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Abstract	This contribution presents statistical properties and the impact of out-of-band emission in OFDM and in SC-FDMA on the link performance. The analysis on the power spectrum density shows that OFDM has the same average power spectrum density as SC-FDMA. Hence, the average out-of-band emission (OOBE) power is equal for both systems. However, the fourth-order moment of frequency-domain symbols of SC-FDMA is larger than for the case of OFDM, which leads to larger variance of OOBE power in SC-FDMA than in OFDM. Several companies raise the issue that larger variance of OOBE power may cause performance degradation. The link simulation demonstrates that SC-FDMA, which has larger OOBE power variance, does not degrade the link performance of the victim system compared to OFDM as long as the average OOBE power is same.
Purpose	Review and discussion
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Out-of-Band Emission of OFDM and SC-FDMA

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Abstract

This contribution presents statistical properties and the impact of out-of-band emission in OFDM and in SC-FDMA on the link performance. The analysis on the power spectrum density shows that OFDM has the same average power spectrum density as SC-FDMA. Hence, the average out-of-band emission (OOBE) power is equal for both systems. However, the fourth-order moment of frequency-domain symbols of SC-FDMA is larger than for the case of OFDM, which leads to larger variance of OOBE power in SC-FDMA than in OFDM. Several companies raise the issue that larger variance of OOBE power may cause performance degradation. The link simulation demonstrates that SC-FDMA, which has larger OOBE power variance, does not degrade the link performance of the victim system compared to OFDM as long as the average OOBE power is same.

Power Spectrum Density of OFDM and SC-FDMA

In this section, the power spectrum of OFDM and SC-FDMA is presented with mathematical expressions. For the simplicity of analysis, the signal without appending cyclic prefix is considered. The continuous time base-band OFDM or SC-FDMA signal can be written as follows:

$$y(t) = \frac{1}{\sqrt{T_s}} \sum_{k=0}^{M-1} X(k) e^{j2\pi q_k \Delta f t}, \ 0 \le t \le T_s \qquad (1)$$

where M is the data block size, q_k is an index for assigned subcarriers, Δf is a subcarrier spacing, and T_s is the symbol duration without cyclic prefix , that is, $T_s = 1/\Delta f$. In OFDM, $X(k)$ represents a QAM symbol, and for SC-FDMA, $X(k)$ is the DFT-spreading output of QAM symbol $x[n]$. That is,

$$X(k) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} x[n] e^{-j2\pi kn/M}, \quad k = 0,1,\ldots,M-1. \qquad (2)$$

Since the mean of $x[n]$ is zero, $E\{X(k)\} = 0$ for both OFDM and SC-FDMA. From (1), the frequency-domain expression of both OFDM and SC-FDMA signals is given by

$$Y(f) = \sqrt{T_s} \sum_{k=0}^{M-1} X(k) \frac{\sin\{\pi(f - q_k \Delta f)T_s\}}{\pi(f - q_k \Delta f)T_s} e^{-j\pi(f - q_k \Delta f)T_s}, \quad q_k \in \left\{\pm 1, \pm 2, \ldots, \pm \tfrac{N}{2}\right\}, \qquad (3)$$

where N is the total number of subcarriers in one OFDM or SC-FDMA symbol, and f denotes a continuous frequency.

The average power spectral density of the OFDM or SC-FDMA signal can be computed from (3) as follows:

$$E\left\{|Y(f)|^2\right\} = T_s \sum_{k=0}^{M-1} \sum_{k'=0}^{M-1} E\left\{X(k)X(k')^*\right\} \cdot \frac{\sin\{\pi(f - q_k \Delta f)T_s\}}{\pi(f - q_k \Delta f)T_s} \cdot \frac{\sin\{\pi(f - q_{k'} \Delta f)T_s\}}{\pi(f - q_{k'} \Delta f)T_s} \cdot e^{j\pi(q_k - q_{k'})}. \qquad (4)$$

Assuming that adjacent QAM symbols are independent, OFDM satisfies the following:

$$E\left\{X(k)X(k')^*\right\} = 0, \text{ if } k \ne k' \qquad (5)$$

Similarly, QAM symbols of SC-FDMA satisfy that $E\left\{x[n]x[n']^*\right\} = 0$ for $n \ne n'$. Hence, the DFT-spread symbol $X(k)$ is also independent across subcarriers as follows:

$$E\{X(k)X(k')^*\} = \frac{1}{M}\sum_{n=0}^{M-1}\sum_{n'=0}^{M-1} E\{x[n]x[n']^*\}\cdot e^{-j2\pi kn/M}\cdot e^{j2\pi k'n'/M}$$
$$= \frac{\sigma_s^2}{M}\sum_{n=0}^{M-1} e^{-j2\pi(k-k')n/M} = \begin{cases}\sigma_s^2 & ;k=k'\\ 0 & ;k\neq k'\end{cases}, \quad (6)$$

where σ_s^2 denotes an average power of QAM symbols. Therefore, both OFDM and SC-FDMA have the same average power spectral density given by

$$E\{|Y(f)|^2\} = T_s\cdot\sigma_s^2\sum_{k=0}^{M-1}\left|\frac{\sin\{\pi(f-q_k\Delta f)T_s\}}{\pi(f-q_k\Delta f)T_s}\right|^2. \quad (7)$$

Next, the mean and the variance of the out-of-band emission power are investigated. From (7), the average OOBE power can be computed as follows:

$$E\left\{\int_\Omega |Y(f)|^2 df\right\} = \int_\Omega E\{|Y(f)|^2\}df$$
$$= T_s\cdot\sigma_s^2\sum_{k=0}^{M-1}\int_\Omega |\mathrm{sinc}\{(f-q_k\Delta f)T_s\}|^2 df = T_s\cdot\sigma_s^2\sum_{k=0}^{M-1}\Lambda(k,k), \quad \Omega=(-\infty,-B)\cap(B,\infty) \quad (8)$$

where Ω represents the out-of-band frequency region, and $\Lambda(k,k')\equiv\int_\Omega \mathrm{sinc}\{(f-q_k\Delta f)T_s\}\cdot\mathrm{sinc}\{(f-q_{k'}\Delta f)T_s\}df$.

It is observed in (8) that the average OOBE power of OFDM is equal to that of SC-FDMA. The variance of the OOBE power is given by

$$Var\left\{\int_\Omega |Y(f)|^2 df\right\} = E\left\{\left|\int_\Omega |Y(f)|^2 df\right|^2\right\} - \left|E\left\{\int_\Omega |Y(f)|^2 df\right\}\right|^2, \quad (9)$$

where

$$E\left\{\left|\int_\Omega |Y(f)|^2 df\right|^2\right\} = T_s^2\sum_{k=0}^{M-1}\sum_{k'=0}^{M-1}\sum_{l=0}^{M-1}\sum_{l'=0}^{M-1} E\{X(k)X(k')^*X(l)X(l')^*\}\Lambda(k,k')\Lambda(l,l')e^{j\pi(q_k-q_{k'}+q_l-q_{l'})}$$
$$= (T_s\cdot\sigma_s^2)^2\cdot\left\{\sum_{k=0}^{M-1}\sum_{l=0}^{M-1}\Lambda(k,k)\Lambda(l,l)+\sum_{k=0}^{M-1}\sum_{k'=0}^{M-1}\Lambda(k,k')\Lambda(k',k)-2\sum_{k=0}^{M-1}\Lambda^2(k,k)\right\}, \quad (10)$$
$$+T_s^2\sum_k E\{|X(k)|^4\}\cdot\Lambda^2(k,k)$$

and $\Lambda(k,k')=\Lambda(k',k)$. By substituting (8) and (10) into (9), the variance of the OOBE power can be rewritten by

$$Var\left\{\int_\Omega |Y(f)|^2 df\right\} = (T_s\cdot\sigma_s^2)^2\cdot\left\{\sum_{k=0}^{M-1}\sum_{k'=0}^{M-1}|\Lambda(k,k')|^2-2\sum_{k=0}^{M-1}\Lambda^2(k,k)\right\}+T_s^2\sum_k E\{|X(k)|^4\}\cdot\Lambda^2(k,k). \quad (11)$$

The equation (11) shows that the variance of the OOBE power depends on the fourth-order moment of the frequency-domain signal $X(k)$, which is given by

$$E\{|X(k)|^4\} = E\left\{\left(X_r(k)^2+X_i(k)^2\right)^2\right\} = E\{X_r(k)^4\}+E\{X_i(k)^4\}+2E\{X_r(k)^2X_i(k)^2\}, \quad (12)$$

where $X_r(k)$ and $X_i(k)$ are the real and imaginary part of $X(k)$, respectively.

While OFDM and SC-FDMA have the same value for $E\{|X(k)|^2\}$ as shown in (5) and (6), the fourth-order moment of $X(k)$ for SC-FDMA is different from the case of OFDM. For OFDM, $X(k)$ is a QAM symbol such that $X_r(k)$ and $X_i(k)$ are independent and identically distributed with zero mean. Hence, the fourth-order moment of $X(k)$ can be computed as follows:

$$E\{|X(k)|^4\} = 2E\{X_r(k)^4\} + 2[E\{X_r(k)^2\}]^2. \quad (13)$$

Assuming that the average QAM symbol energy is set to 1, that is, $\sigma_s^2 = 1$, then for OFDM, $E\{X_r(k)^2\} = 0.5$, and the values of $E\{X_r(k)^4\}$ are 0.25, 0.41, and 0.4405 for QPSK, 16QAM, and 64QAM, respectively. In SC-FDMA, the fourth-order moment of $X(k)$ is given by

$$\begin{aligned} E\{|X(k)|^4\} &= 2E\{X_r(k)^4\} + 2E\{X_r(k)^2 X_i(k)^2\} \\ &= 2\cdot\left[(4-\tfrac{3}{M})\cdot[E\{x_r[n]^2\}]^2 + \tfrac{1}{M}E\{x_r[n]^4\}\right] = 2\cdot\left[\sigma_s^4 - (0.75\sigma_s^4 - E\{x_r[n]^4\})\cdot\tfrac{1}{M}\right], \end{aligned} \quad (14)$$

where $x_r[n]$ and $x_i[n]$ are the real and imaginary part of the QAM symbol $x[n]$, and $E\{|x[n]|^2\} = \sigma_s^2$ for all n. In the equation (14), note that the value of $E\{|X(k)|^4\}$ in SC-FDMA depends on the data block size M, equivalently, a spreading factor. If $M = 1$, that is, for the case of no spreading, $E\{|X(k)|^4\}$ in (14) is equal to the expression for OFDM in (13). Furthermore, the fourth-order moment of $X(k)$ in SC-FDMA increases as the spreading factor increases since values of $E\{x_r[n]^4\}$ are less than $0.75\sigma_s^4$ for all QAM constellations. The detailed derivation of (14) is provided in Appendix A. Table I presents the value of $E\{|X(k)|^4\}$ for each QAM constellation.

Finally, the variance of the OOBE power can be evaluated by

$$Var\left\{\int_\Omega |Y(f)|^2 df\right\} = T_s^2 \cdot \left\{\sigma_s^4 \sum_{k=0}^{M-1}\sum_{k'=0}^{M-1} |\Lambda(k,k')|^2 + (E\{|X(k)|^4\} - 2\sigma_s^4)\sum_{k=0}^{M-1}\Lambda^2(k,k)\right\}. \quad (15)$$

By substituting the values in Table I into (15), it is demonstrated that the variance of the OOBE power in SC-FDMA is larger than in OFDM.

Table I. The fourth-order moment of $X(k)$ when the average QAM symbol energy is set to σ_s^2

	QPSK	16QAM	64QAM
OFDM	σ_s^4	$1.32\sigma_s^4$	$1.381\cdot\sigma_s^4$
SC-FDMA	$(2-1/M)\sigma_s^4$	$(2-0.68/M)\sigma_s^4$	$(2-0.619/M)\sigma_s^4$

Estimation of Power Spectrum using Periodogram

In this section, the power spectrum of OFDM and SC-FDMA and the average and the variance of the OOBE power are estimated by the method called "Periodogram" [1]. The periodogram, the power spectrum density estimate, is obtained as follows: observing N_b OFDM or SC-FDMA bauds, where each baud consists of N subcarriers in the frequency domain and N samples (excluding cyclic prefix) in the time domain, performing N_bN -point FFT over the collected samples, computing the magnitude square of the FFT output, and normalizing it by N_bN.

Figure 1 demonstrates periodogram averages of OFDM and SC-FDMA from 10000 realizations where each periodogram is obtained by using ten OFDM or SC-FDMA bauds. It is assumed that 128 subcarriers around the DC subcarrier are occupied out of 1024 total subcarriers. Note that the subcarrier index in Figure 1 is the ten times oversampled frequency index. Figure 1 shows that the periodogram average of OFDM overlaps the periodogram average of SC-FDMA. That is, OFDM and SC-FDMA have the same average power spectrum density and, accordingly, the average OOBE power is equal for both systems. The variance of the OOBE power can also be estimated from the periodogram. When the estimated average OOBE power is 9.9, the estimated OOBE power variance in SC-FDMA is 6.0, which is larger than the variance of OFDM, 5.1.

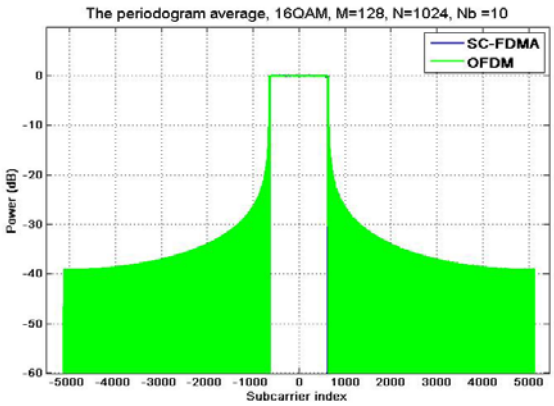


Figure 1 Periodogram averages for SC-FDMA and for OFDM, 16QAM, 128 subcarriers are occupied out of 1024 total subcarriers. 10 OFDM or SC-FDMA bauds are used for estimating the power spectrum.

Impact of Out-Of-Band Emission Power on the System Performance

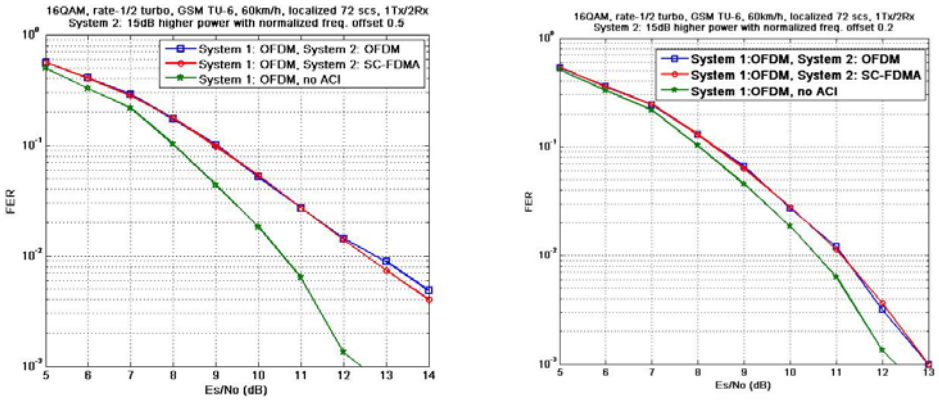
In this section, the impact of out-of-band emission on the link performance is investigated through link-level simulation. Assume that two 5MHz bandwidth systems are deployed in the adjacent frequency band. The victim system (System 1) is OFDM, and the interfering system (System 2) is OFDM or SC-FDMA. 72 contiguous subcarriers located on the band edge of each system are allocated for transmission, and the frequency distance between two allocations is 525 kHz plus a frequency offset less than a subcarrier spacing 15 kHz. System parameters used for simulation are given in Table II. A sub-frame, where one codeword spans, consists of two slots, and each slot with 0.5ms duration has six data blocks and one pilot block. The GSM TU 6-ray channel model with a UE speed of 60km/h is used for both the victim and the interfering systems, and only one active user and two receive antennas are assumed.

Table II. System Parameters

Item	Value/Description
Channel Bandwidth for Each System	5 MHz
Carrier Frequency of System 1	2 GHz
Number of Sub-carriers (*M*)	72 sub-carriers
Sub-carrier Spacing	15 kHz
Cyclic Prefix Length	4.69 μs
Baud/Symbol duration	66.67 μs
Modulation	16-QAM
Coding	Turbo code with constituent convolutional code (r=1/3), Max-log-MAP kernel, 3GPP interleaver, 8 iterations
Channel	GSM TU 6-ray model
Channel Estimation	Two dimensional MMSE filter
Number of Active UEs	1
Number of Tx/Rx Antennas	1Tx/2Rx

Figure 2 demonstrates the FER performance comparison for different adjacent channel interference (ACI)

powers and for different interfering systems. The ACI power depends on both the transmission power and the frequency offset of the interfering system. In simulation, transmitted QAM symbols in the interfering system (System 2) have 15dB higher power than in the victim system (System 1), and frequency offset values of 0.5 and 0.2, which are normalized by a sub-carrier spacing, are considered. Figure 2 shows that the impact of the out-of-band emission on the link performance of System 1 is same for OFDM and SC-FDMA. That is, the FER performance of the victim system does not depend on whether the interfering system is OFDM or SC-FDMA. For the normalized frequency 0.5, the SNR loss at 10% FER due to ACI is 1dB, and the normalized frequency 0.2 results in 0.4dB SNR loss at 10% FER.



(a) Normalized frequeqncy offset: 0.5 (b) Normalized frequency offset: 0.2

Figure 2 The impact of OOBE on the FER performance, 16QAM, rate-1/2 turbo, GSM TU-6 ray channels, 60km/h, localized 72 subcarrier allocation on the band-edge of each system, Transmitted QAM symbols in the interfering system (System 2) have 15dB higher power than in the victim system (System 1).

Conclusion

It has been shown that SC-FDMA and OFDM have the same average out-of-band emission power, and the out-of-band emission power variance is larger in SC-FDMA than in OFDM. The link simulation results show that larger out-of-band emission power variance of SC-FDMA does not degrade the link performance of the victim system compared to the impact of out-of-band emission of OFDM.

Reference

[1] M. H. Hayes, “Statistical Digital Signal Processing and Modeling”, *John Wiley & Sons, Inc*, 1996.

Appendix A

In this Appendix, evaluation of the fourth-order moment of $X(k)$ in SC-FDMA is detailed. The equation (12) shows that $E\{X_r(k)^4\}$, $E\{X_i(k)^4\}$, and $E\{X_r(k)^2 X_i(k)^2\}$ need to be evaluated for the fourth-order moment of $X(k)$. In SC-FDMA, the frequency domain signal $X(k)$, which is the DFT spreading output of QAM symbols $x[n]$ with the average power σ_s^2, can be written by

$$X(k) = X_r(k) + jX_i(k) = \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} a_k[n] + j\left(\frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} b_k[n]\right), \quad k = 0,1,\ldots,M-1, \quad (16)$$

where $a_k[n] = x_r[n]\cos\frac{2\pi kn}{M} + x_i[n]\sin\frac{2\pi kn}{M}$ and $b_k[n] = x_i[n]\cos\frac{2\pi kn}{M} - x_r[n]\sin\frac{2\pi kn}{M}$.

Considering that $a_k[n]$ for all n are independent with a zero mean, $E\{X_r(k)^4\}$ is computed as follows:

$$\begin{aligned} E\{X_r(k)^4\} &= \frac{1}{M^2}\sum_n\sum_{n'}\sum_l\sum_{l'} E\{a_k[n]a_k[n']a_k[l]a_k[l']\} \\ &= \frac{3}{M^2}\left[\sum_n\sum_l E\{a_k[n]^2\}E\{a_k[l]^2\} - \sum_n\left(E\{a_k[n]^2\}\right)^2\right] + \frac{1}{M^2}\sum_n E\{a_k[n]^4\} \end{aligned} \quad (17)$$

The variance of $a_k[n]$ is $0.5\sigma_s^2$, and the fourth-order moment of $a_k[n]$, $E\{a_k[n]^4\}$, is given by

$$E\{a_k[n]^4\} = E\{x_r[n]^4\} + [6(E\{x_r[n]^2\})^2 - 2E\{x_r[n]^4\}]\cos^2\frac{2\pi kn}{M}\sin^2\frac{2\pi kn}{M}. \quad (18)$$

It can be easily shown that the mean, the second-order and the fourth-order moments of $b_k[n]$ are same as those of $a_k[n]$. Hence, $E\{X_r(k)^4\}$ is equal to $E\{X_i(k)^4\}$.

Since $E\{a_k[n]b_k[n]\} = 0$, $E\{X_r(k)^2 X_i(k)^2\}$ is evaluated as follows:

$$\begin{aligned} E\{X_r(k)^2 X_i(k)^2\} &= \frac{1}{M^2}\sum_n\sum_{n'}\sum_l\sum_{l'} E\{a_k[n]a_k[n']b_k[l]b_k[l']\} \\ &= \frac{1}{M^2}\left[\sum_n\sum_l E\{a_k[n]^2\}E\{b_k[l]^2\} - \sum_n E\{a_k[n]^2\}E\{b_k[n]^2\}\right] + \frac{1}{M^2}\sum_n E\{a_k[n]^2 b_k[n]^2\} \end{aligned} \quad (19)$$

In (19), $E\{a_k[n]^2 b_k[n]^2\}$ is given by

$$\begin{aligned} &E\{a_k[n]^2 b_k[n]^2\} \\ &= (E\{x_r[n]^2\})^2(\cos^4\tfrac{2\pi kn}{M} + \sin^4\tfrac{2\pi kn}{M} - 4\cos^2\tfrac{2\pi kn}{M}\sin^2\tfrac{2\pi kn}{M}) + 2E\{x_r[n]^4\}\cos^2\tfrac{2\pi kn}{M}\sin^2\tfrac{2\pi kn}{M}. \\ &= (E\{x_r[n]^2\})^2 - [6(E\{x_r[n]^2\})^2 - 2E\{x_r[n]^4\}]\cos^2\tfrac{2\pi kn}{M}\sin^2\tfrac{2\pi kn}{M} \end{aligned} \quad (20)$$

Finally, the fourth-order moment of $X(k)$ can be computed by using (17)-(20) as follows:

$$\begin{aligned} E\{|X(k)|^4\} &= 2E\{X_r(k)^4\} + 2E\{X_r(k)^2 X_i(k)^2\} \\ &= 2\cdot\left[\tfrac{4}{M}(M-1)\cdot(E\{x_r[n]^2\})^2 + \tfrac{1}{M}E\{x_r[n]^4\} + \tfrac{1}{M}(E\{x_r[n]^2\})^2\right], \\ &= 2\cdot\left[\sigma_s^4 - (0.75\sigma_s^4 - E\{x_r[n]^4\})\cdot\tfrac{1}{M}\right] \end{aligned} \quad (21)$$