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Abstract	Differential precoding can reduce the uplink overhead considerably. Differential precoding works by feeding back the variation in the channel instead of directly feeding back the channel condition. Typically quantizing the channel variations requires much smaller codebook size than quantizing the channel itself.
Purpose	Discussion and Decision
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Differential Precoding Codebook

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1 Introduction

In downlink transmission, Precoding is used to achieve higher spectral efficiencies and lower packet error rate. The uplink overhead is one of the major constraint in design of precoding schemes. In this contribution, we propose the use of differential precoding as a means to reduce the uplink feedback overhead. In differential precoding, the variation in the channel is fed back by using a quantization codebook. Typically, the quantization of the channel variation requires smaller codebook.

In differential Precoding two codebooks are used: a main codebook and a differential codebook. The differential codebook adds more resolution to the main codebook or facilitate the precoder selection from the main codebook. Although joint optimization of the main codebook and differential codebook is desirable to achieve the best performance, it is always possible to design a differential codebook for a given choice of the main codebook. The proposed differential precoding scheme and simulation results are presented in this contribution.

2 Precoding Codebook

In this section, we discuss the precoding codebook design. First, we briefly address the conventional codebook design in which only one precoder index is fed back for the resource blocks (RBs) and different RBs are treated individually. Such codebook can also be used to report a single precoding index for a group of RBs. Next, we present multi-resolution codebook, where for a group of adjacent RBs, the precoder selection algorithm first selects a common precoder index from a low-resolution codebook and then for each RBs within the group a high-resolution codebook is used to fine tune the precoders for each RB. Thus, in this work, the conventional codebook is referred to as the single-resolution codebook.

The concept of multi-resolution codebook may be extended to multiple level of codebook resolution. Nonetheless, in this work, we primarily focus on the multi-resolution codebooks with only two resolution levels: the low-resolution and the high-resolution. It will be shown that by using such multi-resolution codebook similar performance can be achieved with lower feedback load in comparison to a system using a single-resolution codebook. Furthermore, both single-resolution or multi-resolution require almost similar memory to store the codebooks and the complexity of the precoder selection using either of the codebooks is also comparable.

2.1 Single-resolution codebook

We drop the subscript c in this section and let $\mathbf{H}$ denote the channel matrix of any subcarrier. For a resource block, we generally use the channel matrix of the center tone for the sake of precoder selection. For a given transmission rank r and B bits of PMI feedback, the codebook design problem is formulated as finding the set $\mathcal{Q} = \{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_{2^B}\}$ of $N_T \times r$ semi-unitary matrices that is a solution to the optimization problem given by

$$C = \max_{\mathcal{Q}} \mathbb{E}\left[\max_{\mathbf{Q}\in\mathcal{Q}} \log\det\left(\mathbf{I} + \frac{P}{N_T}\mathbf{H}\mathbf{Q}\mathbf{Q}^*\mathbf{H}^*\right)\right]. \quad (1)$$

Conventionally, the maximization problem is solved by equivalently minimizing the chordal distance between the dominant right singular vectors of the channel and the corresponding quantized vectors, where the chordal distance is defined as

$$d_{chordal}(\mathbf{Q}, \mathbf{V}) = \frac{1}{\sqrt{2}}||\mathbf{V}\mathbf{V}^* - \mathbf{Q}\mathbf{Q}^*||_F = \sqrt{N_T - ||\mathbf{V}^*\mathbf{Q}||_F^2}. \quad (2)$$

and $\mathbf{U}\mathbf{D}\mathbf{V}^*$ is the partial SVD of $\mathbf{H}$ obtained by retaining only the r right singular vectors that correspond to the r largest singular values of $\mathbf{H}$. Other distance metric such as Fubini-study metric and p-metric defined between the subspaces $\mathbf{V}$ and $\mathbf{Q}$ on the Grassmanian manifold $G(N_T, r)$ may also be used.

If a single-resolution codebook is used, the precoder $\mathbf{Q}$ for the channel $\mathbf{H}$ is obtained from the codebook $\mathcal{Q}$ such that $\log\det\left(\mathbf{I} + \frac{P}{N_T}\mathbf{H}\mathbf{Q}\mathbf{Q}^*\mathbf{H}^*\right)$ is maximized. This capacity measure may be modified accordingly depending on the type of the receiver structure, e.g., if LMMSE is used.

2.2 Multi-resolution codebook

In a multi-resolution codebook, the codebook $\mathcal{Q} = \{\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_q\}$ consists of q codebooks where the codebook $\mathcal{Q}_{i+1}$ is used to add more resolution to the prior set of codebooks $\mathcal{Q}_1, \ldots, \mathcal{Q}_i$.

By using a multi-resolution codebook, finding the optimal precoder for a given channel realization $\mathbf{H}$ is performed as follows. First, we find the optimal precoder $\mathbf{Q}_1(\mathbf{H}) \in \mathcal{Q}_1$ for a given channel realization $\mathbf{H}$. Then, we find the optimal precoder $\mathbf{Q}_2(\mathbf{H}, \mathbf{Q}_1) \in \mathcal{Q}_2$ that depends both on the previous precoder $\mathbf{Q}_1$ and the channel matrix $\mathbf{H}$. Depending on the required resolution, this action is successively performed to find, e.g., the precoder $\mathbf{Q}_i(\mathbf{H}, \mathbf{Q}_1, \ldots, \mathbf{Q}_{i-1}) \in \mathcal{Q}_i$, $i \leq q$. It should be pointed out that the multi-resolution codebook is optimized based on the channel statistics. In general, different resolution may be optimized for different channel statistics. For example, in an OFDM systems, the first codebook $\mathcal{Q}_1$ may be optimized by considering the statistics of the center tone in any RBs, and the second codebook $\mathcal{Q}_2$ may be optimized by considering the statistics of the adjacent RBs as well.

For a simpler discussion, we assume $q = 2$ and consider only one low-resolution codebook $\mathcal{Q}_1$ and one high-resolution codebook $\mathcal{Q}_2$. In the sequel, we provide two examples of the multi-resolution codebook designs: the multi-resolution codebook based on partitioning and the multi-resolution codebook based on the unitary rotation.

Multi-resolution codebook based on partitioning

For a simple construction of a multi-resolution codebook, we can use a partitioning concept. Let $\mathcal{K}$ denote a single-resolution codebook of size $|\mathcal{K}|$ which has designed to meet the highest required resolution. To generate a multi-resolution codebook, we first find a low-resolution codebook $\mathcal{Q}_1$ by taking $|\mathcal{Q}_1|$ elements form $\mathcal{K}$ which have the maximum minimum distance between each two element. For each element $\mathbf{Q}_i$ of the codebook $\mathcal{Q}_1$, we then find the set of *closest* $|\mathcal{K}|/|\mathcal{Q}_1|$ elements from $\mathcal{K}$ and denote it by $\mathcal{Q}_{2i}$. However, the elements are chosen such that each element of $\mathcal{K}$ belongs to one and only one $\mathcal{Q}_{2i}$. Please note that the set $\mathcal{Q}_2 = \{\mathcal{Q}_{21}, \mathcal{Q}_{22}, \ldots, \mathcal{Q}_{2,|\mathcal{Q}_1|}\}$ eventually includes all the elements of $\mathcal{K}$ but each element is doubly indexed, i.e., an index corresponding to a unique element of $\mathcal{Q}_1$ and an index which enumerate the possible choices of the precoders in $\mathcal{Q}_2$ for a given precoder index in $\mathcal{Q}_1$. Thus, the set $\mathcal{Q}_2$ is divided into $|\mathcal{Q}_1|$ partitions such that each partition $\mathcal{Q}_{2i}$ has equal number of elements. It should also be noted that such partitioning is not unique, because the set of *closest* elements depends on the order that we assign the elements of $\mathcal{K}$ to each partition. Thus, an optimization may be performed to maximize the overall codebook performance by taking the best partitioning.

An alternative approach to design a multi-resolution codebook is to first design a low-resolution codebook $\mathcal{Q}_1$ directly. Next, for each entry of the codebook, say $\mathbf{Q}_i$, we find the set of channel conditions for which this entry is chosen and denote it as $\mathcal{H}(\mathbf{Q}_i)$. Then, we design an optimal codebook $\mathcal{Q}_{2i}$ using only the channel conditions in $\mathcal{H}(\mathbf{Q}_i)$, i.e., solving the optimization problem (1) where the expectation is taken over $\mathcal{H}(\mathbf{Q}_i)$. The size of $\mathcal{Q}_{2i}$ is chosen to meed the desired resolution level. The codebook $\mathcal{Q}_2 = \{\mathcal{Q}_{21}, \mathcal{Q}_{22}, \ldots, \mathcal{Q}_{2,|\mathcal{Q}_1|}\}$ is then constructed as the set of all such designed codebooks.

In some cases, it is beneficial to design a nested multi-resolution codebook. The codebook $\mathcal{Q} = (\mathcal{Q}_1, \mathcal{Q}_2)$ is called *nested multi-resolution codebook* if and only if

$$\forall i, 1 \leq i \leq |\mathcal{Q}_1| : \mathcal{Q}_{2i} \subset \mathcal{Q}_1 \tag{3}$$

One possible design can be obtained by taking the low-resolution codebook $\mathcal{Q}_1 = \mathcal{K}$ and then find the set of $\mathcal{Q}_{2i} \subset \mathcal{K}$ as the $\mathcal{Q}_{2i}|$ closest neighbors of each elements $\mathbf{Q}_i \in \mathcal{Q}_1$ to construct the high-resolution codebook $\mathcal{Q}_2 = \{\mathcal{Q}_{2i}\}_{i=1}^{|\mathcal{Q}_1|}$. In this case, the high-resolution codebook does not increase the resolution for the original RB, but successive application of this codebook for the adjacent RBs considerably improves the resolution of the precoder matrix for the adjacent RBs.

Please note that the representation of the latter codebook constructions requires storing $|\mathcal{Q}_1|$ and $|\mathcal{Q}_2|$ precoders and a set of indices to denote the partitioning in $\mathcal{Q}_2$. Also, the former codebook construction needs storing $|\mathcal{K}|$ precoders plus two set of indices to represent $\mathcal{Q}_1$ and $\mathcal{Q}_2$ codebooks.

Multi-resolution codebook based on unitary rotation

To simplify the codebook storage, we may use functions, $f_i, 1 \leq i \leq |\mathcal{Q}_{2k}|$ which convert each element $\mathbf{Q}_1 \in \mathcal{Q}_1$ to a new precoder of the same size. An example of such function is a unitary rotation, i.e., $\mathbf{Q}_2 = f_i(\mathbf{Q}_1) = \mathbf{F}_i\mathbf{Q}_1$. For the precoder of size $N_T \times r$, the unitary rotations $\mathbf{F}_i$ are of size $N_T \times N_T$. Using such transformation, one needs to store only $|\mathcal{Q}_{2k}|$ unitary matrices that is constant irrespective of k to generate the entire high-resolution codebook $\mathcal{Q}_2$. Thus, the average performance of such generated codebook over all entries of the codebook $\mathcal{Q}_1$ should be considered in the design of the functions f_i. More specifically, we divide the set of all the channel realizations into $|\mathcal{Q}_1|$ partitions $\mathcal{H}(\mathbf{Q}_i), 1 \leq i \leq |\mathcal{Q}_1|$ where the precoder $\mathbf{Q}_i \in \mathcal{Q}_1$ is the optimal precoder.

Thus the codebook design problem is given by

$$\max_{\mathcal{F}} \sum_{i=1}^{|\mathcal{Q}_1|} \mathbb{E}_{\mathcal{H}(\mathbf{Q}_i)} \left[\max_{\mathbf{F} \in \mathcal{F}} \ \log\det\left(\mathbf{I} + \frac{P}{N_T}\mathbf{H}\mathbf{F}\mathbf{Q}_i\mathbf{Q}_i^*\mathbf{F}^*\mathbf{H}^*\right)\right]. \tag{4}$$

where $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \ldots\}$ is the set of unitary matrices that equivalently represent the codebook $\mathcal{Q}_2$.

3 Simulation Results

First, we define the system setup and the codebooks used for the simulations. We assume BS with 4 transmit antennas and a MS with 2 receive antennas. Thus, the precoding schemes use two codebooks of rank 1 and rank 2 with the same size. The rank-1 and rank-2 codebooks for the single-resolution codebook are obtained from IEEE802.16e [1]. An extra feedback bit is also used to feedback the rank of the selected precoder. The PMI of size 3,4,5, or 6 bits represents 8,16,32, or 64 choices for precoders per rank. The single-resolution codebook of size 2^B is denoted by $\mathcal{C}^{(B)}$. The channel model is according to [2] and the other important system parameters are summarized in Table 1.

For the multi-resolution codebook, we use the following cases. For each codebook $\mathcal{C}^{(B)}$, the first set of multi-resolution codebooks $\mathcal{M}^{(B)}$ is obtained by taking the first resolution codebook $\mathcal{M}_1^{(B)} = \mathcal{C}^{(B)}$ and designing a 3-bit unitary rotation codebook $\mathcal{F}^{(B)}$ which equivalently represents $\mathcal{M}_2^{(B)}$.

The second set of multi-resolution codebook $\mathcal{N}^{(B)}$, $B \geq 5$ is designed by using the partitioning approach. We first find $\mathcal{N}_1^{(B)}$ of size $2^{(B-3)}$ by taking the elements from $\mathcal{C}^{(B)}$ which have the maximum minimum distance from each other. Then, we partition the set $\mathcal{C}^{(B)}$ into $2^{(B-3)}$ where each partition contains the set of 8 closest neighbors for each codebook entry $\mathbf{Q} \in \mathcal{N}_1^{(B)}$. Thus, $\mathcal{N}_2^{(B)}$ is the set of $\mathcal{C}^{(B)}$ along with the acquired partitioning.

The third set of multi-resolution codebooks $\mathcal{O}^{(B)}$ is also designed by using the partitioning approach. Here, we take $\mathcal{O}_1^{(B)} = \mathcal{C}^{(B)}$ and then we find the set of 8 closest neighbor of each codebook entry $\mathbf{Q}_i \in \mathcal{N}_1^{(B)}$ to obtain $\mathcal{O}_{2i}^{(B)}$, and we have $\mathcal{O}_2^{(B)} = \{\mathcal{O}_{21}^{(B)}, \mathcal{O}_{21}^{(B)}, \ldots, \mathcal{O}_{2,|\mathcal{C}^{(B)}|}^{(B)}\}$.

Please note that, as a point of comparison, the multi-resolution codebook is designed such that only 3 bits is required to represent the high-resolution information about the precoder in all mentioned cases. Thus, the value B in the codebooks $\mathcal{C}^{(B)}$, $\mathcal{M}^{(B)}$, and $\mathcal{O}^{(B)}$ only refers to the size of low-resolution codebook. However, the size of low-resolution codebook in $\mathcal{N}^{(B)}$ is $B-3$ bits.

We compared the throughput performances of the different codebooks through a link level simulation. We use the precoding over M resource blocks of size 24 subcarrier each. When a single-resolution codebook $\mathcal{C}^{(B)}$ is used, the precoder index for each resource block is found individually and total of $6M$ bits is fed back to the BS. However, when the either of the codebooks $\mathcal{M}^{(B)}$, $\mathcal{N}^{(B)}$, and $\mathcal{O}^{(B)}$ is used, the precoding requires B bits for the center resource block and only 3 bits for the rest of $M-1$ resource blocks, thus, the total number of feedback bits is calculated as $6 + 3(M-1) = 3M + 3$.

Figure 1 shows the link level throughput results comparing the single-resolution codebook $\mathcal{C}^{(B)}$ with three different design of multi-resolution codebooks, i.e., $\mathcal{M}^{(B)}$, $\mathcal{N}^{(B)}$, and $\mathcal{M}^{(B)}$. In this case we consider $M = 11$ resource blocks and only present the result for the codebook size $B = 6$. It is seen that although the performance of $\mathcal{M}^{(B)}$ is not so appealing, the performance of the multi-resolution codebook $\mathcal{O}^{(B)}$ is comparable with that of the original single-resolution codebook $\mathcal{C}^{(B)}$. The performance of the other multi-resolution codebook $\mathcal{N}^{(B)}$ is somewhere in between that of $\mathcal{M}^{(B)}$ and $\mathcal{O}^{(B)}$. Thus, by proper design of the multi-resolution codebook, it is possible to achieve a performance close to a single-resolution codebook but with much lower feedback load. In this case, the feedback load has been reduced to 36 from 66 that is a huge saving of 30 bits.

4 Conclusion

We propose the use of differential codebook for the DL transmission in IEEE802.16m. Differential precoding can reduce the uplink overhead considerably. Differential precoding works by feeding back the variation in the channel instead of directly feeding back the channel condition. Typically quantizing the channel variations requires much smaller codebook size than quantizing the channel itself. Based on the

5 Proposed Text

Please refer to joint contribution C80216m-08_947 for the proposed text to SDD.

References

[1] IEEE Standard 802.16 Working Group. Air interface for broadband wireless access systems. *P802.16Rev2/D3*, Feb. 2008.

[2] IEEE Standard 802.16 Working Group. Project 802.16m evaluation methodology document (EMD). *IEEE 802.16m-08/004r1*, Mar. 2008.

Parameter	Assumption
Access	OFDM
RF carrier frequency	2.0 GHz
Bandwidth	10.0 MHz
Number of paths (Multi-path model)	6
Sub-carrier spacing	10.9375 kHz
Sampling frequency	11.2 MHz
Number of occupied sub-carriers	720
Number of OFDM symbols / frame	47
Frame duration	5ms
Number of subcarriers per slot (RB)	24 x 2 per 2 OFDM symbols
CP length	(1/8)*102.86 micro second
FFT point	1024
Number of antennas at BS	4
Number of antennas at MS	2
Codebook size (in bits)	3,4,5,6 bits for low- resolution, 3 bits for high-resolution
Number of the resource blocks (M)	3,7,11
Channel models	SCM channel model case 1A: 3kmph, L=6
Channel estimation	Ideal channel estimation
MCS	4, 16 and 64 QAM, code rates 1/2, 2/3, 3/4 and 5/6
Quantization codebook	Single-resolution:IEEE802.16e, Multi-resolution:Design

Table 1: Simulation parameters and channel model.

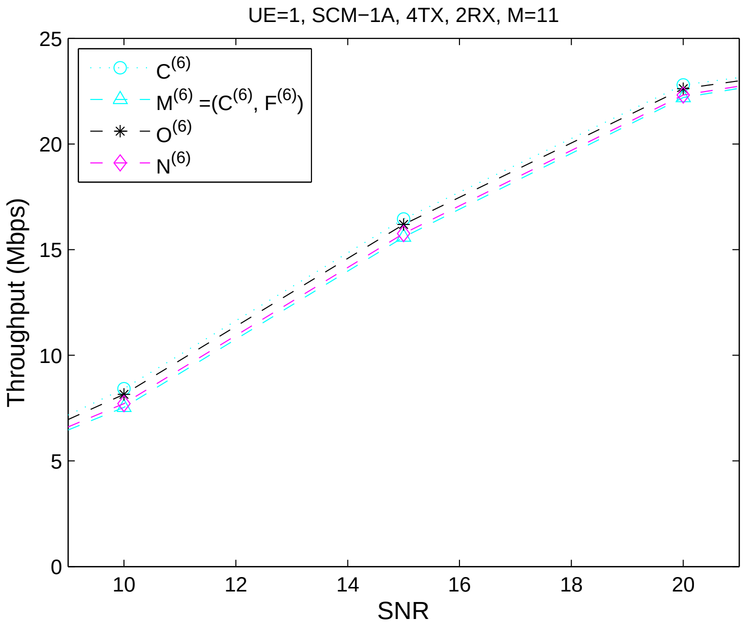

Figure 1: Link level throughput performance over M resource blocks.