Correlation Models for Shadow Fading Simulation

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Purpose:

TGm Evaluation Methodology development

Correlation Models for Shadow Fading Simulation

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Outline

- Shadow Fading Model for System Level Simulation
- Auto-correlation Model for Shadow Fading
- Cross-correlation Model for Shadow Fading

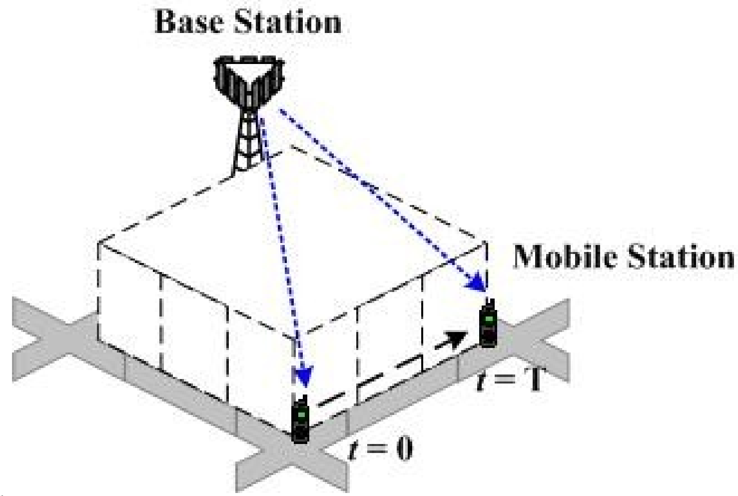
Shadow Fading Model for System Level Simulation

- The shadow fading effect is usually modeled by a **<u>log-normal</u>** random variable in system level simulation
  - Its mean *m* is usually given as 0dB, and the standard deviation σdB depends on the propagation environment (ex. 8dB for urban).
    - This random variable is normal distributed in dB

- When doing the system level simulation, a normal distributed random variable can be dropped from time to time to represent the shadow fading for each radio link.
  - The typical way is to drop the random variable independently for different time and for different radio link
    - i.e. the shadow fading effects for each radio link at each time instance are simulated in uncorrelated manner

Auto-correlation Model for Shadow Fading

- However, the shadow fading effect of <u>the same radio link</u> is highly correlated for the nearby locations.
- A simple and well accepted <u>auto</u>-correlation model was proposed by Gudmundson [1] for this effect:



the correlation of signal samples separated by k sampling instances:

$$R(k) = \sigma^2 a^{|k|}$$

$$a = \varepsilon_D^{vT/D}$$

σ *is the standard deviation of shadow fading model*
a *is the correlation coefficient*
ε_D *is the correlation between two locations separated by distance D*
v *is the mobile velocity*
T *is the sampling interval*

Auto-correlation Model for Shadow Fading

- The aforementioned model was reference by [2] and modified as:

$$r(\nabla x) = e^{-\frac{|\nabla x|}{d_{cor}} \ln 2}$$

d_{cor} de-correlation distance;

its definition here is the distance separation at which the correlation coefficient reduces to ½ .

Δx the distance between two observation locations

ρ the correlation coefficient

- Simulation Methodology:
  - If L_1 is the log-normal component at position P_1, L_2 is for P_2 which is $\triangle x$ away from P_1. Then L_2 is normally distributed in dB with mean $\rho(\triangle x)L_1$ and variance $(1-\rho(\triangle x)^2)\sigma^2$.
    - *derived in next page*

Auto-correlation Model for Shadow Fading

- Consider X and Y are Normal distributed random variables. Given Y=y and correlation coefficient as ρ, then the distribution of (X|Y=y) can be obtained as [3]:

μ_1, μ_2 *and* σ_1^2, σ_2^2 are the mean and variance of X and Y

$$\mathrm{f_X}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$= \frac{\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{X,Y}^2}} \cdot \left[\frac{\exp\left(-\frac{(y-\mu_2)^2}{2\sigma_2^2}\right)}{\sqrt{2\pi\sigma_2^2}}\right]^{-1}$$

$$= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{y-\mu_2}{\sigma_2}\right)^2 - (1-\rho_{X,Y}^2)\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$$

$$= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\frac{x-\mu_1}{\sigma_1} - \rho_{X,Y}\frac{y-\mu_2}{\sigma_2}\right]^2\right\}$$

$$= \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{-\frac{\left[x-\mu_1-\rho_{X,Y}\frac{\sigma_1}{\sigma_2}\left(y-\mu_2\right)\right]^2}{2\sigma_1^2(1-\rho_{X,Y}^2)}\right\}$$

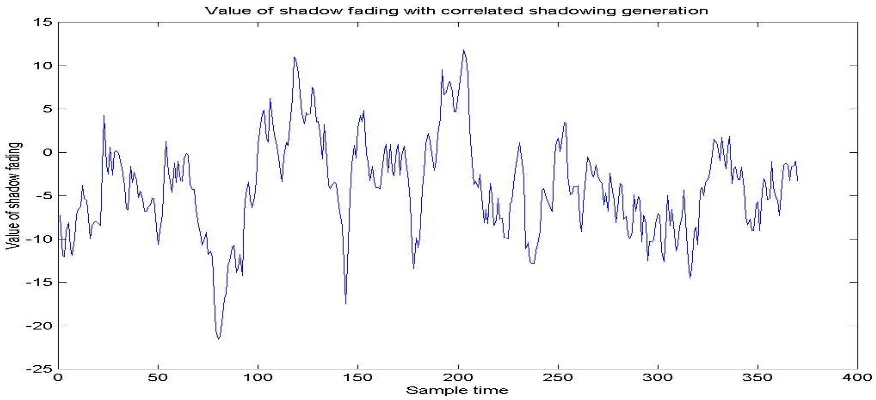
the mean of $f_X(x \mid y)$ is $\mu_1 + \rho_{X,Y}\frac{\sigma_1}{\sigma_2}\left(y-\mu_2\right)$, with variance $\sigma_1^2(1-\rho_{X,Y}^2)$.

Consider μ_1=μ_2=0 and σ_1=σ_2=σ , then the mean of $f_X(x \mid y)$ is $\rho_{X,Y}\left(y-\mu_2\right)$ and variance $\sigma_1^2(1-\rho_{X,Y}^2)$

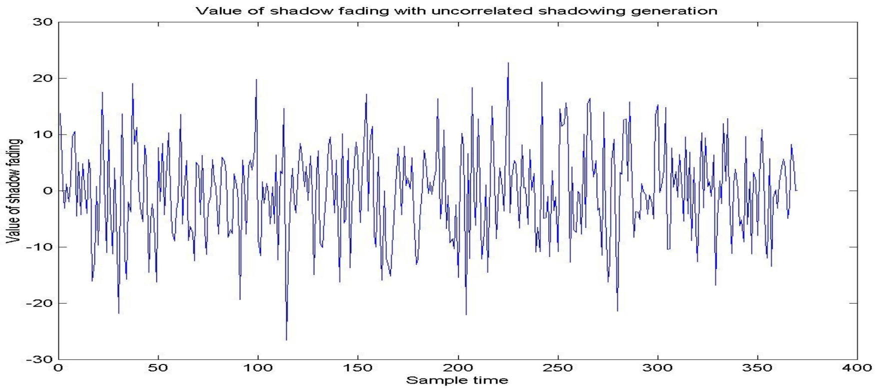
Auto-correlation Model for Shadow Fading

- Example:
  – Shadow fading effect simulated with and without auto-correlation model

Correlated shadow fading (with auto-correlation[2])



Uncorrelated shadow fading

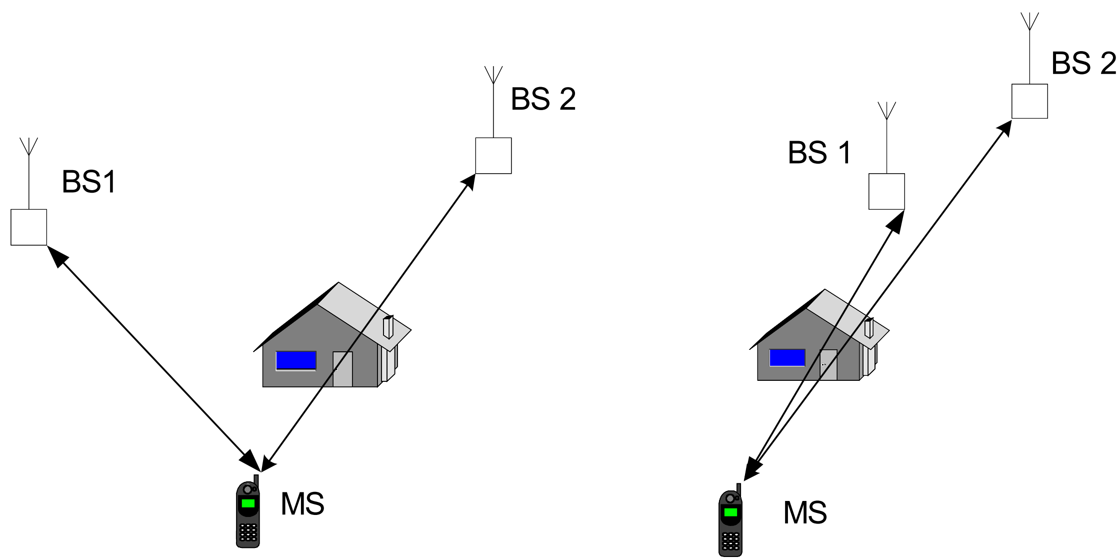


Cross-correlation Model for Shadow Fading

- As for the shadow fading effect among different radio link, the cross-correlation indicates the similarity of the propagation environment.

- For example, the shadow fading effects for the two radio links are less correlated in the left figure then in right one.

Longer common propagation path induce higher correlation



Cross-correlation Model for Shadow Fading

- Cross-Correlation Model for Shadow Fading[4] :

  θ: *Angle-of-Arrival Difference (AAD)*

  ρ: *correlation coefficient*

$$\rho(\theta) = \begin{cases} 0.8 - \dfrac{|\theta|}{150} & \text{if } |\theta| \le 60^{o} \\ 0.4 & \text{if } |\theta| > 60^{o} \end{cases}$$

Cross-correlation Model for Shadow Fading

- Simulation Methodology:
  1. Generate the uncorrelated random variables $\boldsymbol{X}=[\boldsymbol{x_1},\boldsymbol{x_2}\ldots\boldsymbol{x_N}]$
     - *each x_n is Normal distributed with zero mean and stdσ_n in dB*

  1. Obtain the correlation coefficient matrix $\boldsymbol{C}$

$$\Gamma = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1N} \\ r_{21} & 1 & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N1} & r_{N2} & \cdots & 1 \end{bmatrix}$$

and

$$\Gamma = CC^T$$

C can be obtained by Cholesky decomposition
(note: it takes high computation power)

  1. Then cross-correlated shadow fading effect $\boldsymbol{Y}=[\boldsymbol{y_1},\boldsymbol{y_2}\ldots\boldsymbol{y_N}]$ can be obtained by

$$\boldsymbol{Y}=\boldsymbol{CX}$$

References

[1] Gudmundson, M., “Correlation Model for Shadow Fading in Mobile Radio Systems,” Electronics Letters, pp.2145-2146, vol. 27, No 23, November 1991.

[2] ETSI TR 101.112 v3.2.0, “Selection procedures for the choice of radio transmission technologies of the UMTS”, April 1998.

[3] I-Kang Fu, “A Dynamic Simulation Platform for Heterogeneous Multiple Access Systems,” Master Thesis, Department of Electrical Engineering, National Chung Cheng University, Taiwan, R.O.C., July 2002.

[4] Thomas Klingenbrunn, Preben Mogensen “Modelling Cross-Correlated Shadowing in Network Simulations”, IEEE VTC ,1999, pp.1407-1411.