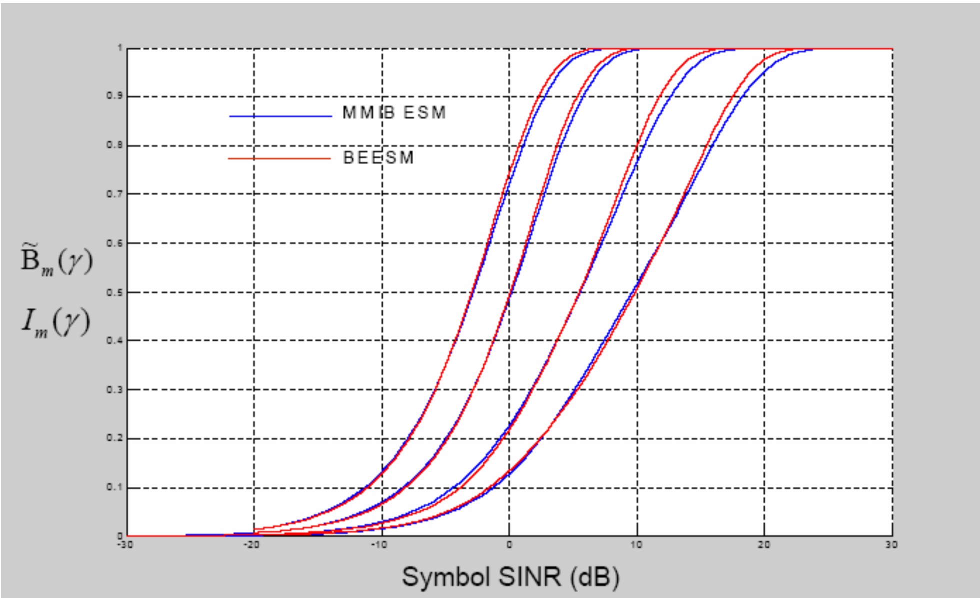

Figure b Matching between MMIB ESM and BEESM

It is quite important to point out that both MMIB-ESM and BEESM are derived based on certain heuristic ideas, one is constraint channel capacity, the other is Chernoff bound. It is unclear so far which one could theoretically performance better than the other. Fortunately, they are quite similar to each other with a tiny SINR offset. That's one of the reason we suggest to reserve an exact scaling factor α *for further investigation and comparison.*

Meanwhile, both MMIB-ESM and BEESM only remove the impact of different modulation schemes from the calculation of MMIB or BE. The impact of different coding rate (may be coding type as well) has not been taken into consideration. The scaling factor α *can also possibly be used to tackle this problem.*