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Abstract	A different feedback scheme is depicted for closed-loop MIMO beamforming.
Purpose	Discuss and adopt in 802.16m
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Codebook Design for IEEE 802.16m MIMO Schemes

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Abstract

Differential feedback is considered in the SDD for closed-loop MIMO beamforming. The proposed differential schemes in the contributions exploit both time and space correlations for feedback overhead reduction and performance increase. They can be categorized into rotation based [1-5] and codeword hopping based [6]. In this contribution, we summarize the differential schemes. In addition, we clarify the rotation based scheme in [1] and make comparisons with the other schemes. The results demonstrate that rotation based schemes outperform the codeword hopping scheme [6] in all scenarios. The reason is that the performance of the codeword hopping scheme is always below that of the employed base codebook while the rotation based scheme doesn’t have such a limitation.

1. Signal Model

The assumed system model is

$$\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}\mathbf{s} + \mathbf{n}, \qquad (1)$$

where $\mathbf{n}$ is the complex AWGN with variance N_0; $\mathbf{s}$ is the $N_s \times 1$ transmitted vector with unit power; N_s is the number of spatial streams; $\mathbf{y}$ is the received vector; $\mathbf{H}$ is the channel matrix of size $N_r \times N_t$; $\hat{\mathbf{V}}$ is the beamforming matrix (or vector) of size $N_t \times N_s$. N_r and N_t are the numbers of the receive and transmit antennas respectively.

2. Differential Feedback

There exist strong correlations between beamforming matrixes in adjacent frequencies and frames, and the correlation can be exploited to reduce feedback overhead and increase beamforming accuracy. For example, the channel correlation for samples 5 ms apart is about 0.7 for 3 km/h mobile speed. Any information sent in the previous feedback doesn’t need to be sent again and only the difference between the current beamforming matrix and the previous one needs feeding back. There are two approaches to describe the difference, i.e.

rotation based and codeword hopping based.

2.1. Rotation Based

We take the example of time domain correlation to depict the rotation based scheme. The SS first rotates the center of a differential codebook to $\hat{\mathbf{V}}(t-1)$, which is the beamforming matrix used for the $(t-1)$-th frame, and then computes and quantizes the difference (called differential matrix) between $\mathbf{V}(t)$ the ideal beamforming matrix for the t-th frame and $\hat{\mathbf{V}}(t-1)$. Equivalently, the SS first subtracts $\hat{\mathbf{V}}(t-1)$ from $\mathbf{V}(t)$ and then quantizes the remaining using the differential codebook, also known as the polar cap codebook. This is illustrated in Figure 1. The BS receives the quantization index and rotates $\hat{\mathbf{V}}(t-1)$ by the differential matrix for reconstructing the beamforming matrix. The detailed operations of this scheme denoted by rotation scheme I are listed below.

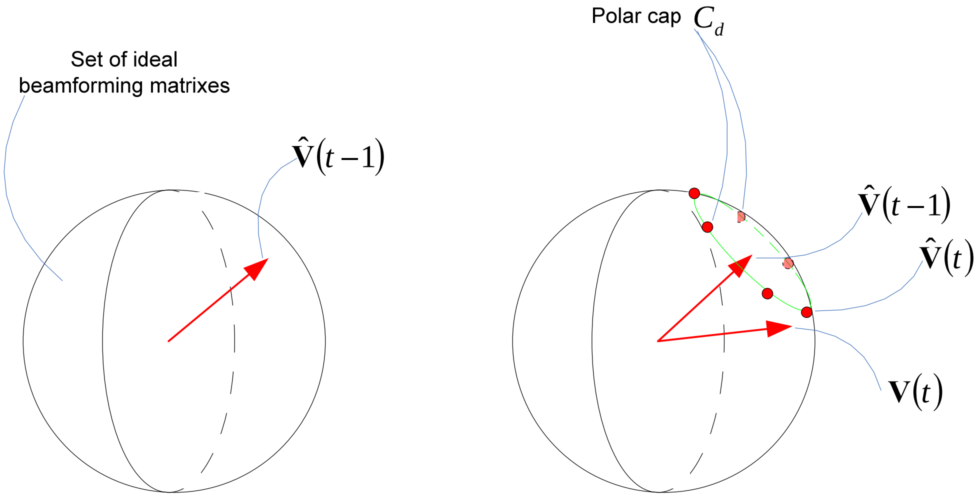

Figure 1. Polar cap, $\hat{\mathbf{V}}(t-1)$, and $\mathbf{V}(t)$.

<u>Rotation scheme I</u>

Differentiation at SS:

$$\mathbf{D} = \mathbf{Q}^H(t-1)\,\mathbf{V}(t), \tag{2}$$

Quantization at SS:

$$\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \left\| \mathbf{D}^H \mathbf{D}_i \right\|_F , \tag{3}$$

Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \mathbf{Q}(t-1)\hat{\mathbf{D}}, \quad (4)$$

Beamforming at BS:

$$\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}(t)\mathbf{s} + \mathbf{n}. \quad (5)$$

where $\mathbf{D}$ is $N_t \times N_s$ and is the differential matrix; $\mathbf{V}(t)$ and $\hat{\mathbf{V}}(t)$ are $N_t \times N_s$ and they are the ideal and quantized beamforming matrixes for frame t, respectively; $\mathbf{Q}(t-1)$ is a $N_t \times N_t$ rotation matrix determined by $\hat{\mathbf{V}}(t-1)$; $\mathbf{D}_i$ is the codeword matrix of the differential codebook (i.e. the polar cap codebook) denoted by C_d. The polar cap codebook only covers a small portion of the whole space of the ideal beamforming matrix as illustrated in Figure 1. Various polar cap codebooks are proposed [1-5]. The quantization in (3) maximizes the received signal power and other criteria such as channel capacity or MSE can be also used. The sanity check of the scheme denoted by rotation scheme I is simple. Assuming small quantization error i.e. $\hat{\mathbf{D}} \approx \mathbf{D}$, substitution of (2) into (4) gives $\hat{\mathbf{V}}(t) \approx \mathbf{Q}(t-1)\mathbf{Q}^H(t-1)\mathbf{V}(t) = \mathbf{V}(t)$.

Rotation scheme I is used in [1-3] and a variant is used [4-5]. In the variant, $N_s = N_t$ is assumed. Namely, the SS always provides feedback for the maximum number of spatial streams even though only one or two streams are usually sent. With this assumption, $\mathbf{D}$ and its quantized version $\hat{\mathbf{D}}$ are $N_t \times N_t$ square matrices. Therefore, all the matrices at the right hand sides of (2), (3), and (4) can exchange order as shown in the operations below.

Rotation scheme II

Differentiation at SS:

$$\mathbf{D} = \mathbf{V}(t)\mathbf{Q}^H(t-1), \quad (6)$$

Quantization at SS:

$$\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \left\| \mathbf{D}^H \mathbf{D}_i \right\|_F, \quad (7)$$

Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \hat{\mathbf{D}}\mathbf{Q}(t-1), \quad (8)$$

Beamforming at BS:

$$\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}(t)\mathbf{s} + \mathbf{n}. \tag{9}$$

There are pros and cons for the variant. On the positive side, the polar cap codebook doesn't vary with the number of spatial streams. However, the performance is poorer than the original scheme, i.e. (2)-(5) for the same feedback overhead. The reason is that $\mathbf{D}$ is of a smaller dimension in the original scheme than its variant, $N_t \times N_s$ vs. $N_t \times N_t$. For the same number of quantization codewords in C_d, a smaller dimension results in a denser codebook and thus smaller quantization errors. For example, for 4 transmit antennas with a single spatial stream, $\mathbf{D}$ in the variant is a 4×4 unitary matrix that has 12 degrees of freedom. In contrast, $\mathbf{D}$ in the original scheme is 4×1 and has only 6 degrees of freedom. For a 3-bit codebook, 8 codewords are distributed to 4×4 and 4×1 for the variant and the original respectively, as illustrated in Figure 2. In the figure, the green circle represents the space of the 4×1 differential matrix $\mathbf{D}$ that is the only useful space for the beamforming of the single stream. On the right of the figure, rotation scheme II deploys part of the codewords outside the green circle and has fewer usable codewords for the needed subspace than rotation scheme I. Therefore, the denser codebook of rotation scheme I results in smaller quantization errors.

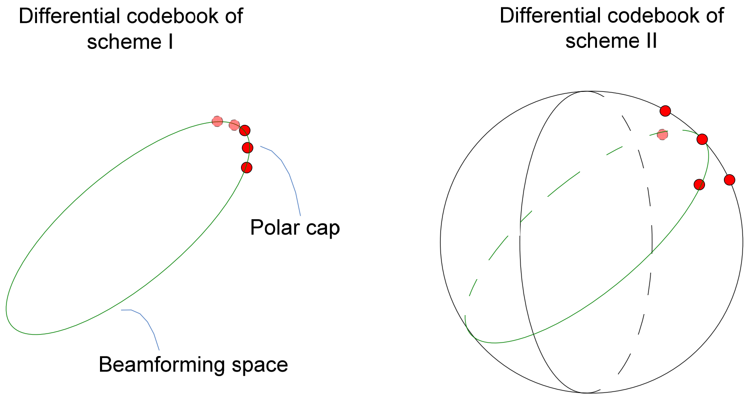

Figure 2. Codeword locations of rotation scheme I and II.

The computation of the square matrix $\mathbf{Q}(t-1)$ needs to be the same at both SS and BS. To fully exploit the correlation from the previous feedback $\hat{\mathbf{V}}(t-1)$, we need the first N_s columns of $\mathbf{Q}(t-1)$ to span the same subspace as that spanned by $\hat{\mathbf{V}}(t-1)$. We propose a low-complexity computation of $\mathbf{Q}(t-1)$. Define the Householder matrix $\mathbf{\Omega}_\mathbf{x}$ of vector $\mathbf{x}$ as

$$\mathbf{\Omega}_\mathbf{x} = \begin{cases} \mathbf{I} - \dfrac{2}{\|\mathbf{w}\|^2}\mathbf{w}\mathbf{w}^H & \text{for } \|\mathbf{w}\|, \|\mathbf{x}\| > 0 \\ \mathbf{I} & \text{otherwise} \end{cases} \tag{10}$$

where $\mathbf{w} = e^{-j\theta}\dfrac{\mathbf{x}}{\|\mathbf{x}\|} - \mathbf{e}_1$; θ is the phase of the first entry of $\mathbf{x}$; $\mathbf{e}_1 = [1 \quad 0 \quad \cdots \quad 0]^T$. For $N_s = 1$, $\hat{\mathbf{V}}(t-1)$ is an

$N_t \times 1$ vector and

$$\mathbf{Q}(t-1) = \mathbf{\Omega}_{\hat{\mathbf{V}}(t-1)}. \qquad (11)$$

For $N_S = 2$, $\hat{\mathbf{V}}(t-1)$ is first converted to a unitary matrix with a zero at the lower left corner as

$$\tilde{\mathbf{V}}(t-1) = \hat{\mathbf{V}}(t-1)\begin{bmatrix} & 1 \\ 1 & \end{bmatrix}\mathbf{\Omega}_{\mathbf{a}}\begin{bmatrix} & 1 \\ 1 & \end{bmatrix} \qquad (12)$$

where $\mathbf{a} = \begin{bmatrix} \hat{\mathbf{V}}(t-1)_{N_t,2} & \hat{\mathbf{V}}(t-1)_{N_t,1} \end{bmatrix}^H$; $\hat{\mathbf{V}}(t-1)_{i,j}$ is the entry of $\hat{\mathbf{V}}(t-1)$ on the i-th row and j-th column. It should be noted that both $\tilde{\mathbf{V}}(t-1)$ and $\hat{\mathbf{V}}(t-1)$ are unitary. $\tilde{\mathbf{V}}(t-1)$ is decomposed by two Householder transformations as

$$\tilde{\mathbf{V}}(t-1) = \begin{bmatrix} \mathbf{\Omega}_{\mathbf{b}} & \\ & 1 \end{bmatrix}\begin{bmatrix} 1 & \\ & \mathbf{\Omega}_{\mathbf{c}} \end{bmatrix}\begin{bmatrix} e^{j\theta_1} & \\ & e^{j\theta_2} \\ & \\ & \end{bmatrix}, \qquad (13)$$

where $\mathbf{b}$ and $\mathbf{c}$ are $(N_T - N_S + 1)\times 1$ unit vectors f; $\mathbf{b} = \begin{bmatrix} \tilde{\mathbf{V}}(t-1)_{1,1} \\ \vdots \\ \tilde{\mathbf{V}}(t-1)_{N_T-1,1} \end{bmatrix}$. The computations of (11)-(13), $\mathbf{b}$, and $\mathbf{c}$ are illustrated in Figure 3. The square matrix $\mathbf{Q}(t-1)$ is computed as

$$\mathbf{Q}(t-1) = \begin{bmatrix} \mathbf{\Omega}_{\mathbf{b}} & \\ & 1 \end{bmatrix}\begin{bmatrix} 1 & \\ & \mathbf{\Omega}_{\mathbf{c}} \end{bmatrix}. \qquad (14)$$

For $N_s = 3$ and $N_t = 4$, the beamforming feedback is for the norm vector of the subspace spanned by the ideal beamforming matrix and the operation is the same as $N_s = 1$ case.

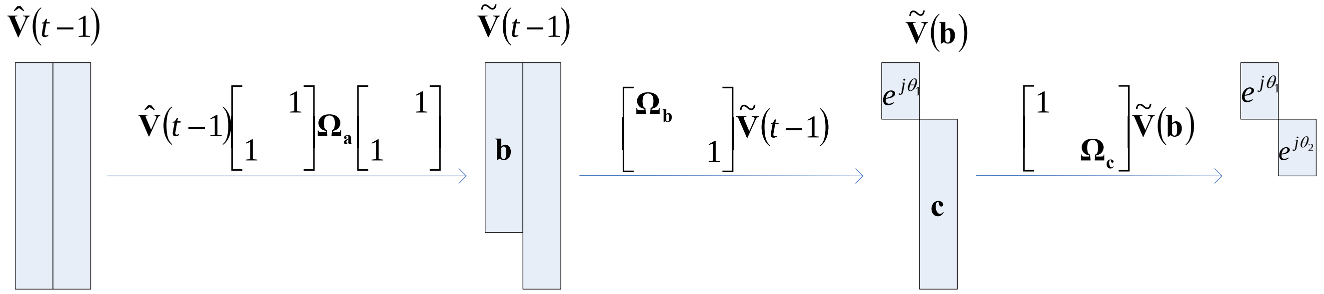

Figure 3. Computations of (11)-(13) and vectors $\mathbf{b}$ and $\mathbf{c}$.

The polar cap codebook for 4×1 is listed. The angle between a outer codeword and the center of the cap is 30 degrees.

Table 1. Polar cap codebook for 4×1 3-bit.

	Codeword vectors

$\mathbf{d}_1$	$[1 \quad 0 \quad 0 \quad 0]^T$
$\mathbf{d}_2$	$[0.8660 \quad 0.3229 + 0.3797i \quad -0.0096 - 0.0016i \quad 0.0373 + 0.0082i]^T$
$\mathbf{d}_3$	$[0.8660 \quad 0.0672 - 0.1647i \quad -0.1390 - 0.4282i \quad 0.0335 + 0.1205i]^T$
$\mathbf{d}_4$	$[0.8660 \quad -0.4448 + 0.1805i \quad 0.0379 - 0.0427i \quad 0.1082 + 0.0677i]^T$
$\mathbf{d}_5$	$[0.8660 \quad 0.0918 - 0.1985i \quad 0.1412 + 0.2267i \quad 0.1608 + 0.3240i]^T$
$\mathbf{d}_6$	$[0.8660 \quad 0.0073 - 0.1214i \quad -0.1744 + 0.1388i \quad 0.2578 - 0.3450i]^T$
$\mathbf{d}_7$	$[0.8660 \quad -0.0553 - 0.0087i \quad -0.2615 + 0.1477i \quad -0.3949 + 0.0265i]^T$
$\mathbf{d}_8$	$[0.8660 \quad 0.0088 - 0.0613i \quad 0.4032 - 0.0381i \quad -0.2082 - 0.1969i]^T$

2.1. Codeword Hopping Based

The other approach for differential feedback is the codeword hopping scheme as shown in Figure 4 and depicted in [6]. A base codebook is employed and a neighbor list is built for each codeword. The 8 closest codewords around each codeword are noted in the list. If the current beamforming matrix is a codeword, then the next beamforming matrix will hop to one of the 8 neighbor codewords in the list. If the channel varies slightly between adjacent feedbacks, for the next beamforming matrix, the best codeword in the whole base codebook can be found in the neighbor list mostly likely. However, even at low speed such as 3 km/h, there is a certain probability that the best codeword is not in the neighbor list. In such a case, an approximate codeword in the neighbor list is fed back and the performance degrades. Therefore, it is apparent that the performance of the codeword hopping scheme is always worse than that of the one-shot feedback using the whole base codebook. This is the limitation of codeword hopping. Compared with the rotation scheme, the advantage of codeword hopping is that no rotation operation and no differential codebook are needed.

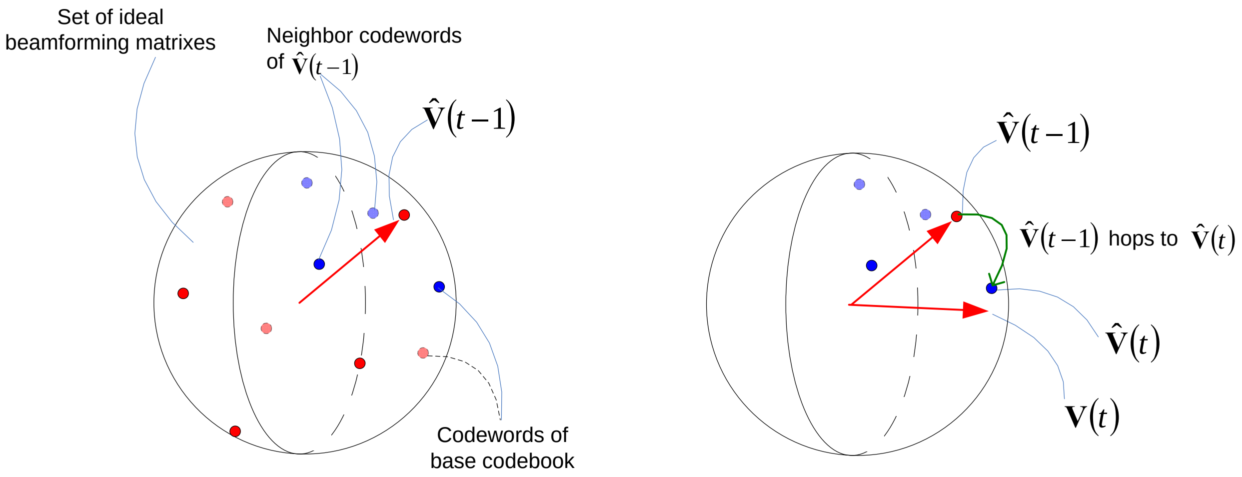

Figure 4. Codeword hopping.

3. System Level Simulations

We make three comparisons. First of all, we compare the differential schemes [1][4] with non-differential schemes. Secondly, we compare the rotation based schemes [1][4] with the codeword hopping scheme [6]. Finally, the two rotation based schemes [1][4] are compared. Both single-user MIMO (SU-MIMO) and multiuser MIMO (MU-MIMO) are simulated. The antenna configuration is 4 transmit and 2 receive. The transmit antenna spacing is 4 and 0.5 wavelengths for weakly and highly correlated channels respectively. The differential scheme consists of one initial step and nine differential steps for each feedback period. A base codebook of 4-bit or 6-bit is used in the initial step and a differential codebook of 4-bit or 3-bit is used for the differential steps. The feedback rate is every 5 ms. The feedback is error free. Spectrum efficiency (SE) of the system is computed.

In Table 2, Table 3, and Table 4, the schemes are compared in SU-MIMO. Rotation based differential scheme [1] is compared to the 16e 6-bit codebook in Table 2. The 16e 6-bit codebook is used for the initial step in the differential scheme. The feedback overheads for the differential scheme and 16e 6-bit codebook are 3.3 bits and 6 bits per feedback, respectively. The differential scheme reduces the feedback overhead by 45%. In both uncorrelated and highly correlated channels, the rotation based differential scheme outperforms the non-differential scheme in terms of throughput and overhead. Since the throughput performance of the codeword hopping based scheme [6] is below the non-differential scheme for the same codebook, the rotation based scheme [1] outperforms the codeword hopping based scheme [6]. Another comparison between differential and non-differential schemes is made in Table 3 for a DFT codebook [4]. As expected, the differential scheme outperforms the non-differential for all correlation scenarios. The improvement due to differential feedback decreases as the channel correlation increases for rotation scheme II with DFT codebook [4]. In Table 4, two rotation based differential schemes [1] and [4] are compared. For the reason in Section 2.1, rotation scheme I [1] outperforms rotation scheme II [4] in terms of throughput and overhead. Scheme [1] reduces the overhead of scheme [4] by 18%.

Table 2. Spectrum efficiency of 802.16e 6-bit codebook and 3-bit differential codebook for SU-MIMO.

	Uncorrelated channels	Highly Correlated channels
16e 6-bit codebook (b/s/Hz)	6.7217	7.2527
Rotation scheme I, 3-bit differential [1] (b/s/Hz)	6.7706	7.5414
Codeword hopping, 3-bit differential [6] (b/s/Hz)	< 6.7217	< 7.2527
SE gain of rotation scheme I [1] over codeword hopping based [6]	> 0.73%	> 3.98%

Table 3. Spectrum efficiency of DFT 4-bit codebook and 4-bit differential codebook for SU-MIMO.

	Uncorrelated channels	Weakly correlated channels	Highly correlated channels
DFT 4-bit codebook (b/s/Hz)	6.4855	6.8481	7.5046
Rotation scheme II, 4-bit differential [4] (b/s/Hz)	6.7177	7.0519	7.5060
SE gain of differential [4] over non-differential	3.58%	2.98%	0.02%

Table 4. Spectrum efficiency and feedback overhead of two rotation based schemes.

	Uncorrelated channels	Highly correlated channels
Rotation scheme I [1] (b/s/Hz)	6.7706	7.5414
Rotation scheme II [4] (b/s/Hz)	6.7177	7.5060
SE gain of [1] over [4]	0.79%	0.47%
Overhead comparison: [1] vs. [4]	3.3 bits : 4 bits	3.3 bits : 4 bits

In Table 5 and Table 6, the schemes are compared in MU-MIMO. Rotation based differential scheme [1] is compared to the 16e 6-bit codebook in Table 5. The 16e 6-bit codebook is used for the initial step in the differential scheme. The feedback overheads for the differential scheme and 16e 6-bit codebook are 3.3 bits and 6 bits per feedback, respectively. The differential scheme reduces the feedback overhead by 45%. In all correlation scenarios, the rotation based differential scheme outperforms the non-differential scheme in terms of throughput and overhead. Since the throughput of the codeword hopping based scheme [6] is below the non-differential scheme for the same codebook, the rotation based scheme [1] again outperforms the codeword hopping based scheme [6]. In Table 6, two rotation based differential schemes [1] and [4] are compared. The original rotation scheme I [1] employs a 6-bit transformed codebook for the initial step followed by 3-bit differential feedbacks and the average overhead is 3.53 bits per feedback. The variant rotation scheme II [4] employs a 4-bit DFT codebook for the initial step followed by 4-bit differential feedbacks and the average overhead is 4 bits per feedback. Rotation scheme I [1] outperforms rotation scheme [4] in terms of both throughput and overhead. Scheme [1] reduces the overhead of scheme [4] by 12%.

Table 5. Spectrum efficiency of 802.16e 6-bit codebook and 3-bit differential codebook for MU-MIMO.

	Uncorrelated channels	Weakly correlated channels	Highly correlated channels
16e 6-bit codebook (b/s/Hz)	6.3643	6.903	7.6895
Rotation scheme I, 3-bit differential [1] (b/s/Hz)	6.6172	7.5492	9.766
Codeword hopping, 3-bit differential [6] (b/s/Hz)	< 6.3643	< 6.903	< 7.6895
SE gain of rotation scheme I [1] over codeword hopping [6]	> 3.97%	> 9.36%	> 27.0%

Table 6. Spectrum efficiency and feedback overhead of two rotation based schemes.

	Uncorrelated channels	Weakly correlated channels	Highly correlated channels
Rotation scheme I [1] (b/s/Hz)	6.6993	8.0027	11.3129
Rotation scheme II [4] (b/s/Hz)	6.6766	7.7787	10.1239
SE gain of [1] over [4]	0.34%	2.88%	11.74%
Overhead comparison: [1] vs. [4]	3.53 bits : 4 bits	3.53 bits : 4 bits	3.53 bits : 4 bits

4. Conclusion

Differential schemes reduce the overhead of the non-differential schemes. Between two types of differential schemes, rotation based scheme outperforms codeword hopping based scheme. In addition, the rotation based scheme [1] performs better than the other rotation based scheme [4] because [1] employs a compacter codebook.

5. Reference

[1]. IEEE C802.16m-08/1182r3, “Codebook Design for IEEE 802.16m MIMO Schemes.”

[2]. IEEE C802.16m-08/1109, “DL SU-MIMO codebooks.”

[3]. IEEE C802.16m-08/1095r1, “Differential Precoding Codebook.”

[4]. IEEE C802.16m-08/1187, “Evaluation of CL SU and MU MIMO Codebooks.”

[5]. IEEE C802.16m-09/0024, “Simplified Differential Feedback Method for IEEE 802.16m CL SU/MU-MIMO.”

[6]. IEEE C802.16m-08/1074r1, “Evaluation of Codebook and Differential Feedback for DL Closed-Loop SU-MIMO.”

6. Proposed Text

11.8.2.1.3 Feedback for SU-MIMO

[add one sentence at the end of line 32 in page 103]

The differential mode: the feedback from a mobile station provides a differential knowledge of the short-term channel information. This feedback represents information that is used along with other feedback information known at the base station for determining a new precoder. Rotation based scheme shall be supported.

11.8.2.2.3.2 CSI feedback

[add one sentence at the end of line 6 in page 105]

The differential mode: the feedback from a mobile station provides a differential knowledge of the short-term channel information. This feedback represents information that is used along with other feedback information known at the base station for determining a new precoder. Rotation based scheme shall be supported.