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Re:	Call for contributions for 802.16m Evaluation Methodology
Abstract	Provides an approach to model the channel estimation errors in MIMO system simulations
Purpose	For discussion and approval by 802.16 TGm
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Channel Estimation Error Modeling for System Simulations

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Purpose

As one of the most important form of actual receiver impairments, channel estimation (CE) error at the receiver (MS) and its modeling is critical to the evaluation of system throughput. In general, the CE error may impact the receiver performance differently for different types of receiver processing. The key is to establish a common frame work that can be built upon to accommodate various receiver processing techniques. Also important is to recognize the fact that CE error depends on these factors typically:

- Type of channel estimator (e.g., MMSE, LS)
- Time-frequency pilot pattern (e.g., pilot number and positions, often power-boosted as well)
- Design parameters (e.g., assumed SNR for MMSE filter coefficients, 2D or two 1D MMSE, time-domain filter assumptions)
- Actual channel behavior (e.g., Delay spread, Doppler)

- Specifics of CE implementation

In the current EVM document [3], a method of modeling channel estimation impairment is proposed, but for SIMO only. It is a bit unclear on how the post-processing (MRC in this SIMO case) is considered.

In this contribution, we propose a more generalized approach to model the channel estimation error. The parameterized approach allows us to model CE error under any receiver processing such as MRC and MIMO, while it is also flexible enough to allow proponents to simply tailor two parameters to accommodate different CE implementations and pilot patterns. By sharing these parameters, it is also easy to verify/reproduce results from each other, a process that can be separated from the discussion of whether the CE is reasonable or not. In addition, the proposal gives a simulation procedure on how to incorporate the estimation error in the MI or EESM based system simulations.

Channel Estimation Modeling

In this section, we will show how different types of CE can affect error modeling. We will start with a regular pilot pattern to illustrate the concept and then extends to more generic context.

LS Approximation[1] (Uniform Pilot Pattern)

When uniform and periodic pilot structure is available (for example, preamble), a DFT based channel estimation approach may be used (just as an example). In this case, the following approximation can be applied to channel estimation mean squared error (MSE).
If the pilot subcarriers are assumed to be uniformly spaced on every K subcarriers, the received signal after being divided by the pilot symbol at the *pilot subcarrier* can be expressed as

$$\mathbf{Y} = \mathbf{Fh} + \mathbf{n} \tag{1.1}$$

where **h** is a 1xL vector of the channel taps in the time-domain (i.e., L taps) , **n** is the complex AWGN noise vector on the pilot subcarriers and **F** is the P by L DFT matrix (where $P = N/K,\ N = \text{ FFT Size}$) given by

$$\mathbf{F}(m,n) = \exp^{-j\frac{2\pi mn}{P}} \tag{1.2}$$

Note the LS channel estimator requires P>L and the estimate is given by

$$\begin{aligned} \hat{h} &= (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H Y \\ &= h + (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{n} \\ \hat{\mathbf{H}} &= \Phi\hat{h} = \mathbf{H} + \Phi(F^H F)^{-1}F^H\mathbf{n} \end{aligned} \tag{1.3}$$

where Φ is the N by L DFT matrix, given by

$$\Phi(m,n) = \exp^{-j\frac{2\pi mn}{N}} \tag{1.4}$$

The MSE of this channel estimate is then given by

$$\begin{aligned} &E[|\mathbf{H} - \hat{\mathbf{H}}|^2] \\ &= E[(\Phi(F^H F)^{-1}F^H\mathbf{n})(\Phi(F^H F)^{-1}F^H\mathbf{n})^H] \\ &= \sigma^2\Phi(F^H F)^{-1}\Phi^H \\ &= \frac{\sigma^2}{P}\Phi\Phi^H = \frac{L}{P}diag(\sigma^2) \end{aligned} \tag{1.5}$$

We denote this approximation to channel estimation error as follows

$$\xi_{LS} = \frac{L}{P}\sigma^2 \tag{1.6}$$

[1] Reference [2] provides the derivation that is reproduced here for convenience.

MMSE Approximation

In this section, we briefly describe computation of estimation error when a channel estimator uses *N* pilots. Denote H_i as the true channel coefficient at data location i, $\hat{H}_i$ as the corresponding estimates, and $\vec{H}_p = (H_{p_1}, H_{p_2}, ..., H_{p_N})$ as the vector of N "noisy" pilots channel estimates obtained by simply dividing the received pilot signals by the pilot symbols. After the linear MMSE "smoothing", the estimate is given by

$$\hat{H}_i = C_{\sigma_d} \vec{H}_p^T \tag{1.7}$$

where C_{σ_d} is the vector of the Wiener coefficients optimized under the assumed noise variance σ_d^2. The MMSE filter can be obtained from standard Wiener theory as

$$C_{\sigma_d} = (R_\infty + \sigma_d^2 I)^{-1} V \tag{1.8}$$

where $R_\infty = E[H_p^* H_p^T]$ is the autocorrelation matrix of pilot subcarriers under no noise (the channel power are normalized such that $R_\infty(1,1) = 1$ above for simplicity only), $V = E[H_i H_p^*]$ is the cross correlation vector of a data subcarrier with the pilot subcarriers, and σ_d^2 is the noise variance derived from an assumed SNR (pilot boosting needs to be appropriately accounted for) . The resulting mean square error with an assumed noise power of σ_d^2 at a data location i in the data zone on which CE is performed, can be obtained as

$$\begin{aligned} \xi_i^\sigma &= E[|H - \hat{H}_i|^2] \\ &= E[|H|^2] - 2\,\mathrm{Re}\{C_{\sigma_d}^H V\} + C_{\sigma_d}^H (R_\infty + \sigma^2 I) C_{\sigma_d} \\ &= E[|H|^2] - 2\,\mathrm{Re}\{C_{\sigma_d}^H V\} + C_{\sigma_d}^H R_\infty C_{\sigma_d} + \sigma^2 C_{\sigma_d}^H C_{\sigma_d} \\ &= 1 - 2\,\mathrm{Re}\{C_{\sigma_d}^H V\} + C_{\sigma_d}^H R_\infty C_{\sigma_d} + \sigma^2 C_{\sigma_d}^H C_{\sigma_d} \\ &= MSE_{i,\infty}^{\sigma_d} + \gamma_i^{\sigma_d} \sigma^2 \end{aligned} \tag{1.9}$$

where $MSE_{i,\infty}^{\sigma_d}$ (sum of the first three terms) and $\gamma_i^{\sigma_d}$ (the last term) are, respectively, the asymptotic mean square error and post-processing noise gain at data subcarrier i, where the super script σ_d denotes that the MMSE filter coefficients are derived based on the assumption of a specific noise variance. The asymptotic MSE, $MSE_{i,\infty}^{\sigma_d}$,contains all factors other than noise amplification and it is mainly the interpolation error due to different mismatches in a practical channel estimation design (such as the mismatch between the assumed frequency-domain correlation and the actual one, the mismatch between assume noise power and the actual value, or any modeling error). It is clear that the channel estimation error can be parameterized as follows

$$\xi_{MMSE} = a + (9/16) b \sigma^2 \tag{1.10}$$

where a is the mean asymptotic MSE due to SNR assumption mismatch and b is the mean noise gain of the filter, where the mean is taken over the data subcarriers. The factor 9/16 is introduced to account for the pilot boosting defined in IEEE 802.16e [1]. Note that the error in (1.11) is relative to a "normalized" channel (i.e., $R_\infty(1,1) = 1$), while it is generally understood that the first term (i.e.,"a") will be proportional to the actual mean channel power, i.e.,

$$\xi_{MMSE} = aE\left|H\right|^2 + b\sigma^2 / B \tag{1.12}$$

where B is the pilot subcarrier power boosting compared to data.

The Impact of Estimation Errors in Link Performance Modelling

Let us denote the channel estimation error at subcarrier i as

$$H_i - \hat{H}_i = e_i \tag{1.13}$$

We assume that e_i is complex Gaussian and uncorrelated to η_i, which is the AWGN noise component of the received signal. The received vector can be written as

$$y_i = \hat{H}_i s_i + e_i s_i + \eta_i \tag{1.14}$$

The channel estimation error can be treated as a component that contributes as an additional source of distortion independent of the noise component. Then, the decoder SNR with channel estimation error is given by

$$\begin{aligned} SNR_{est} &= \frac{E[|\hat{H}|^2]}{\sigma_e^2 + \sigma^2} \\ &\approx \frac{E[|H|^2]}{\sigma_e^2 + \sigma^2} \\ &= \frac{E[|H|^2]}{aE[|H|^2] + [1 + (1/B)b]\sigma^2} \end{aligned} \tag{1.15}$$

where $\sigma_e^2 = \xi_{MMSE}$ as defined previously.

Note that above SNR denotes the long term averaged SNR. However, the MI or EESM method are used to better predict the coded link performance because they account for the SNR variation across subcarriers. In this case, a different expression is required for obtaining effective SNR metrics. The following per-subcarrier decoder SNR metric can be defined

$$\begin{aligned} SNR_i &= \frac{|\hat{H}_i|^2}{aE[|H|^2] + [1 + (9/16)b]\sigma^2} \\ &\approx \frac{|H_i|^2}{aE[|H|^2] + [1 + (1/B)b]\sigma^2} \end{aligned} \tag{1.16}$$

Note that for either LS or MMSE channel estimator, the estimation error typically varies across the subcarriers, especially at the edges of the band or the edges of the clusters if the estimator operates on a per-cluster basis. For simplicity, we could model the error as uniformly distributed across the entire band.

Extension to SIMO/MIMO

SIMO

For the single stream 1x2 SIMO case, the received data signal is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{1.17}$$

Here, $E[|h_{ij}|^2] = E_s$, the signal power, but $E[|s_1|^2] = 1$ to retain normalization of the total transmit power at 1. With channel estimation, it can be modified as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
$$= \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} e_{11}s_1 + n_1 \\ e_{21}s_1 + n_2 \end{bmatrix}$$
$$= \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \quad (1.18)$$
$$\triangleq \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix}$$

We then have

$$\sigma_{ce,i}^2 = E[|n_i^{ce}|^2] = MSE_{1,n_r} + \sigma_{n_i}^2 \quad (1.19)$$

where σ_{ce}^2 is now the effective combined noise variance to be used in the MRC combining equations after appropriate scaling. MSE_{n_t,n_r} is the MSE on transmit antenna n_t and receive antenna n_r

2x2 MIMO

Here we provide modified signal expressions with channel estimation. They can be adapted to general $N \times M$ MIMO configuration. The received signal on data subcarriers is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (1.20)$$

where $E[|h_{ij}|^2] = 1$. Further $E[|s_1|^2] = E[|s_2|^2] = 1/2$ to normalize the total transmit power (i.e. the sum of the transmit power over both antennas) to 1. Note that this does provide an implicit pilot boosting, since pilots are transmitted in SISO mode on each antenna, but this factor is recognized in the derivation which follows. With channel estimation, the above expression can be modified to

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
$$= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} e_{11}s_1 + e_{12}s_2 + n_1 \\ e_{21}s_1 + e_{22}s_2 + n_2 \end{bmatrix}$$
$$= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \quad (1.21)$$
$$\triangleq \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix}$$

which separates the known component of signal and the error due to channel estimation. Further, the last expression neglects the minor degradation in received signal component, since the loss of performance can primarily be attributed to the increase in effective noise variance. We have

$$\sigma_{ce,n_r}^2 = E[|n_{n_r}^{ce}|^2] = \frac{1}{2} MSE_{1,n_r} + \frac{1}{2} MSE_{2,n_r} + \sigma_{n_r}^2 \quad (1.22)$$

MSE could potentially be different on the different transmit antennas with time processing or if different pilot patterns are used, but typically can be assumed to be the same.

With the above modified signal model, the approach is then similar to that with ideal channel estimation. For example, the post processing SNRs can be computed starting from this model and then input to link abstraction methods

Proposed Text

Proposed text to be added to 4.5.7

-----------*Begin Proposed Text* ---

Channel estimation error can be modeled in a two-step approach as follows:

Step 1: The channel estimation MSE is modeled as

$$MSE = a + (1/B)b\sigma^2 \tag{1.23}$$

where a, b are parameters that represent asymptotic interpolation error and noise gain respectively. B represents the power boosting of pilot over data.

Step 2: Obtain Post Processing SNRs for a given transmission mode and receiver type as follows

SISO

The per subcarrier SNR is modeled as

$$SNR_i = \frac{|H_i|^2}{aE[|H|^2] + [1 + (1/B)b]\sigma^2} \tag{1.24}$$

SIMO

For the single stream 1x2 SIMO case, the received data signal is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{1.25}$$

Here, $E[|h_{ij}|^2] = E_s$, the signal power, but $E[|s_1|^2] = 1$ to retain normalization of the total transmit power at 1. With channel estimation, it can be modified as

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \\ &= \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} e_{11}s_1 + n_1 \\ e_{21}s_1 + n_2 \end{bmatrix} \\ &= \begin{bmatrix} \hat{h}_{11} \\ \hat{h}_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \\ &\triangleq \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} s_1 + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \end{aligned} \tag{1.26}$$

We then have

$$\sigma_{ce,i}^2 = E[|n_i^{ce}|^2] = MSE_{1,n_r} + \sigma_{n_i}^2 \tag{1.27}$$

where σ_{ce}^2 is now the effective combined noise variance to be used in the MRC combining equations after appropriate scaling. MSE_{n_t,n_r} is the MSE on transmit antenna n_t and receive antenna n_r

2x2 MIMO

Here we provide modified signal expressions with channel estimation. *They can be adapted to general* $N \times M$ *MIMO configuration*. The received signal on data subcarriers is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{1.28}$$

where $E[|h_{ij}|^2]=1$. Further $E[|s_1|^2]=E[|s_2|^2]=1/2$ to normalize the total transmit power (i.e. the sum of the transmit power over both antennas) to 1. Note that this does provide an implicit pilot boosting, since pilots are transmitted in SISO mode on each antenna, but this factor is recognized in the derivation which follows. With channel estimation, the above expression can be modified to

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \\ &= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} e_{11}s_1 + e_{12}s_2 + n_1 \\ e_{21}s_1 + e_{22}s_2 + n_2 \end{bmatrix} \\ &= \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \\ &\triangleq \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1^{ce} \\ n_2^{ce} \end{bmatrix} \end{aligned} \tag{1.29}$$

which separates the known component of signal and the error due to channel estimation. Further, the last expression neglects the minor degradation in received signal component, since the loss of performance can primarily be attributed to the increase in effective noise variance. We have

$$\sigma_{ce,n_r}^2 = E[|n_{n_r}^{ce}|^2] = \frac{1}{2}MSE_{1,n_r} + \frac{1}{2}MSE_{2,n_r} + \sigma_{n_r}^2 \tag{1.30}$$

MSE could potentially be different on the different transmit antennas with time processing or if different pilot patterns are used, but typically can be assumed to be the same.

With the above modified signal model, the approach is then similar to that with ideal channel estimation. The post processing SNRs are computed starting from this model and then input to link abstraction methods

Table Format

Filter Design Set	Permutation/MIMO Mode	Pilot Pattern	Doppler (Kmph)	Other Detail	Model Parameters
1	PUSC – STC Zone	Common	3	Time Filtering over T symbols	(a,b)
2					
3					
4					

Table 1 – Modes and Parameters for Channel Estimation Model

An example table is provided above. When system results are provided in a contribution with channel estimation schemes turned on, it would be sufficient to provide the above table of parameters. Different filter designs could correspond to different permutation modes like PUSC, AMC, different pilot patterns like common pilots or dedicated pilots, SNRs, Doppler, channels etc. The parameterization can be implementation dependent and is recommended to be provided with the simulation results when channel estimation is used. Though they are specific to individual implementations, they have enough information to harmonize or calibrate results.

Obtaining Parameters from Simulation

The parameters can be derived for each filter design set (i.e., a fixed channel estimation filters) by

1. Running the channel estimator at a set of SNRs.
2. Storing the MSE of channel estimation at each of these SNRs.
3. Performing a simple linear least squares curve fit to this data.

The parameters can be obtained from a full link simulation or a simple simulation with channel estimation.
-----------*End Proposed Text* ---

Simulation Results

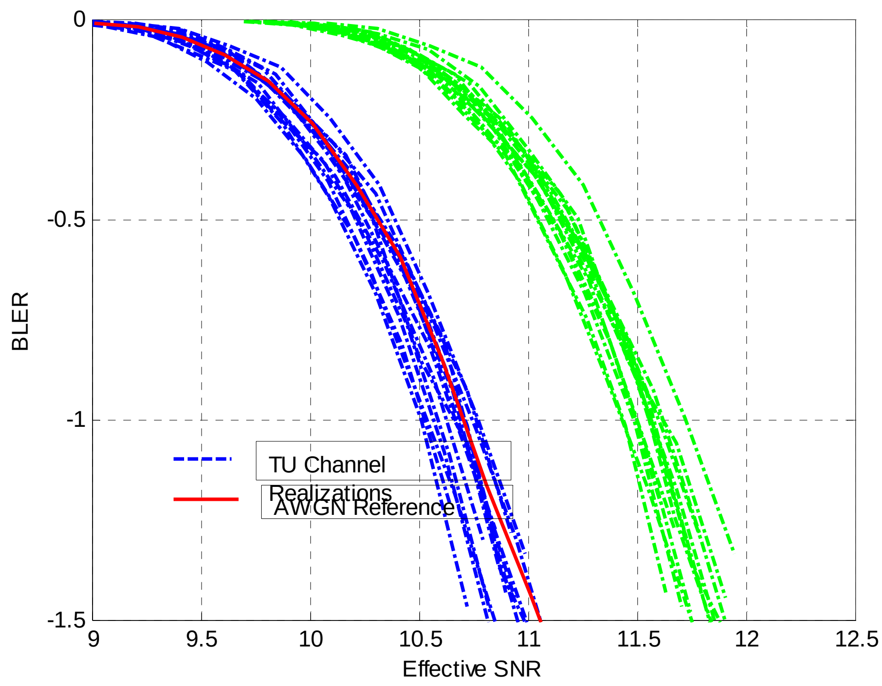

Figure 1 - 16QAM R-3/4, PCE

Legend – Green Curves show prediction if MSE is not compensated in MMIB computation.

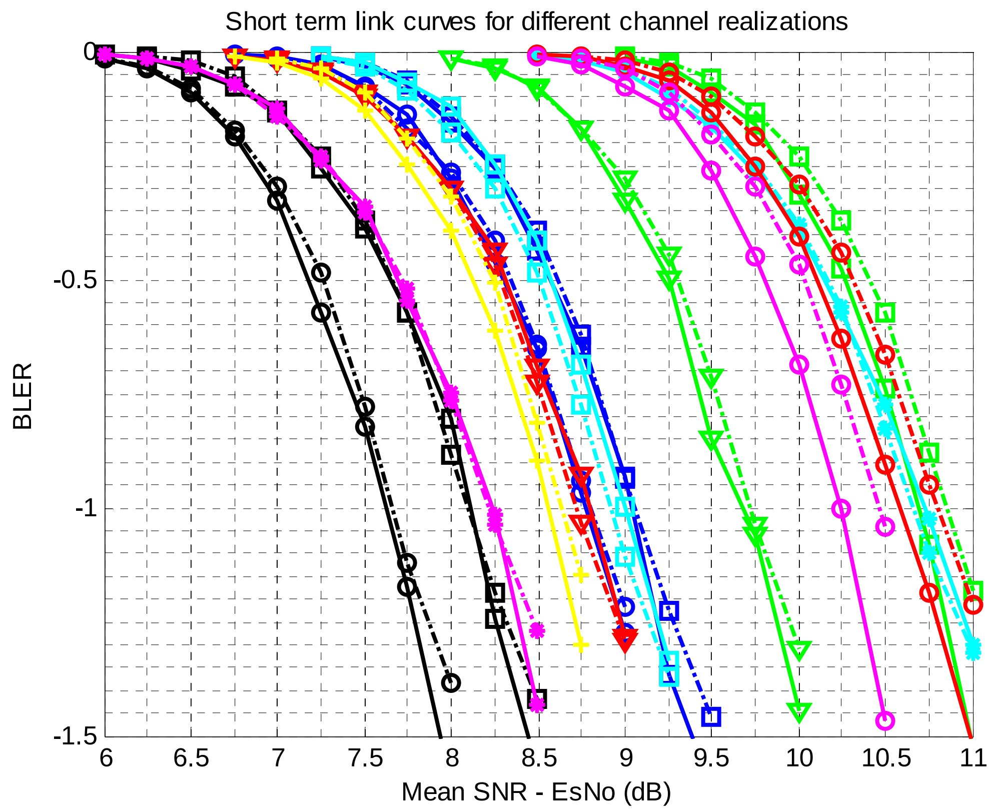

Figure 2 -16QAM R-3/4 Short term link curves, PCE

Legend: Dotted - Prediction, Solid – Actual Simulation, Many Realizations

References

[1] IEEE Std 802.16 – 2004, "IEEE Standard for Metropolitan Area Networks - Part 16: Air Interface for Fixed Broadband Wireless Systems"

[2] C30-20040823-061, "Derivation of equations used in the computation of symbol SNR and equivalent SNR for OFDM based transmission", 3GPP2.

[3] IEEE 802.16m Evaluation Methodology 037r1.