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Re:	IEEE 802.16m-07/031 - Call for Comments on the 802.16m Evaluation Methodology Document.
Abstract	This document contains proposed text for the IEEE 802.16m-07/080r3 Draft Evaluation Methodology. The text defines per tone SINR computations for proposals with Cyclic Delay Diversity implementations.
Purpose	To review and adopt the proposed text in the next revision of the Evaluation Methodology Document.
Notice	
Release	
Patent Policy	

The Impact of Cyclic Delay Diversity on Per-tone SINR Computations

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1 Introduction

Cyclic delay diversity (CDD) is a spatial diversity technique that may be implemented within a coded OFDM system equipped with two or more transmit antennas [1,2]. The basic idea of a CDD implementation is to transform spatial transmit diversity into sub-carrier frequency diversity. A CDD implementation will result in an increase in a channel's frequency selectivity or equivalently a decrease in a channel's coherence bandwidth. As shown in the literature [1,2], the increased frequency selectivity can be exploited by a receiver's deinterleaving and decoding to improve link-level performance.

This contribution shows CDD's impact on computing received per-tone SINR for coded OFDM-based systems. Per-tone SINR computations used in system evaluations should account for the artificially induced frequency selective associated with a CDD implementation. This contribution proposes a new subsection for the current evaluation document [3] that accounts for proposed CDD implementations.

2 CDD Impact on Per-tone SINR Computations

2.1 Transmitted Base Station Signal

Figure 1 illustrates a exemplary baseband model of a BS transmitter equipped with a CDD implementation and mobile station receiver. For simplicity interference and noise blocks are not shown. All baseband signal processing operations including coding and the IFFT operation are subsumed by the OFDM Signal Source block. The time-domain OFDM signal is split into N_T antenna branches. The top branch carries the original time-domain signal, the other branches carry cyclically shifted versions of the original time-domain signal. The cyclic shift blocks multiply the OFDM signal by cyclic shifts $e^{-j2\pi n\delta_m/N}$ where the shift values in samples are defined as

$$\delta_m = m,\ m = 0, 1, 2, \ldots, N_T - 1 \tag{1}$$

Other shift values may be used but the maximal cyclic shift is bounded by the size of the IFFT. Note that CDD is independent of the cyclic prefix and allows an increase in channel frequency selectivity without an increase in cyclic prefix length [1,2].

Let N denote IFFT/FFT length, k denote discrete time, n a discrete frequency or sub-carrier index, complex-number $x^{(0)}(k)$ a time-domain sample, complex-number $X^{(0)}(n)$ a frequency-domain sample, and N_g the cyclic prefix length. The time-domain representation of a baseband OFDM symbol transmitted from the mth antenna branch is the length $(N + N_g)$ row-vector

$$\begin{aligned} \mathbf{x}_m &= \left[\begin{array}{cc} \mathbf{x}_m^{CP} & \mathbf{x}_m^{D} \end{array} \right] \\ m &= 0, 1, \ldots, N_T - 1 \end{aligned} \tag{2}$$

where the length N row-vector

$$\mathbf{x}_m^D = \underbrace{\left[\begin{array}{cccc} x^{(0)}(-\delta_m) & x^{(0)}(1-\delta_m) & \ldots & x^{(0)}(N-1-\delta_m) \end{array} \right]}_{\text{OFDM Data Symbol}} \tag{3}$$

contains time-domain data values and the length N_g row-vector

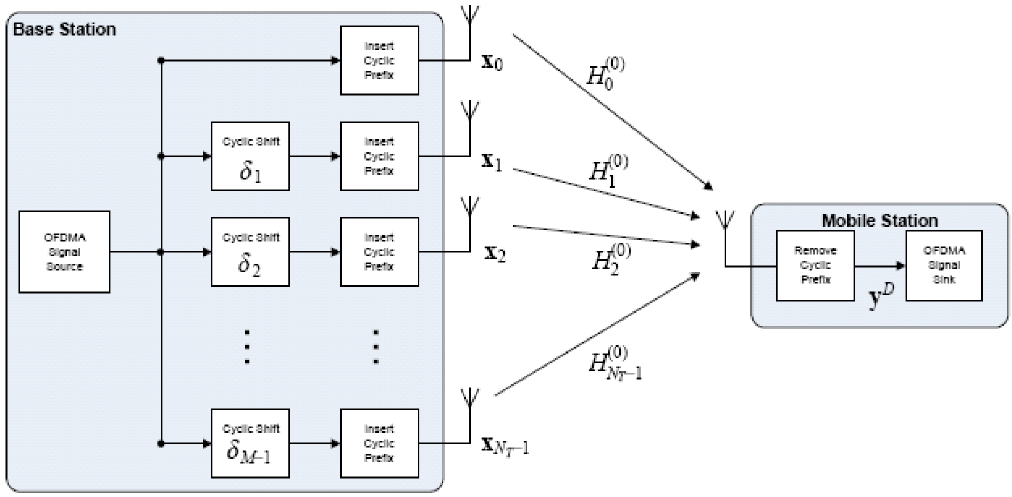

Figure 1: Simple conceptual block diagram of a CDD implementation.

$$
\begin{aligned}
\mathbf{x}_m^{CP} &= \left[\ x^{(0)}(-\delta_m - N_g) \quad \dots \quad x^{(0)}(-\delta_m - 1)\ \right] \\
&= \underbrace{\left[\ x^{(0)}(N - \delta_m - N_g) \quad \dots \quad x^{(0)}(N - 1 - \delta_m)\ \right]}_{\text{Cyclic Prefix}}
\end{aligned} \tag{4}
$$

the time-domain cyclic prefix values. Elements in $\mathbf{x}_m^D$ and $\mathbf{x}_m^{CP}$ are defined as

$$
\begin{aligned}
x^{(0)}(k - \delta_m) &= \frac{\sqrt{P_{tx}^{(0)}/N_T}}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-j2\pi n \delta_m/N} X^{(0)}(n) e^{j2\pi nk/N} \\
k &= 0, 1, \dots, N-1 \text{ and } m = 0, 1, \dots, N_T - 1
\end{aligned} \tag{5}
$$

Note that for equation (5) we have used the IFFT time shift property

$$
x^{(0)}(k - \delta_m) \overset{IFFT}{\longleftrightarrow} e^{-j2\pi n\delta_m/N} X^{(0)}(n) \tag{6}
$$

and for $\mathbf{x}_m^D$ and $\mathbf{x}_m^{CP}$ in equations (3) and (4) the IFFT periodicity property $x[-k] = x[N-k]$. The scaling factor $\sqrt{P_{tx}^{(0)}/N_T}$ is used so that the total transmit power from all N_T branches equals $P_{tx}^{(0)}$; this value is the same as that used for the SISO case in subsection 4.5.1 of [3].

2.2 Received Mobile Subscriber Signal

Assume that the guard interval of a received OFDM symbol has been removed and that the cyclic prefix is greater than the channel delay spread. The time-domain representation of the received data part of the OFDM symbol (see Figure 1) is then

$$
\mathbf{y}^D = \left[\ y^{(0)}(0) \quad y^{(0)}(1) \quad \dots \quad y^{(0)}(N-1)\ \right] \tag{7}
$$

where

$$
\begin{aligned}
y^{(0)}(k) &= \sqrt{P_{tx}^{(0)} P_{loss}^{(0)}} \sum_{m=0}^{N_T-1} \frac{H_m^{(0)}(k)}{\sqrt{N_T}} x^{(0)}(k-\delta_m) + \sum_{j=1}^{N_I} \sqrt{P_{tx}^{(j)} P_{loss}^{(j)}} H^{(j)}(k) x^{(j)}(k) + u^{(0)}(k) \qquad (8)\\
k &= 0, 1, \ldots, N-1
\end{aligned}
$$

The notation used is that defined in the current evaluation document [3]: N_I is the number of interferers, $P_{tx}^{(j)}$ is the total transmit power from jth BS (per sector) or MS, $P_{loss}^{(j)}$ is the distance dependent path loss including shadowing, antenna gain/loss and cable losses from the jth BS (per sector) or MS, $H^{(j)}(n)$ is the channel gain for the nth sub-carrier and jth user/sector, $X^{(j)}(n)$ is the frequency-domain symbol transmitted by the jth user/sector on the nth sub-carrier, $U^{(0)}(n)$ is the receiver thermal noise for the nth sub-carrier, modeled as AWGN with zero mean and variance σ^2.

Applying an FFT to $\mathbf{y}^D$ gives the frequency-domain representation of the received OFDM symbol

$$
\mathbf{Y}^D = \left[\begin{array}{cccc} Y^{(0)}(0) & Y^{(0)}(1) & \ldots & Y^{(0)}(N-1) \end{array} \right] \qquad (9)
$$

where

$$
\begin{aligned}
Y^{(0)}(n) &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\sqrt{P_{tx}^{(0)} P_{loss}^{(0)}} \sum_{m=0}^{N_T-1} \frac{H_m^{(0)}(k)}{\sqrt{N_T}} x^{(0)}(k-\delta_m) \right) e^{-j2\pi nk/N} \qquad (10)\\
&+ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\sum_{j=1}^{N_I} \sqrt{P_{tx}^{(j)} P_{loss}^{(j)}} H^{(j)}(k) x^{(j)}(k) \right) e^{-j2\pi nk/N}\\
&+ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} u^{(0)}(k) e^{-j2\pi nk/N}
\end{aligned}
$$

The first FFT in equation (10) above can be written as

$$
\begin{aligned}
& \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\sqrt{P_{tx}^{(0)} P_{loss}^{(0)}} \sum_{m=0}^{N_T-1} \frac{H_m^{(0)}(k)}{\sqrt{N_T}} x^{(0)}(k-\delta_m) \right) e^{-j2\pi nk/N} \qquad (11)\\
&= \frac{\sqrt{P_{tx}^{(0)} P_{loss}^{(0)}}}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\frac{1}{\sqrt{N_T}} \sum_{m=0}^{N_T-1} H_m^{(0)}(k) x^{(0)}(k-\delta_m) \right) e^{-j2\pi nk/N}\\
&= \sqrt{P_{tx}^{(0)} P_{loss}^{(0)}} \left(\frac{1}{\sqrt{N_T}} \sum_{m=0}^{N_T-1} H_m^{(0)}(n) e^{-j2\pi n\delta_m/N} \right) X^{(0)}(n)\\
&= \sqrt{P_{tx}^{(0)} P_{loss}^{(0)}} \tilde{H}^{(0)}(n) X^{(0)}(n)
\end{aligned}
$$

where

$$
\tilde{H}^{(0)}(n) = \frac{1}{\sqrt{N_T}} \sum_{m=0}^{N_T-1} H_m^{(0)}(n) e^{-j2\pi n\delta_m/N} \qquad (12)
$$

denotes the *effective or composite channel frequency gain* that subsumes both CDD transmit diversity and the physical channel. The second and third FFTs in equation (10) can be written as

$$\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\left(\sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}(k)x^{(j)}(k)\right)e^{-j2\pi nk/N}=\sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}(n)X^{(j)}(n) \quad (13)$$

and

$$\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}u^{(0)}(k)e^{-j2\pi nk/N}=U^{(0)}(n) \quad (14)$$

Hence by substitution we can write equation (10) as

$$Y^{(0)}(n)=\sqrt{P_{tx}^{(0)}P_{loss}^{(0)}}\tilde{H}^{(0)}(n)X^{(0)}(n)+\sum_{j=1}^{N_I}\sqrt{P_{tx}^{(j)}P_{loss}^{(j)}}H^{(j)}(n)X^{(j)}(n)+U^{(0)}(n) \quad (15)$$

To better see the induced frequency selectivity associated with CDD implementation we give an simple example for a two-element antenna array. For simplicity assume $P_{tx}P_{loss}=1$ and a channel with $H_m^{(0)}(n)=\sqrt{N_T}$ for all m then $\tilde{H}^{(0)}(n)=1+e^{-j2\pi n\delta_m/N}$. The magnitude-squared channel gain for a two-element antenna with one antenna delayed δ_m is then

$$\begin{aligned}|\tilde{H}^{(0)}(n)|^2 &= (1+e^{-j2\pi n\delta_m/N})(1+e^{j2\pi n\delta_m/N}) \\ &= 2\left[1+\cos\left(\frac{2\pi n\delta_m}{N}\right)\right],\ n=0,1,\ldots,N-1\end{aligned} \quad (16)$$

Figure 2 shows plots for $\delta_m=1,2,$ and 4 and $N=256$. Note that the maxima equal N_T^2 and that a larger value for δ_m results in higher frequency selectivity or a smaller coherence bandwidth.

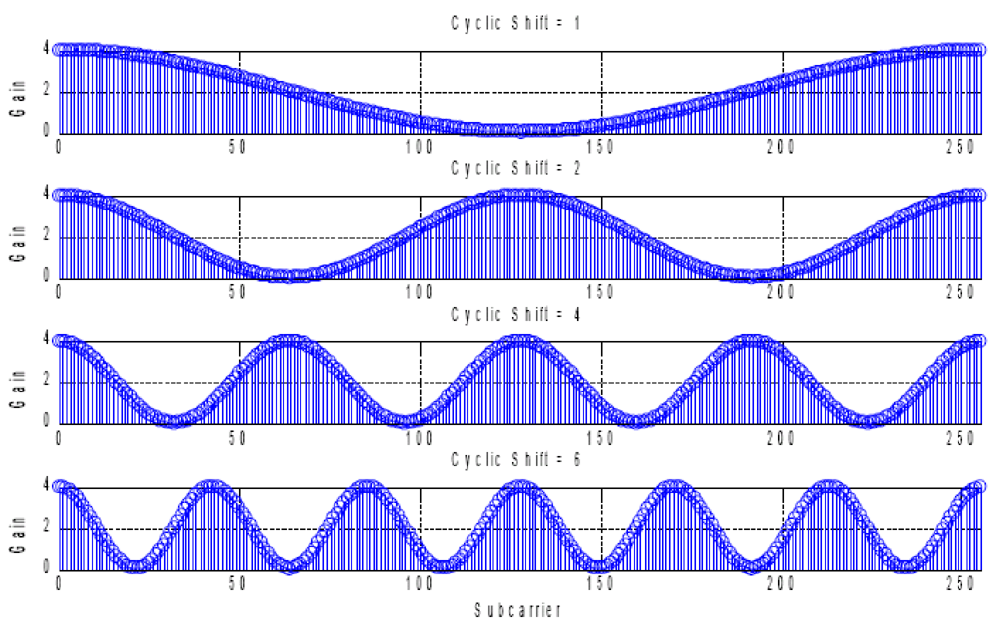

Figure 2: Simple example of composite magnitude-squared channel gain for a two antenna CDD implementation with $\tilde{H}^{(0)}(n)=1+e^{-j2\pi n\delta_m/N}$.

As another more general example we use N_T antenna elements with equally spaced delays $\delta_m=0,1,\ldots,N_T-1$. The magnitude-squared channel gain is

$$|\tilde{H}^{(0)}(n)|^2 = \left(\sum_{m=0}^{N_T-1} e^{-j\delta_m 2\pi n/N}\right)\left(\sum_{k=0}^{N_T-1} e^{j\delta_k 2\pi n/N}\right) \tag{17}$$
$$= \left(\frac{1-e^{-j2\pi n N_T/N}}{1-e^{-j2\pi n/N}}\right)\left(\frac{1-e^{j2\pi n N_T/N}}{1-e^{j2\pi n/N}}\right)$$
$$= \frac{1-\cos(2\pi n N_T/N)}{1-\cos(2\pi n/N)}$$

whose maxima of N_T^2 occur at $n = 0$ and $n = 255$. Figure 3 shows plots for $N_T = 3$ and 4. Note that as N_T increases the peaks at $n = 0$ and $n = 255$ dominate and the other sub-carriers become negligible.

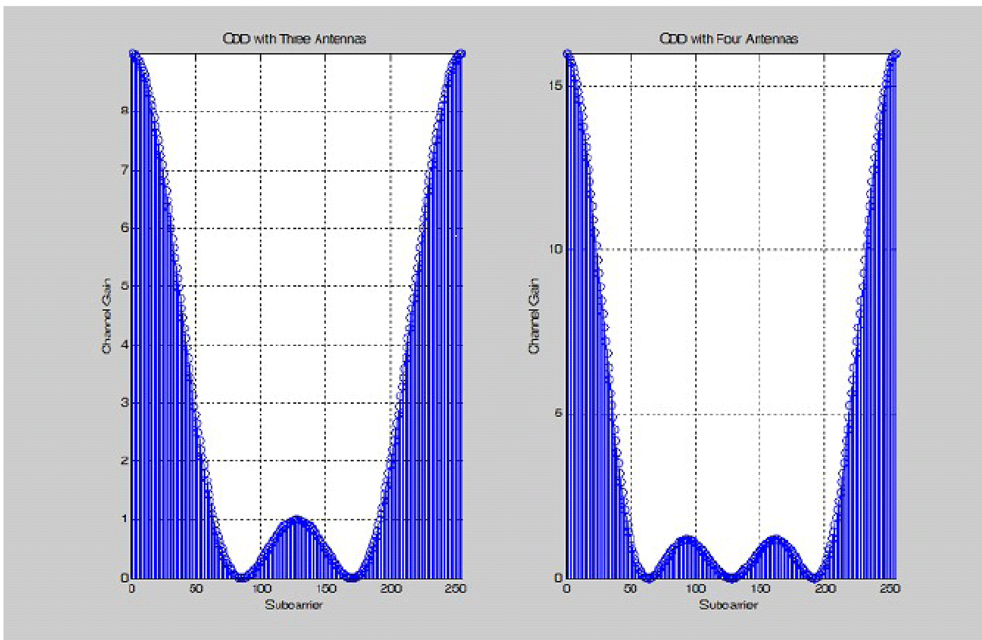

Figure 3: Composite magnitude-squared channel gain for three and four antenna CDD implementations with equal cyclic shifts and $\tilde{H}^{(0)}(n) = \sum_{m=0}^{N_T-1} e^{-j2\pi n\delta_m/N}$.

3 Proposed Text

Add subsection 3.1 of this document somewhere within section 4.5 of IEEE C802.16m-07/089r3:

3.1 Per-tone Post Processing SINR for MISO and MIMO with CDD

For MISO (multi-input, single-output) and MIMO proposals with CDD implementations the effective CDD or composite channel gains should be used for per tone SINR computations. For example, the nth tone post processing SINR for a MISO system with a CDD implementation may be defined as

$$SINR^{(0)}(n) = \frac{P_{tx}^{(0)} P_{loss}^{(0)} \left|\tilde{H}^{(0)}(n)\right|^2}{\sigma^2 + \sum_{j=1}^{N_I} P_{tx}^{(j)} P_{loss}^{(j)} \left|H^{(j)}(n)\right|^2}$$

where

$$\tilde{H}^{(0)}(n) = \frac{1}{\sqrt{N_T}} \sum_{m=0}^{N_T-1} H_m^{(0)}(n) e^{-j2\pi n \delta_m / N}$$

is defined as the effective CDD or composite channel gain that incorporates the physical channel gains $H_m^{(0)}(n)$ and the artificially induced frequency selective associated with a CDD cyclic shifts $e^{-j2\pi n \delta_m / N}$. Example cyclic shift values are

$$\delta_m = m, \; m = 0, 1, 2, \ldots, N_T - 1$$

where the cyclic shift $\delta_0 = 0$ is assumed to be the reference antenna in a CDD implementation.

4 References

1. G. Bauch and J. S. Malik. Orthogonal frequency division multiple access with cyclic delay diversity. In ITG Workshop on Smart Antennas, Munich, Germany, March 2004.

2. M. Bosserty, A. Huebnery, F. Schuehleiny, H. Haasz, and E. Costaz, On Cyclic Delay Diversity in OFDM Based Transmission Schemes, 7th International OFDM-Workshop, Hamburg, Germany, 2002.

3. IEEE C802.16m-07/080r3, Draft IEEE 802.16m Evaluation Methodology Document, 2007-08-28.