Undetected Error Analysis HEC-16 & HEC-32 Polynomials (8B10B Update)

Nirmal R. Saxena
Paritosh Kulkarni
Chip Engines, Inc.
Sunnyvale, CA 94086

Outline

- Problem Statement
  - HEC-16 vs HEC-32
- Coding Preliminaries
  - Weight Distribution of Codes
  - Undetected Error Probability Formula
  - 8B/10B Error Weight Distribution & Escapes
- Undetected Error Probability Results
  - Packets without Scrambling
  - Packets with Scrambling
  - Packets with 8B/1B Coding
- Conclusions

Problem Statement

Does HEC-16 Provide Adequate Error Coverage ?

- 16 Byte (128 bits) RPR Header
- Realistic Fiber Bit Error Rates ($10^{-9} \sim 10^{-13}$)
- Generator Polynomial

  $X^{16}+X^{12}+X^{5}+1$

Do We Need HEC-32 ?

- Generator Polynomial

  $X^{32}+X^{26}+X^{23}+X^{22}+X^{16}+X^{12}+X^{11}+X^{10}+X^{8}+X^{7}+X^{5}+X^{4}+X^{2}+X+1$

Approach– Determine Undetected Error Probability

- HEC-16, Code Length L = 144 (128+16), Check Bits r = 16
- HEC-32, Code Length L = 160 (144+32), Check Bits r = 32

Coding Preliminaries

Bit Error Rate– ϵ

Weight Distribution Enumerator N[L,w]

- For A Given Generator Polynomial

N[L,w] – Number of Length *L* Codewords with *w* ones

Probability of Undetected Error

$$\sum_{w=1}^{w=L} N(L,w)(1-\epsilon)^{L-w}\epsilon^{w}$$

Coding Preliminaries

Undetected Error Probability

- Computationally More Efficient to Use Dual Codes
- Every (L,r) Code Has a (L,L-r) Dual Code
  - See Reference [1] for Definition of Dual Codes
- Dual Code Weight Enumerator M[L,w]
  - See Reference [2] for Algorithms to Compute M[L,w]

Probability of Undetected Error

$$\sum_{w=1}^{w=L} N(L,w)(1-\epsilon)^{L-w}\epsilon^{w}$$

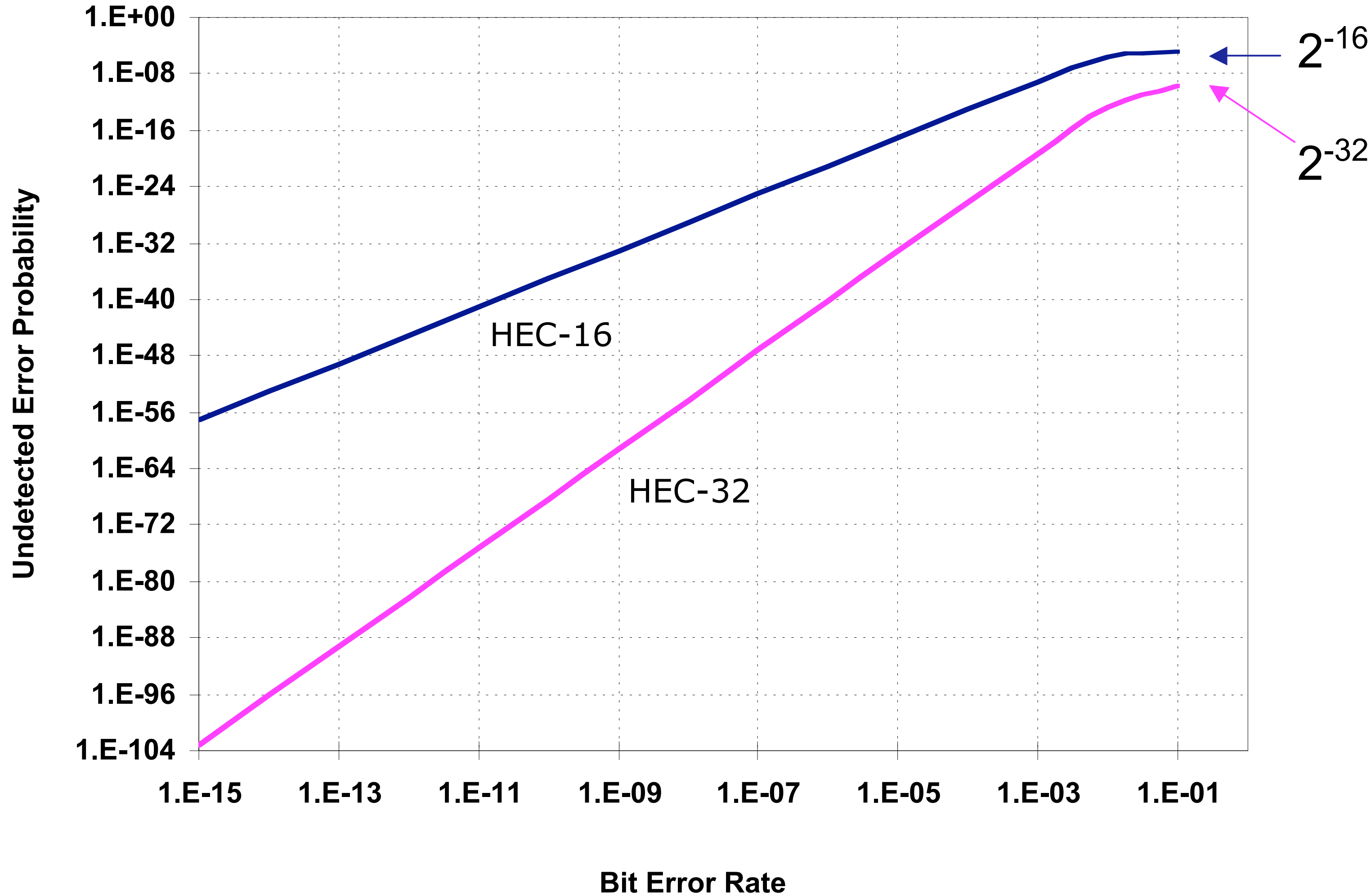
$$= 2^{-r}\sum_{w=0}^{w=L} M(L,w)(1-2\epsilon)^{w} - (1-\epsilon)^{L}$$

Results without Scrambling Consideration

Undetected Error Probability

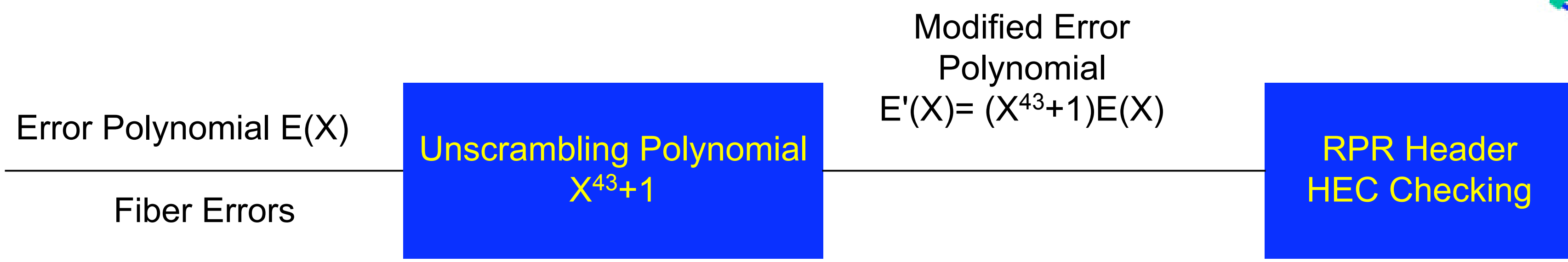
Bit Error Rate	HEC-16	HEC-32
10^{-15}	8.31×10^{-58}	6.00×10^{-104}
10^{-14}	8.31×10^{-54}	6.00×10^{-97}
10^{-13}	8.31×10^{-50}	6.00×10^{-90}
10^{-12}	8.31×10^{-46}	6.00×10^{-83}
10^{-11}	8.31×10^{-42}	6.00×10^{-76}
10^{-10}	8.31×10^{-38}	6.00×10^{-69}
10^{-09}	8.31×10^{-34}	6.00×10^{-62}
10^{-08}	8.31×10^{-30}	5.99×10^{-55}
10^{-07}	8.31×10^{-26}	5.99×10^{-48}
10^{-06}	8.31×10^{-22}	5.99×10^{-41}
10^{-05}	8.29×10^{-18}	5.99×10^{-34}
10^{-04}	8.19×10^{-14}	5.93×10^{-27}
10^{-03}	7.22×10^{-10}	5.36×10^{-20}
10^{-02}	2.12×10^{-06}	1.91×10^{-13}
10^{-01}	1.53×10^{-05}	2.32×10^{-10}

HEC-16 and HEC-32 Comparison



Results with Scrambling Artifacts

Scrambling Artifacts

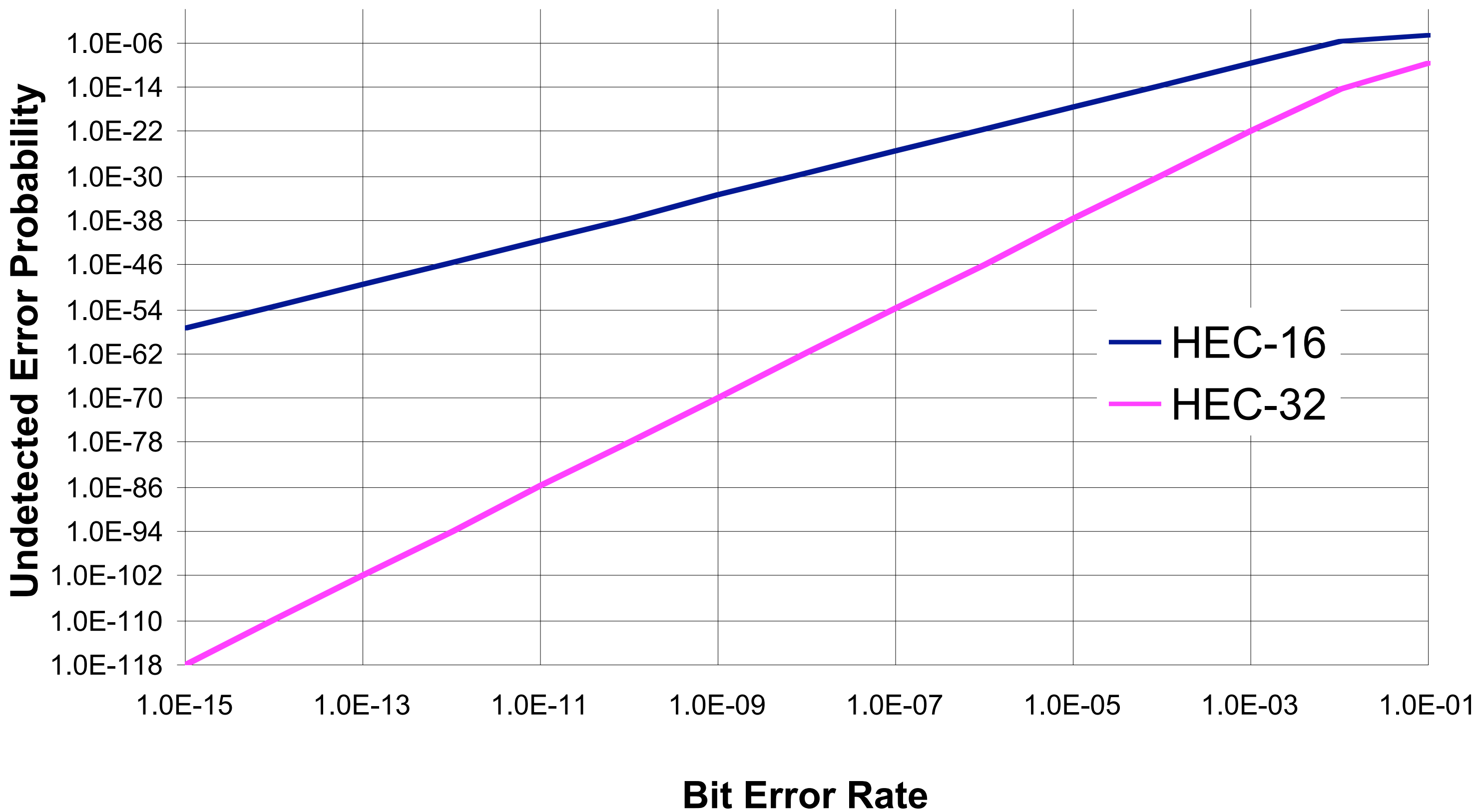


- Bit Errors on Fiber Can Be Represented By Error Polynomial E(X)
  - Unscrambling Modifies the Error Polynomial By A Multiplication Factor
- Undetected Error Probability Analysis Approach
  - Need Only Consider Error Polynomials E(X) of Degree Up To L-43
  - For E(X) with Degree Greater Than L-43
    - Errors Spill into Payload of RPR Packet
    - Detected By FCS-32 of RPR Packet (Up To Detection Probability)
- HEC-16 & HEC-32 Properties
  - HEC-16 shares a Common Factor (1+X) with $(1+X^{43})$
  - HEC-32 is Primitive Polynomial and Relatively Prime to $(1+X^{43})$
  - Undetected Error Probability Calculation Includes These Properties

Undetected Error Probability

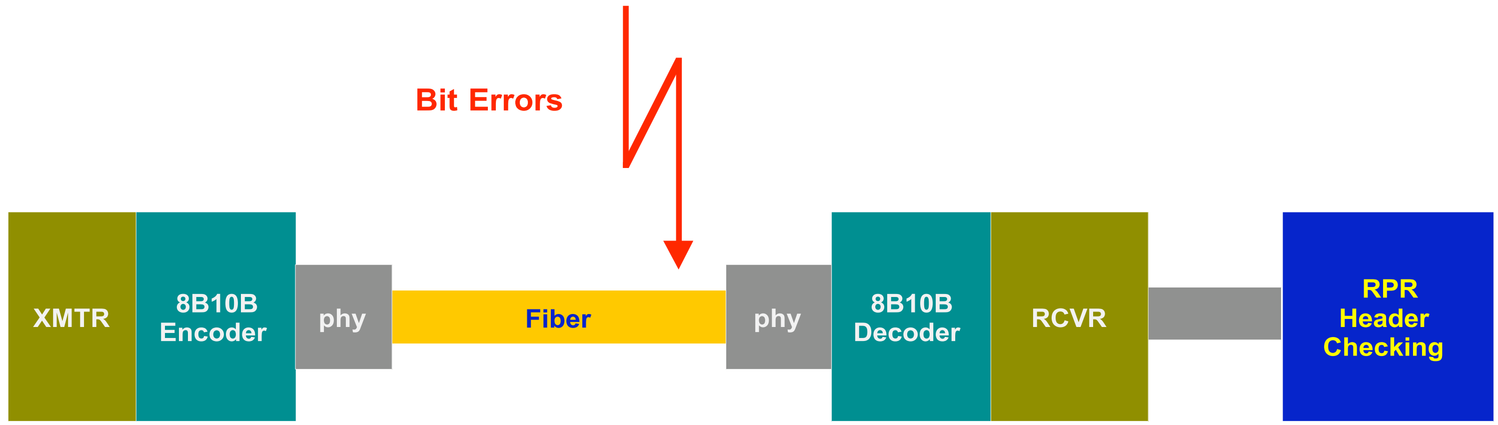
Bit Error Rate	HEC-16	HEC-32
10^{-15}	2.94×10^{-58}	1.45×10^{-118}
10^{-14}	2.94×10^{-54}	1.45×10^{-110}
10^{-13}	2.94×10^{-50}	1.45×10^{-102}
10^{-12}	2.94×10^{-46}	1.45×10^{-94}
10^{-11}	2.94×10^{-42}	1.45×10^{-86}
10^{-10}	2.94×10^{-38}	1.45×10^{-78}
10^{-09}	2.94×10^{-34}	1.45×10^{-70}
10^{-08}	2.94×10^{-30}	1.45×10^{-62}
10^{-07}	2.94×10^{-26}	1.45×10^{-54}
10^{-06}	2.94×10^{-22}	1.45×10^{-46}
10^{-05}	2.94×10^{-18}	1.45×10^{-38}
10^{-04}	2.91×10^{-14}	1.44×10^{-30}
10^{-03}	2.68×10^{-10}	1.32×10^{-22}
10^{-02}	1.21×10^{-06}	5.61×10^{-15}
10^{-01}	3.08×10^{-05}	2.10×10^{-10}

HEC-16 and HEC-32 Comparison



Results with 8B/10B Artifacts

8B/10B Error Analysis



00000000 →	100111 0100 →	100111 0101 →	01000000	RCVR Escape Disparity OutOfSync
	RD- => RD-	RD- => RD+		
00111011 →	001001 1001 →	001101 1001 →	00101100	RCVR Escape Disparity OutOfSync
	RD+ => RD-	RD+ => RD+		
00111011 →	001001 1001 →	001101 0001 →	Illegal Code	RCVR Detect
	RD+ => RD-	RD+ => RD-		

Analysis Approach & Observations

- Odd Number of Bit Errors (1, 3, …)
  - Cause OutOfSync XMTR & RCVR Disparity
  - Detected at the RCVR
- Only Data Code (Dx.y) Errors Considered
  - Control Code Errors (Kx.y) Not Part of the Analysis
  - Gives An Upper Bound on Undetected Probability
- Combinatorial Weight Distribution Calculation
  - Error Weight Distribution Exhaustively Enumerated
  - Errors that Escape RCVR Evenly Distributed Over 8-bits
  - Simple Formula for Undetected Error Probability
- Simulation Based Estimation
  - Exhaustive Error Injection of 1, 2, 3, 4, 5 Bit Errors

Undetected Error Probability Formula

$$G(z) = \left(\sum_{i=0}^{10} A_i z^i\right)^L$$

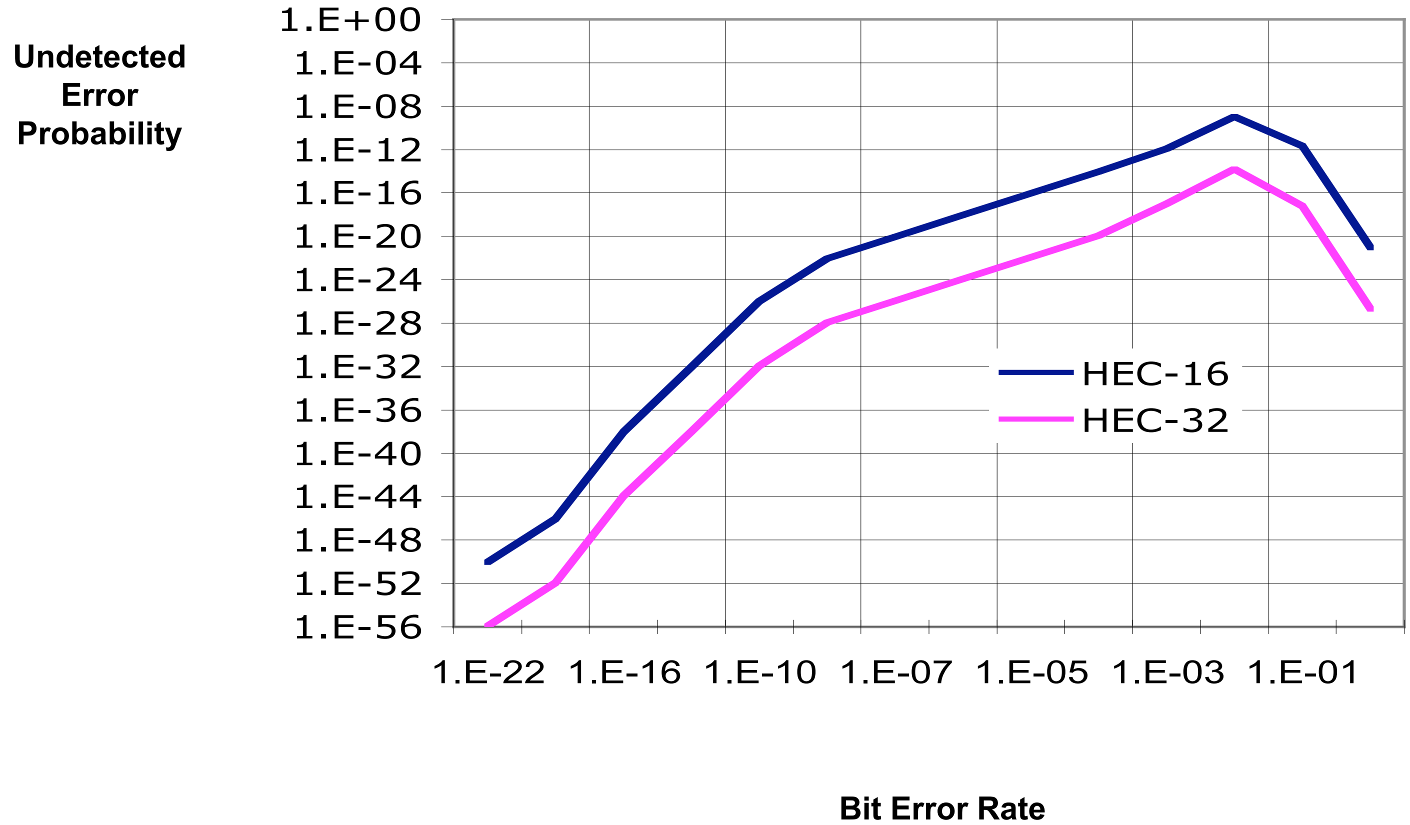
$$Un\det ectError\Pr obability = \frac{(1 - \square)^L \left(G\left(\frac{\square}{1 - \square}\right) - 1\right)}{2^r}$$

HEC-16: L=18, r=16
HEC-32: L=20, r=32

Undetected Error Probability (8b10b)

Bit Error Rate	HEC-16	HEC-32
10^{-22}	9.53×10^{-51}	1.16×10^{-56}
10^{-20}	9.53×10^{-47}	1.16×10^{-52}
10^{-16}	9.53×10^{-39}	1.16×10^{-44}
10^{-13}	9.53×10^{-33}	1.16×10^{-38}
10^{-10}	9.53×10^{-27}	1.16×10^{-32}
10^{-08}	9.53×10^{-23}	1.16×10^{-28}
10^{-07}	9.53×10^{-21}	1.16×10^{-26}
10^{-06}	9.53×10^{-19}	1.16×10^{-24}
10^{-05}	9.52×10^{-17}	1.16×10^{-22}
10^{-04}	9.42×10^{-15}	1.25×10^{-20}
10^{-03}	1.31×10^{-12}	1.04×10^{-17}
10^{-02}	1.07×10^{-09}	1.63×10^{-14}
10^{-01}	2.17×10^{-12}	6.12×10^{-18}
0.5	8.80×10^{-22}	2.11×10^{-27}

HEC-16 vs HEC-32 (8b10b)



Conclusions

- HEC-16 Provides More than Adequate Protection

- For Realistic Fiber Error Rates (10^{-9} ~ 10^{-13})
  – With or Without Scrambling or 8B10B Artifacts
  – Undetected Error Probability is of Order 10^{-24} ~ 10^{-34}

- Even for Signal Degrade Conditions (BER ~ 10^{-6})
  – Undetected Error Probability for HEC-16 is of Order 10^{-18}

References

1. F. J. MacWilliams and N.J.A Sloane– *The Theory of Error-Correcting Codes*, North Holland, Elsevier Science Publishers, 1986.
2. N. R. Saxena, P. Franco, and E.J. McCluskey, “Simple Bounds on Serial Signature Analysis Aliasing for Random Testing,” IEEE Transactions on Computers, Vol. 41, No. 5, pp. 638-645, May 1992.
3. IEEE 802.3 Std, 8B/10B Code Tables