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Re:	IEEE 802.16m-08/016 - Call for Contributions on Project 802.16m System Description Document (SDD), shoot for “Hybrid ARQ (protocol and timing)” topic.
Abstract	We apply a signal constellation rearrangement to the Chase combining scheme for the HARQ scheme supported in IEEE802.16e. We present the basic idea of the proposed scheme and evaluate the packet error rate performance. The performance of the proposed scheme can be drastically improved for both 16QAM and 64QAM.
Purpose	For 802.16m discussion and adoption
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Enhanced HARQ scheme with Signal Constellation Rearrangement

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1. Introduction

Chase combining (CC) [1] is one of the HARQ schemes supported in IEEE802.16e [2]. In this contribution, we present a HARQ bit-mapping scheme, called “signal constellation rearrangement (CoRe)” for CC in the case of 16QAM and 64QAM [3][4].

The proposed scheme “CC with CoRe” shows a significant performance gain with respect to the original CC without CoRe. Therefore, CoRe should be included in the system description document (SDD).

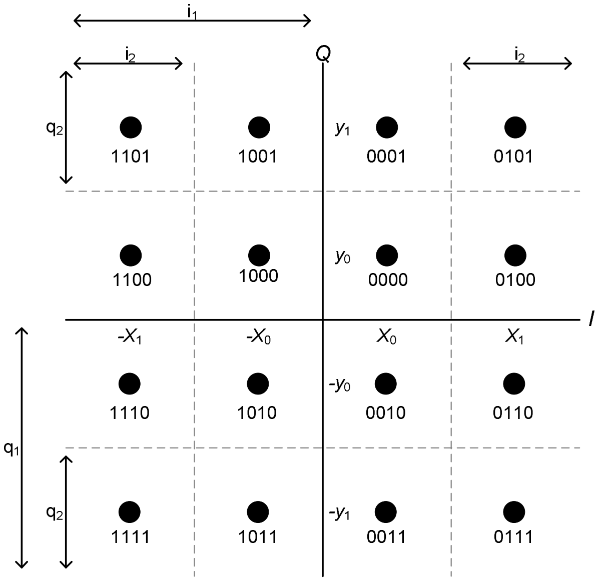
2. Signal constellation rearrangement for quadrature amplitude modulation

2.1. Basic idea

The 2^M-QAM modulated symbol consists of M-bits. For the 2^M-QAM with M larger than 2 (e.g. 8QAM (PSK),16QAM, 64QAM, 256QAM,…), the reliabilities of the bits Gray-mapped onto the modulated symbol vary from the most significant bits (MSBs) i_1 and q_1 to the least significant bits (LSBs) i_2 and q_2 as shown in Figure 1 and in the Appendix. As the reliability variations between these two different kinds of bits increase, the error rate performance is getting worse with respect to having equal bit reliabilities. Considering a HARQ scheme retransmitting (at least partially) identical symbols and employing an identical signal constellation and mapping for all transmissions, the variations in bit reliabilities increase over retransmissions (i.e. the original CC). Particularly, this is the case when soft-combining the received packets by maximal ratio combining (MRC) at modulation symbol level or by adding LLRs at bit level. By rearranging the signal constellations for retransmissions the proposed scheme averages out the bit reliabilities over the retransmissions.

2.1.1. Signal constellation rearrangement for 16QAM and 64QAM

Figure 1 shows the signal constellation in the case of 16QAM supported in IEEE802.16e. The bit i_1, i_2, q_1 and q_2, is located to the 1st bit, 2nd bit, 3rd bit and 4th bit (i.e. $i_1i_2q_1q_2$) within a symbol. Due to Gray-mapping bit i_1 and q_1 is on average more reliable than i_2 and q_2.



Symbol mapping : $i_1i_2q_1q_2$
i_1 and i_2 for In-phase channel
q_1 and q_2 for Quadrature channel

i_1 and q_1 : Most Significant Bit(MSB)
- ones and zeros are mapped to half spaces (symmetry)
- Reliability is depends on the bit content of i_2 and q_2

i_2 and q_2 : Least Significant Bit(LSB)
- ones and zeros are mapped to inner/outer rows /columns
- Reliability is independent from the bit content

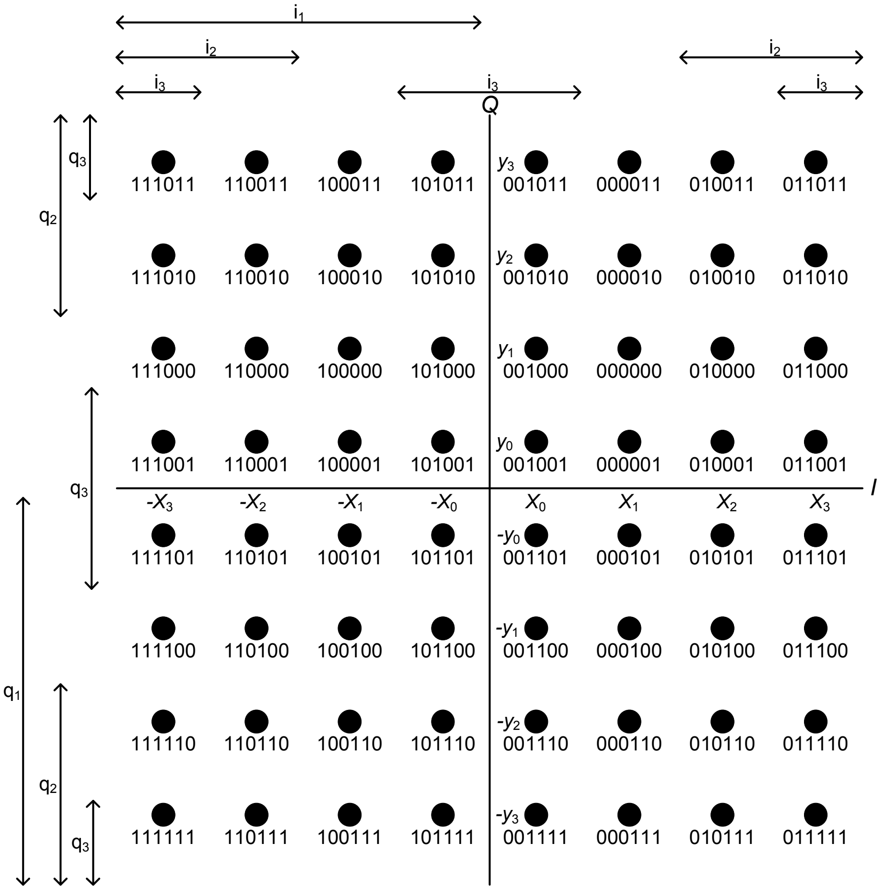
Figure 1. Signal constellation with Gray-mapping in the case of 16QAM

As we present in the previous section 2.1, there are variations in the mean bit reliabilities depending on the significance of the bit within a symbol. Therefore, we use the simple rearrangement rule such as reordering and inversion of the logical bit values according to Table 1 to average out the bit reliabilities over the retransmissions.

Table 1. Rearrangement rule for each transmission in the case of 16QAM

Transmission No.	Bit pattern	Explanation
1	$i_1i_2q_1q_2$	- None
2	$i_2\bar{i}_1q_2\bar{q}_1$	- Swapping i_1 with i_2 and q_1 with q_2/logical inversion of i_1 and q_1
3	$i_2i_1q_2q_1$	- Swapping i_1 with i_2 and q_1 with q_2
4	$i_1\bar{i}_2q_1\bar{q}_2$	- Logical inversion of i_2 and q_2
Further transmission		- Repeatedly using the signal constellations form 1st-4th transmissions

The same rearrangement rule as 16QAM can be applicable for 64QAM. In this case there are 3 levels of bit reliabilities. Figure 2 shows the signal constellation in the case of 64QAM supported in IEEE802.16e and Table 2 shows the rearrangement rule for each transmission in the case of 64QAM.



Symbol mapping : $i_1i_2i_3q_1q_2q_3$
i_1, i_2 and i_3 for In-phase channel
q_1, q_2 and q_3 for Quadrature channel

i_1 and q_1 : High Reliability
- ones and zeros are mapped to half spaces (symmetry)

i_2 and q_2 : Medium Reliability
- ones and zeros are mapped to rows /columns 3-4-5-6(inner)/1-2-7-8(outer)

i_3 and q_3 : Low Reliability
- ones and zeros are mapped to rows /columns 1-4-5-8/2-3-6-7

Figure 2. Signal constellation with Gray-mapping in the case of 64QAM

Table 2. Rearrangement rule for each transmission in the case of 64QAM

Transmission No.	Bit pattern	Explanation
1	$i_1i_2i_3q_1q_2q_3$	- None
2	$i_2i_3i_1q_2q_3q_1$	- 1-bit circular shift for the in-phase and orthogonal components individually
3	$i_3i_1i_2q_3q_1q_2$	- 2-bits circular shift for the in-phase and orthogonal components individually
4	$i_1\bar{i}_2\bar{i}_3q_1\bar{q}_2\bar{q}_3$	- Logical inversion of i_2, i_3 , q_2 and q_3
5	$i_2\bar{i}_3\bar{i}_1q_2\bar{q}_3\bar{q}_1$	- 1-bit circular shift for the in-phase and orthogonal components individually - Logical inversion of i_1, i_3 , q_1 and q_3
6	$i_3\bar{i}_1\bar{i}_2q_3\bar{q}_1\bar{q}_2$	- 2-bits circular shift for the in-phase and orthogonal components individually - Logical inversion of i_1, i_2 , q_1 and q_2
Further transmission		- Repeatedly using constellations form 1st-6th transmissions

3. Simulation results

Table 3 provides a list of simulation parameters.

Table 3. Simulation parameters

Parameter	Value
Carrier frequency	2.5Ghz
System bandwidth	10MHz
FFT size	1024
Sub-carrier frequency spacing(f_s)	10.94kHz
Useful symbol interval (T_s=1/f_s)	91.4usec
Guard interval (T_g=T_s/8)	11.4usec
Number of information bits for packet	128bits(16QAM) / 192bits(64QAM)
Antenna configuration	1-by-1
Channel coding	Turbo coding (original rate = 1/3)
MCS	16QAM/64QAM, R=1/2
Channel model	AWGN
Channel estimation	Ideal
Maximum number of transmissions	4(16QAM) / 6(64QAM)

3.1. Comparison between the original chase combining and the chase combining with constellation rearrangement in the case of 16QAM

Figure 3 shows the packet error rate (PER) performance as a function of the average received signal energy per information bit-to-AWGN power spectrum density ratio (E_b/N_0) with the number of transmission as a parameter in the case of 16QAM. The performance gain for PER=10^{-1} of the CC with CoRe over the original CC is about 1.4dB for the 2nd transmission, about 2.1dB for the 3rd transmission and about 2.9dB for the 4th transmission. The performance gain is increasing in proportion to the number of transmissions, because the effect of averaging out the reliabilities is improved.

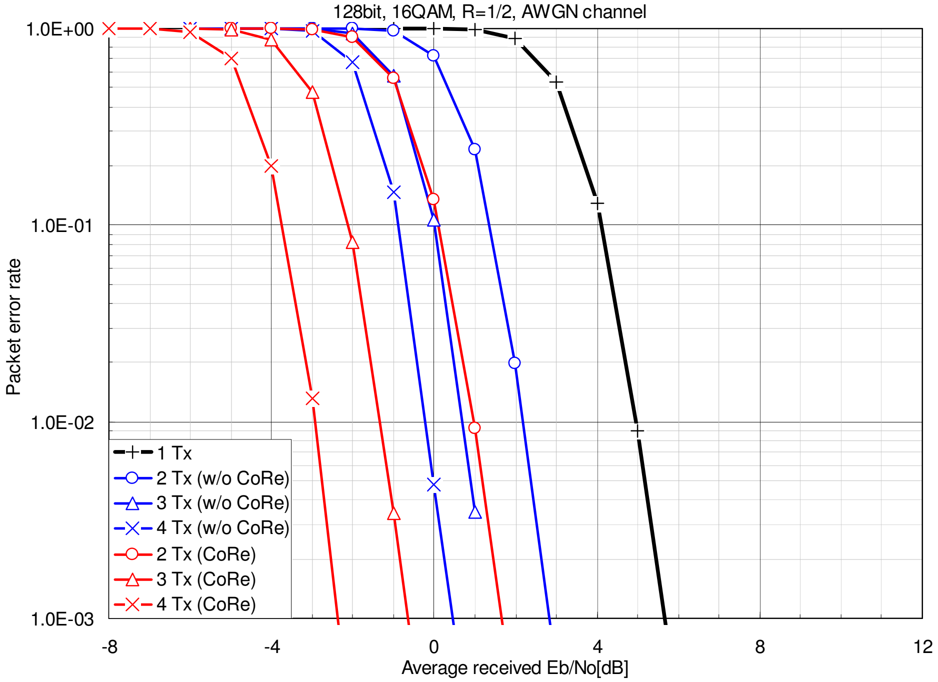


Figure 3. Packet error rate vs. average received E_b/N_0 in the case of 16QAM

3.2. Comparison between the original Chase combining and the Chase combining with constellation rearrangement in the case of 64QAM

Figure 4 shows the PER performance as a function of the average received E_b/N_0 with the number of transmission as a parameter in the case of 64QAM. The performance gain for PER=10^{-1} of the CC with CoRe over the original CC is about 2.1dB for the 2nd transmission, about 3.2dB for the 3rd transmission, about 4.5dB for the 4th transmission, about 5.4dB for the 5th transmission and about 6.0dB for the 6th transmission. The performance gain is improved proportionally to the number of transmissions for the same reason as for the 16QAM case.

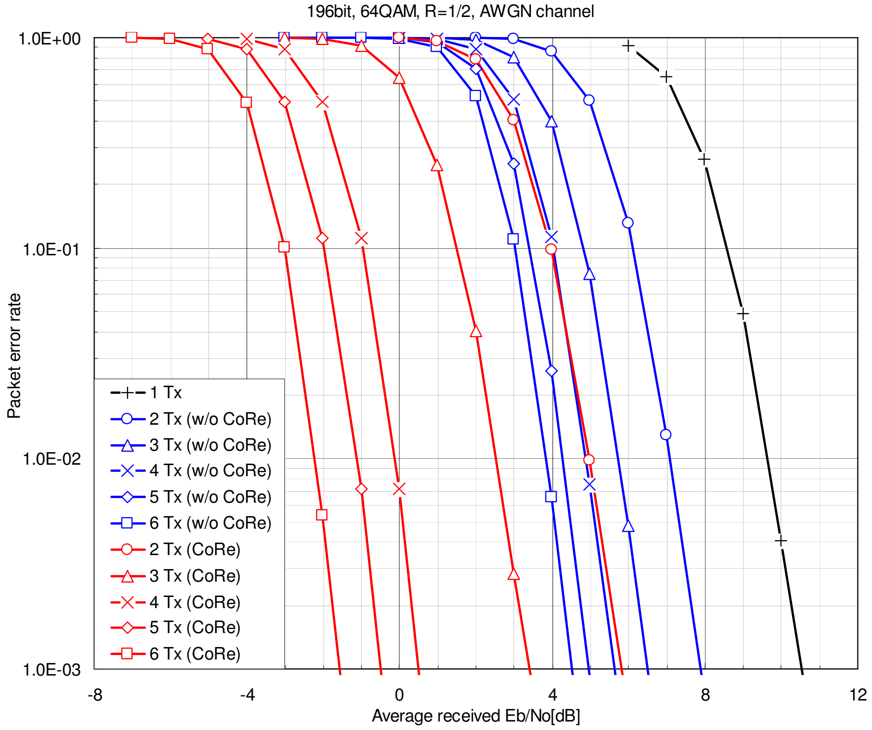


Figure 4. Packet error rate vs. average received E_b/N_0 in the case of 64QAM

3.3. Performance gain with constellation rearrangement comparison between 16QAM-case and 64QAM-case

Table 4 shows a performance gain for PER=10^{-1} of the CC with CoRe over the original CC both in the case of 16QAM and 64QAM. From Table X it can be observed that a performance gain in the case of 64-QAM is larger than that in the case of 16QAM for each transmission. In a word, CoRe is more effective in higher modulation level (or higher coding rate).

Table.4 Diversity gain with CoRe comparison between 16QAM-case and 64QAM-case for PER=10^{-1}

Number of transmissions	16QAM	64QAM
1st transmission	0 dB	0 dB
2nd transmission	1.4 dB	2.1 dB
3rd transmission	2.1 dB	3.2 dB
4th transmission	2.9 dB	4.5 dB
5th transmission		5.4 dB
6th transmission		6.0 dB

4. Conclusion

In this contribution a Chase combining scheme with signal constellation rearrangement is presented. By averaging out the bit reliabilities over the retransmissions, CC with CoRe shows a significantly improved packet error rate performance compared to the original CC for both 16QAM and 64QAM. The performance gain is improving with an increasing number of retransmissions, especially in the case of 64QAM.

We proved the efficiency of applying the CoRe to the CC. Therefore, CC with CoRe shall be described in the SDD as shown below.

--------------------------- *Begin Proposed Text* --

X. HARQ

A Chase combining scheme with signal constellation rearrangement for 16QAM and 64QAM modulation shall be used.

--------------------------*End of Text Proposal* ---

5. Reference

[1] D. Chase, “Code combining: A maximum-likelihood decoding approach for combining an arbitrary number of noisy packets,” IEEE Trans. Commun., Vol. COM-33, pp. 385-393, May 1985.

[2] IEEE Std. 802.16e-2005, “Part 16: Air Interface for Broadband Wireless Access Systems,” Approved 7 December 2005.

[3] 802.16m-07/292r1, ITRI, “Enhanced HARQ technique using Constellation Rearrangement”

[4] R1-01-0237, Panasonic, “enhanced HARQ method with signal constellation rearrangement, ” 3GPP TSG RAN WG1, Las Vegas, USA February 27- March 2, 2001.

[5] S. Le Goff, A. Glavieux, C. Berrou, “Turbo-codes and high spectral efficiency modulation,” *IEEE SUPERCOMM/ICC* ’94, vol.2, pp.645-649, 1994

[6] Ch. Wengerter, A. Golitschek Edler von Elbwart, E. Seidel, G. Velev, M.P. Schmitt, “Advanced hybrid ARQ technique employing a signal constellation rearrangement,” *IEEE VTC 2002 Fall*, vol. 4, pp. 2002-2006 , 2002.

6. Appendix

One possible implementation described in Section 2.2 is to use the same the signal constellation for all transmissions as shown in Figure 1 and reorder and inverse the logical bit values according to Table 1. There is another implementation that the ordering of the bit-to-symbol mapping is identical for all transmissions, however, the signal constellations for each transmission are changed. In this section we analyze the advantageous effect of the *LLR* averaging for the quadrature amplitude modulation, based on this implementation in the case of 16QAM and 64QAM. It is noted that this implementation can achieve an identical averaging of the bit reliabilities as described in Section 2.

The *LLR* which is a soft-metric for the reliability of a demodulated bit b(=0 or 1) from a received modulation symbol $r = x + jy$ is defined as follows[5],

$$LLR(b_p) = \ln \frac{\Pr(b=1 \mid r)}{\Pr(b=0 \mid r)}, \quad (1)$$

where p in b_p denotes the bit position. As can be seen from Figure 1, the mappings of the in-phase component bits and the quadrature component bits on the signal constellation are orthogonal. Therefore, it is sufficient to focus on the in-phase component bits i_1 and i_2. The same conclusions then apply for q_1 and q_2. The *LLR* is given by the following equations [6],

$$LLR(b_p) = \ln\frac{\Pr(b=1\,|\,r)}{\Pr(b=0\,|\,r)} = \ln\left[\frac{\sum\limits_{b=1}\exp\left(-K(x-x_k^{(1)})^2\right)}{\sum\limits_{b=0}\exp\left(-K(x-x_k^{(0)})^2\right)}\right], (1)$$

where x denotes the in-phase component of the normalized received modulation symbol r, $x_k^{(b)}$ denotes the in-phase component of the normalized transmit modulation symbol and K is a factor proportional to the signal-to-noise ratio(SNR).

By using the following approximation,

$$\ln_j\left[\left(-\sum_{b=1}\exp\left(z_j\right)\right)\right] \approx \max\left(-z_j\right) = \min(z_j)\ , (2)$$

equation (1) can be approximated by the following equation,

$$\begin{aligned} LLR(b_p) &\approx K\left[\min_{b=0}\left|x-x_k^{(0)}\right|^2 - \min_{b=1}\left|x-x_k^{(1)}\right|^2\right] \\ &= K\left[\min_{b=0}\left(x_k^{(0)2} - 2x_k^{(0)}x\right) - \min_{b=1}\left(x_k^{(1)2} - 2x_k^{(1)}x\right)\right] \end{aligned}. (3)$$

6.1. 16QAM case

Under the assumption of a uniform signal constellation $x_1 = 3x_0$ in the case of 16QAM equations (1) can be fairly good approximated by the following equations.

$$LLR(b=i_1) \approx \begin{cases} -8Kx_0x - 8Kx_0{}^2 & ,(x < -2x_0) \\ -4Kx_0x & ,(-2x_0 \le x < 2x_0) \\ -8Kx_0x + 8Kx_0{}^2 & ,(2x_0 \le x) \end{cases}$$

$$LLR(b=i_2) \approx \begin{cases} 4Kx_0x - 8Kx_0{}^2 & ,(x < 0) \\ -4Kx_0x - 8Kx_0{}^2 & ,(0 \le x) \end{cases} , (4)$$

Table 5 shows the mean *LLR*s for bits mapped on the in-phase component of the signal constellation for the mapping in the case of 16QAM as shown in figure 1 according to equation (3) (substituting $4Kx_0^2$ by A).

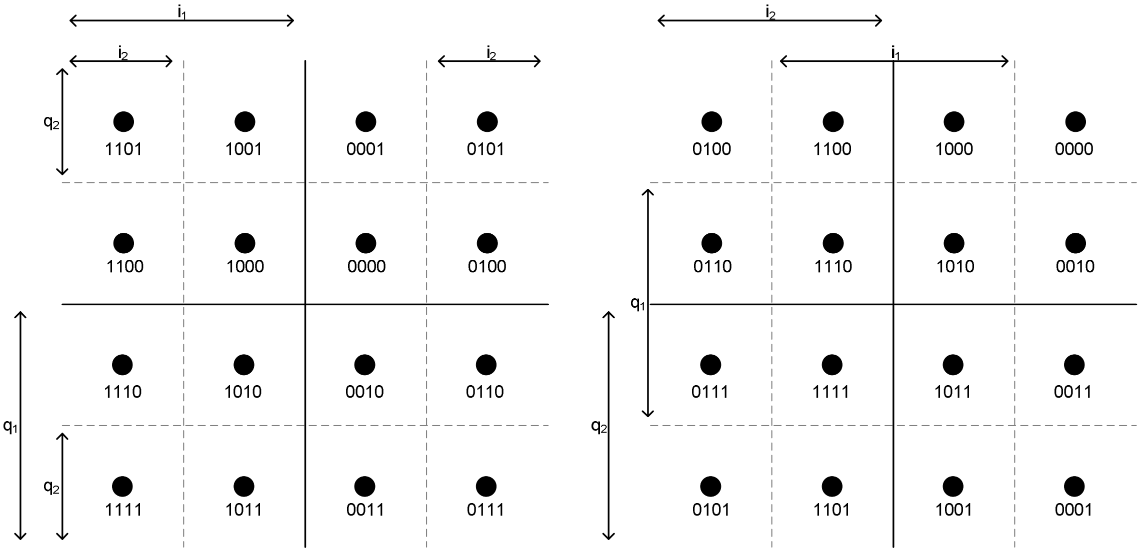
In case of transmitted modulation symbols $01q_1q_2$ and $11q_1q_2$, where q_1 and q_2 are arbitrary, the magnitude of the mean *LLR* (i_1) is higher than of the mean *LLR* (i_2). This represents that the actual *LLR* for i_1 depends on the content of i_2; e.g. in Figure 1 i_1 has a higher mean reliability in case the logical value for i_2 is equal to “1” (the leftmost and rightmost columns). Hence, assuming a uniform distribution of transmitted modulation symbols, on average, 50 % of the MSBs i_1 have about three times the magnitude in *LLR* of i_2.

Table 5. Mean *LLR*s for bits mapped on the in-phase component of the signal constellation for the mapping in figure 1 accoding to equation (3)

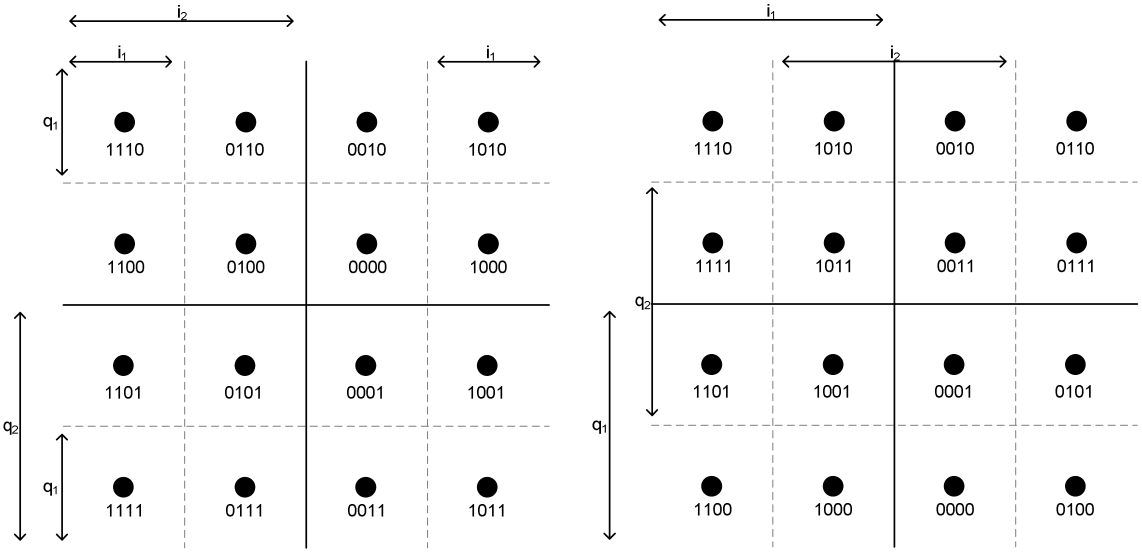
Symbol($i_1i_2q_1q_2$)	Mean value of x	Mean *LLR* (i_1)	Mean *LLR* (i_2)
$00q_1q_2$	x_0	$-4Kx_0^2 = -A$	$-4Kx_0^2 = -A$
$01q_1q_2$	x_1	$-16Kx_0^2 = -4A$	$4Kx_0^2 = A$
$10q_1q_2$	$-x_0$	$4Kx_0^2 = A$	$-4Kx_0^2 = -A$
$11q_1q_2$	$-x_1$	$16Kx_0^2 = 4A$	$4Kx_0^2 = A$

From Table 5 it can be observed that there are variations in the mean bit reliabilities for multilevel modulation formats depending on the significance of the bit within a symbol and on the content of the transmitted modulation symbol.

Figure 5 shows the signal constellation in the case of 16QAM for each transmission and Table 6 shows *LLR*s for bits mapped on the in-phase component of the signal constellation according to the mappings for the 1st transmission to 4th transmission. Assuming a superior decoding performance for uniformly distributed *LLR*s of all transmitted bits, the objective is to find mapping rules for retransmissions with the constellations from Figure 5 that equalize the mean *LLR*s after combining. Using the mappings for requested retransmissions according to Table 6 the soft-combined *LLR*s after requested retransmissions are averaged. This is shown in Table 7 for an AWGN channel, where the soft-combined *LLR*s for i_1 and i_2 after each transmission are given with and without applying the CoRe. It is noted that exactly the same principle applies to the q_1 and q_2 bits mapped onto the quadrature component. From Table 7 it can be observed that in case of 16QAM for an AWGN channel the averaging is perfect after 4 requested transmissions, hence, a total of 4 different mappings is sufficient.



(a) 1^{st} transmission (b) 2^{nd} transmission



(c) 3^{rd} transmission (d) 4^{th} transmission

Figure 5. The signal constellation in the case of 16QAM for each transmission

Table 6. *LLR*s for bits mapped on the in-phase component of the signal constellation according to mapping according to mapping the 1^{st} transmission to 4^{th} transmission

Symbol ($i_1i_2q_1q_2$)	1^{st} transmission		2^{nd} transmission		3^{rd} transmission		4^{th} transmission	
	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_2)	Mean *LLR* (i_2)	Mean *LLR* (i_1)	Mean *LLR* (i_2)
$00q_1q_2$	-A	-A	-A	-4A	-A	-A	-4A	-A
$01q_1q_2$	-4A	A	-A	4A	-A	A	-A	A
$10q_1q_2$	A	-A	A	-A	A	-4A	4A	-A
$11q_1q_2$	4A	A	A	A	A	4A	A	A

Table 7. Cumulative *LLR*s for bits mapped on the in-phase component of the signal constellation for an AWGN channel

Transmission No.	**Corresponding figure**	**Symbol ($i_1i_2q_1q_2$)**	**With CoRe**		**Without CoRe**	
			Mean *LLR* (i_1)	**Mean *LLR* (i_2)**	**Mean *LLR* (i_1)**	**Mean *LLR* (i_2)**
1	Figure 5(a)	$00q_1q_2$	-A	-A	-A	-A
		$01q_1q_2$	-4A	A	-4A	A
		$10q_1q_2$	A	-A	A	-A
		$11q_1q_2$	4A	A	4A	A
2	Figure 5(b)	$00q_1q_2$	-2A	-5A	-2A	-2A
		$01q_1q_2$	-5A	5A	-8A	2A
		$10q_1q_2$	2A	-2A	2A	-2A
		$11q_1q_2$	5A	2A	8A	2A
3	Figure 5(c)	$00q_1q_2$	-3A	-6A	-3A	-3A
		$01q_1q_2$	-6A	6A	-12A	3A
		$10q_1q_2$	3A	-6A	3A	-3A
		$11q_1q_2$	6A	6A	12A	3A
4	Figure 5(d)	$00q_1q_2$	-7A	-7A	-4A	-4A
		$01q_1q_2$	-7A	7A	-16A	4A
		$10q_1q_2$	7A	-7A	4A	-4A
		$11q_1q_2$	7A	7A	16A	4A

6.2. 64QAM case

Under the assumption of a uniform signal constellation $x_1 = 3x_0$, $x_2 = 5x_0$, $x_3 = 7x_0$ for 64QAM, equations (1) can be fairly good approximated by the following equations.

$$LLR(b=i_1) \approx \begin{cases} -16Kx_0x - 48Kx_0^{\ 2} & ,(x < -6x_0) \\ -12Kx_0x - 24Kx_0^{\ 2} & ,(-6x_0 \le x < -4x_0) \\ -8Kx_0x - 8Kx_0^{\ 2} & ,(-4x_0 \le x < -2x_0) \\ -4Kx_0x & ,(-2x_0 \le x < 2x_0) \\ -8Kx_0x + 8Kx_0^{\ 2} & ,(2x_0 \le x < 4x_0) \\ -12Kx_0x + 24Kx_0^{\ 2} & ,(4x_0 \le x < 6x_0) \\ -16Kx_0x + 48Kx_0^{\ 2} & ,(6x_0 \le x) \end{cases}$$

$$LLR(b=i_2) \approx \begin{cases} -8Kx_0x - 40Kx_0^{\ 2} & ,(x < -6x_0) \\ -4Kx_0x - 16Kx_0^{\ 2} & ,(-6x_0 \le x < -2x_0) \\ -8Kx_0x - 24Kx_0^{\ 2} & ,(-2x_0 \le x < 0) \\ 8Kx_0x - 24Kx_0^{\ 2} & ,(0 \le x < 2x_0) \\ 4Kx_0x - 16Kx_0^{\ 2} & ,(2x_0 \le x < 6x_0) \\ 8Kx_0x - 40Kx_0^{\ 2} & ,(6x_0 \le x) \end{cases} \quad , (5)$$

$$LLR(b=i_3) \approx \begin{cases} -4Kx_0x - 24Kx_0^{\ 2} & ,(x < -4x_0) \\ 4Kx_0x + 8Kx_0^{\ 2} & ,(-4x_0 \le x < 0) \\ -4Kx_0x + 8Kx_0^{\ 2} & ,(0 \le x < 4x_0) \\ 4Kx_0x - 24Kx_0^{\ 2} & ,(4x_0 \le x) \end{cases}$$

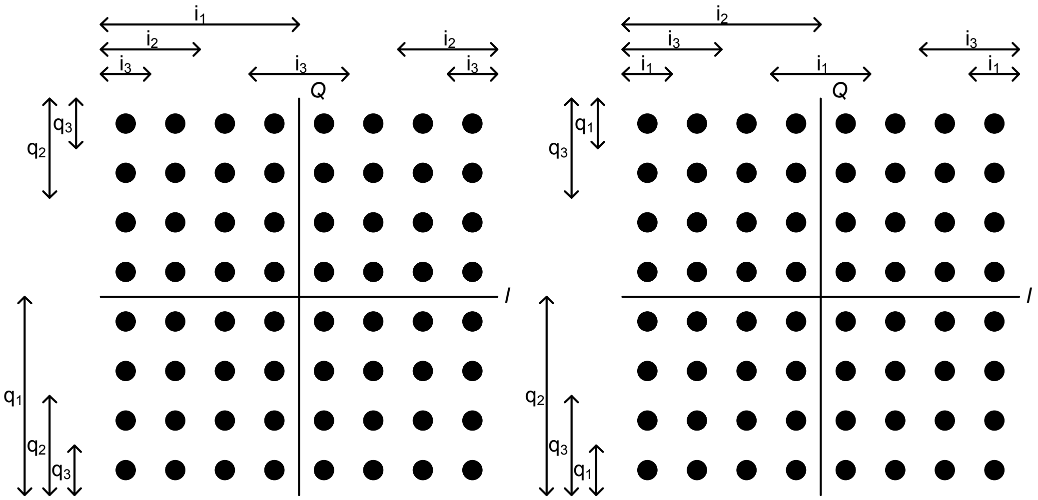
Table 8 shows the mean *LLR*s for bits mapped on the in-phase component of the signal constellation for the mapping in the case of 64QAM as shown in figure 2 according to equation (4) (substituting $4Kx_0^2$ by A).

The *LLR* of i_1 and i_2 depends on the content of the other bits. On the other hand, The magnitude in *LLR* of i_3 is always constant.

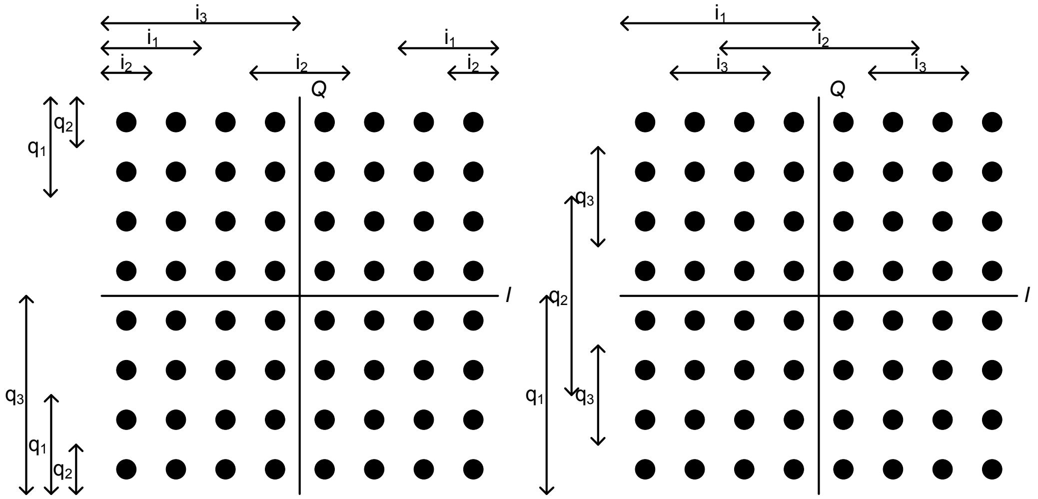
Table 8. Mean *LLR*s for bits mapped on the in-phase component of the signal constellation for the mapping in Figure 2 accoding to equation (4)

Symbol($i_1i_2i_3q_1q_2q_3$)	Mean value of x	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)
$000q_1q_2q_3$	x_1	$-16Kx_0^2 = -4A$	$-4Kx_0^2 = -A$	$-4Kx_0^2 = -A$
$001q_1q_2q_3$	x_0	$-4Kx_0^2 = -A$	$-16Kx_0^2 = -4A$	$4Kx_0^2 = A$
$010q_1q_2q_3$	x_2	$-36Kx_0^2 = -9A$	$4Kx_0^2 = A$	$-4Kx_0^2 = -A$
$011q_1q_2q_3$	x_3	$-64Kx_0^2 = -16A$	$16Kx_0^2 = 4A$	$4Kx_0^2 = A$
$100q_1q_2q_3$	$-x_1$	$16Kx_0^2 = 4A$	$-4Kx_0^2 = -A$	$-4Kx_0^2 = -A$
$101q_1q_2q_3$	$-x_0$	$4Kx_0^2 = A$	$-16Kx_0^2 = -4A$	$4Kx_0^2 = A$
$110q_1q_2q_3$	$-x_2$	$36Kx_0^2 = 9A$	$4Kx_0^2 = A$	$-4Kx_0^2 = -A$
$111q_1q_2q_3$	$-x_3$	$64Kx_0^2 = 16A$	$16Kx_0^2 = 4A$	$4Kx_0^2 = A$

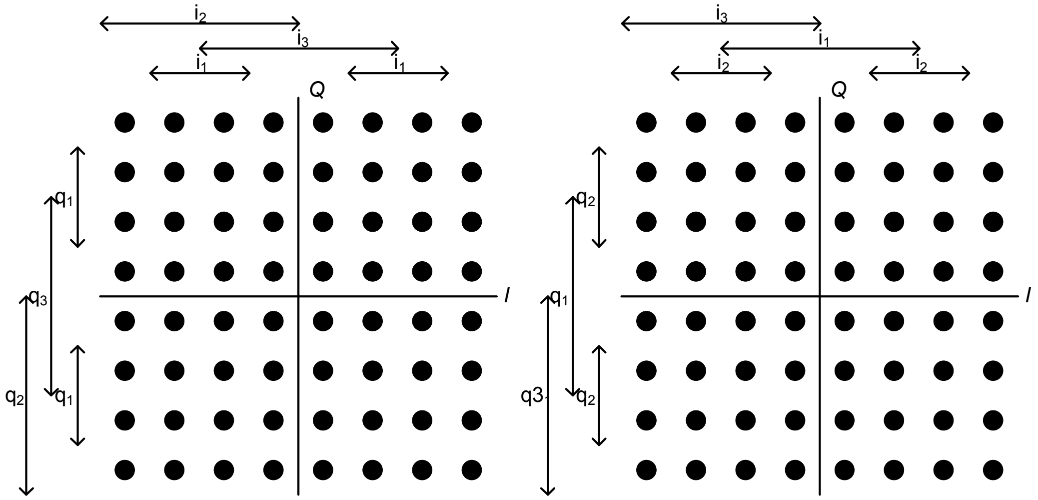
Figure 6 shows the signal constellation in the case of 64QAM for each transmission and Table 8 shows *LLR*s for bits mapped on the in-phase component of the signal constellation according to the mappings for the 1st transmission to 6th transmission. Assuming a superior decoding performance for uniformly distributed *LLR*s of all transmitted bits, the objective is to find mapping rules for retransmissions with the constellations from Figure 6 that equalize the mean *LLR*s after combining. Using the mappings for requested retransmissions according to Table 8 the soft-combined *LLR*s after requested retransmissions are averaged. This is shown in Table 9 for an AWGN channel, where the soft-combined *LLR*s for i_1, i_2 and i_3 after each transmission are given with and without applying the CoRe. It is noted that exactly the same principle applies to the q_1, q_2 and q_3 bits mapped onto the quadrature component. From Table 9 it can be observed that in case of 64QAM for an AWGN channel the averaging is perfect after 6 requested transmissions, hence, a total of 6 different mappings is sufficient.



(a) 1st transmission (b) 2nd transmission



(c) 3rd transmission (d) 4th transmission



(e) 5th transmission (f) 6th transmission

Figure 6. The signal constellation in the case of 64QAM for each transmission

Table 8. ***LLR*****s for bits mapped on the in-phase component of the signal constellation according to mapping according to mapping the 1st transmission to 4th transmission**

Symbol $(i_1i_2i_3q_1q_2q_3)$	1st transmission			2nd transmission			3rd transmission		
	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)
000$q_1q_2q_3$	-4A	-A	-A	-A	-4A	-A	-A	-A	-4A
001$q_1q_2q_3$	-A	-4A	A	-A	-9A	A	-A	-A	4A
010$q_1q_2q_3$	-9A	A	-A	-A	4A	-A	-4A	A	-A
011$q_1q_2q_3$	-16A	4A	A	-A	9A	A	-4A	A	A
100$q_1q_2q_3$	4A	-A	-A	A	-A	-4A	A	-A	-9A
101$q_1q_2q_3$	A	-4A	A	A	-16A	4A	A	-A	9A
110$q_1q_2q_3$	9A	A	-A	A	A	-4A	4A	A	-16A
111$q_1q_2q_3$	16A	4A	A	A	16A	4A	4A	A	16A

Symbol $(i_1i_2i_3q_1q_2q_3)$	4th transmission			5th transmission			6th transmission		
	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)
000$q_1q_2q_3$	-16A	-4A	-A	-A	-16A	-4A	-4A	-A	-16A
001$q_1q_2q_3$	-9A	-A	A	-A	-A	4A	-4A	-A	16A
010$q_1q_2q_3$	-A	4A	-A	-A	16A	-4A	-A	A	-9A
011$q_1q_2q_3$	-4A	A	A	-A	A	4A	-A	A	9A
100$q_1q_2q_3$	16A	-4A	-A	A	-9A	-A	4A	-A	-A
101$q_1q_2q_3$	9A	-A	A	A	-4A	A	4A	-A	A
110$q_1q_2q_3$	A	4A	-A	A	9A	-A	A	A	-4A
111$q_1q_2q_3$	4A	A	A	A	4A	A	A	A	4A

Table 9. Cumulative ***LLR*****s for bits mapped on the in-phase component of the signal constellation for an AWGN channel**

Transmission No.	Corresponding figure	Symbol $(i_1i_2q_1q_2)$	With CoRe			Without CoRe		
			Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)	Mean *LLR* (i_1)	Mean *LLR* (i_2)	Mean *LLR* (i_3)
1	Figure 6(a)	000$q_1q_2q_3$	-4A	-A	-A	-4A	-A	-A
		001$q_1q_2q_3$	-A	-4A	A	-A	-4A	A
		010$q_1q_2q_3$	-9A	A	-A	-9A	A	-A
		011$q_1q_2q_3$	-16A	4A	A	-16A	4A	A
		100$q_1q_2q_3$	4A	-A	-A	4A	-A	-A
		101$q_1q_2q_3$	A	-4A	A	A	-4A	A
		110$q_1q_2q_3$	9A	A	-A	9A	A	-A
		111$q_1q_2q_3$	16A	4A	A	16A	4A	A
2	Figure 6(b)	000$q_1q_2q_3$	-5A	-5A	-2A	-8A	-2A	-2A
		001$q_1q_2q_3$	-2A	-13A	2A	-2A	-8A	2A
		010$q_1q_2q_3$	-10A	5A	-2A	-18A	2A	-2A
		011$q_1q_2q_3$	-17A	13A	2A	-32A	8A	2A
		100$q_1q_2q_3$	5A	-2A	-5A	8A	-2A	-2A

		101$q_1q_2q_3$	2A	-20A	5A	2A	-8A	2A
		110$q_1q_2q_3$	10A	2A	-5A	18A	2A	-2A
		111$q_1q_2q_3$	17A	20A	5A	32A	8A	2A
3	Figure 6(c)	000$q_1q_2q_3$	-6A	-6A	-6A	-12A	-3A	-3A
		001$q_1q_2q_3$	-3A	-14A	6A	-3A	-12A	3A
		010$q_1q_2q_3$	-14A	6A	-3A	-27A	3A	-3A
		011$q_1q_2q_3$	-21A	14A	3	-48A	12A	3A
		100$q_1q_2q_3$	6A	-3A	-14A	12A	-3A	-3A
		101$q_1q_2q_3$	3A	-21A	14A	3A	-12A	3A
		110$q_1q_2q_3$	14A	3A	-21A	27A	3A	-3A
		111$q_1q_2q_3$	21A	21A	21A	48A	12A	3A
4	Figure 6(d)	000$q_1q_2q_3$	-22A	-10A	-7A	-16A	-4A	-4A
		001$q_1q_2q_3$	-12A	-15A	7A	-4A	-16A	4A
		010$q_1q_2q_3$	-15A	10A	-4A	-36A	4A	-4A
		011$q_1q_2q_3$	-25A	15A	4A	-64A	16A	4A
		100$q_1q_2q_3$	22A	-7A	-15A	16A	-4A	-4A
		101$q_1q_2q_3$	12A	-22A	15A	4A	-16A	4A
		110$q_1q_2q_3$	15A	7A	-22A	36A	4A	-4A
		111$q_1q_2q_3$	25A	22A	22A	64A	16A	4A
5	Figure 6(e)	000$q_1q_2q_3$	-23A	-26A	-11A	-20A	-5A	-5A
		001$q_1q_2q_3$	-13A	-16A	11A	-5A	-20A	5A
		010$q_1q_2q_3$	-16A	26A	-8A	-45A	5A	-5A
		011$q_1q_2q_3$	-26A	16A	8A	-80A	20A	5A
		100$q_1q_2q_3$	23A	-16A	-16A	20A	-5A	-5A
		101$q_1q_2q_3$	13A	-26A	16A	5A	-20A	5A
		110$q_1q_2q_3$	16A	16A	-23A	45A	5A	-5A
		111$q_1q_2q_3$	26A	26A	23A	80A	20A	5A
6	Figure 6(f)	000$q_1q_2q_3$	-27A	-27A	-27A	-24A	-6A	-6A
		001$q_1q_2q_3$	-17A	-17A	27A	-6A	-24A	6A
		010$q_1q_2q_3$	-17A	27A	-17A	-54A	6A	-6A
		011$q_1q_2q_3$	-27A	17A	17A	-96A	24A	6A
		100$q_1q_2q_3$	27A	-17A	-17A	24A	-6A	-6A
		101$q_1q_2q_3$	17A	-27A	17A	6A	-24A	6A
		110$q_1q_2q_3$	17A	17A	-27A	54A	6A	-6A
		111$q_1q_2q_3$	27A	27A	27A	96A	24A	6A