Efficient Estimation of Bit Error Rates and Eye Diagrams in Equalizer Enhanced Links

Kasyapa Balemarthy, Stephen E. Ralph, Steven W. McLaughlin

stephen.ralph@ece.gatech.edu

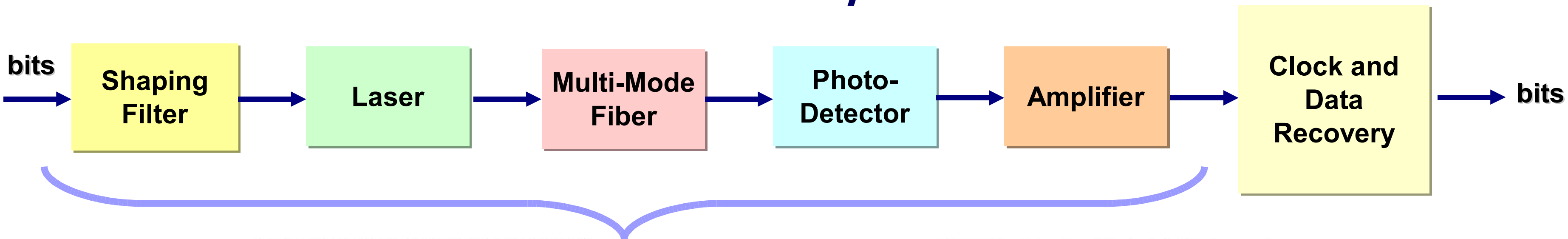
Georgia Institute of Technology

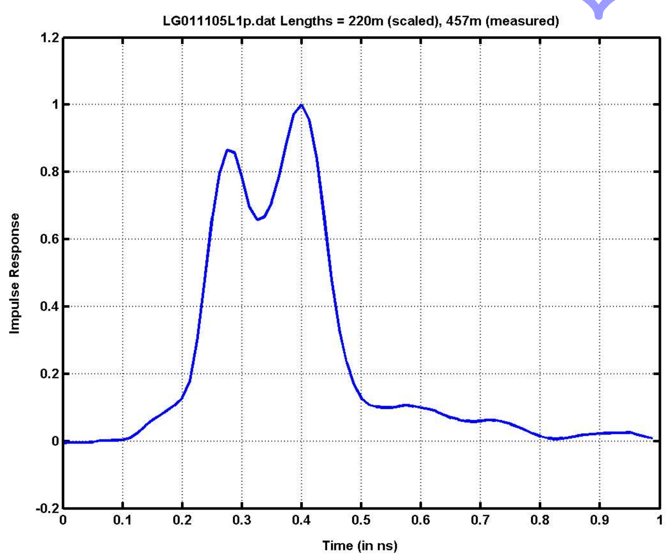
School of Electrical and Computer Engineering

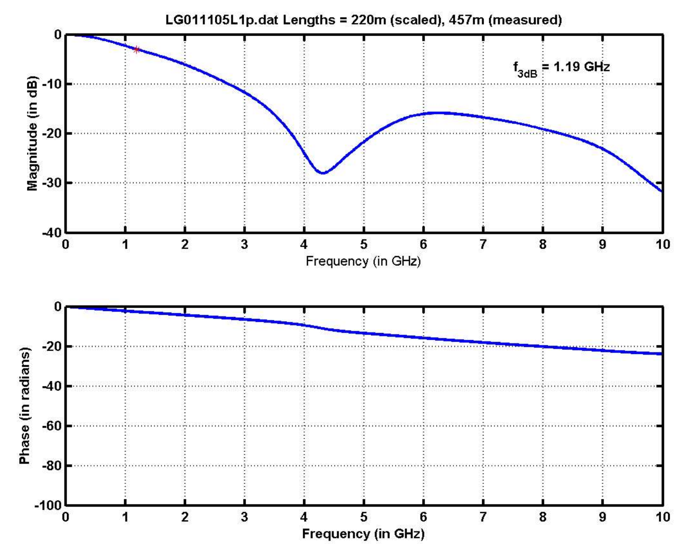
Atlanta, GA

Georgia Institute of Technology

Bit Error Rate Estimation in 10Gb/s LANs







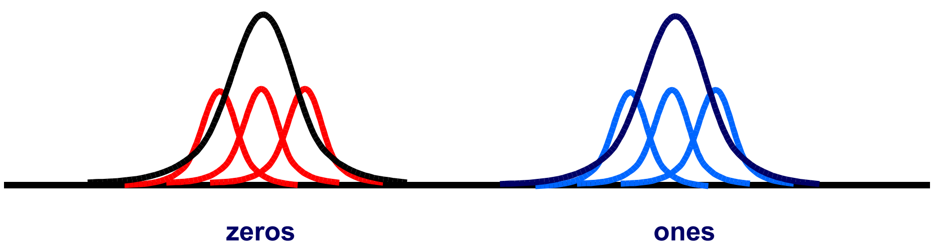
- Monte Carlo simulations are prohibitively long at a BER of 10^{-12} :
  - At least 100 errors for reliable BER estimation for an AWGN Channel
    - Need to transmit at least 10^{14} bits
  - Requires about 2.78 hours of real-time data at 10Gb/s
- Need to resort to BER estimation techniques

Method 1: Gaussian Approx. Approach

- **Model the ISI and the noise term together as AWGN**
- Assume 1's and 0's to be equally likely and independent
- Estimate means and standard deviations
  - at the zero rail (m_0, σ_0)
  - at the one rail (m_1, σ_1)
- Optimum Bit Error Rate (BER) and Threshold are then given by:

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\left[\frac{m_1 - m_0}{\sigma_1 + \sigma_0}\right]\right) \qquad D = \frac{m_0\sigma_1 + m_1\sigma_0}{\sigma_1 + \sigma_0}$$

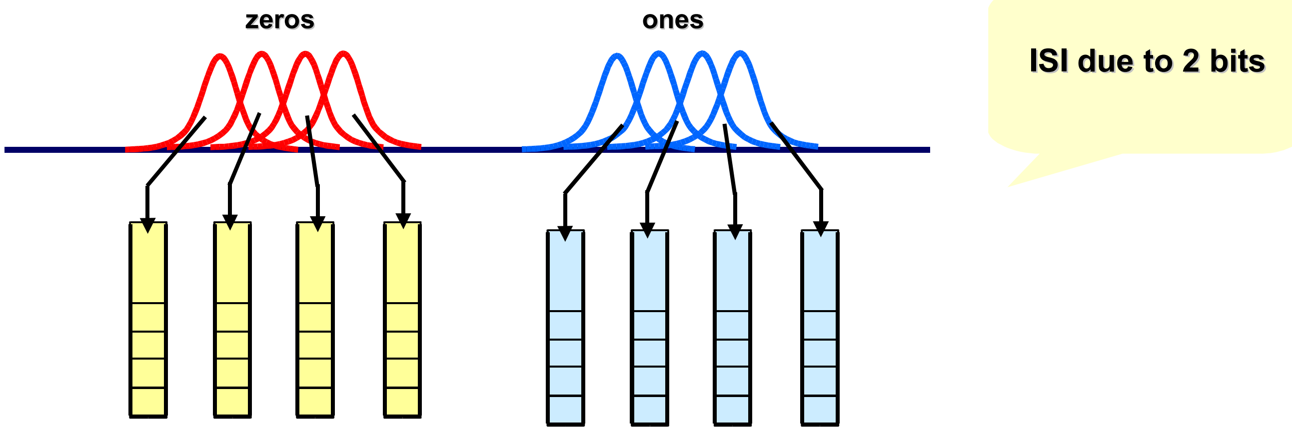
- **But in practice, input to decision device is the sum of Gaussians**
  - each corresponding to one combination of neighboring bits



  - Hence BER estimate is not correct

Method 2: ISI Pattern Approach

- **Model input to the decision device as a sum of Gaussians**
  - **Mean of each Gaussian depends on the adjacent bits**

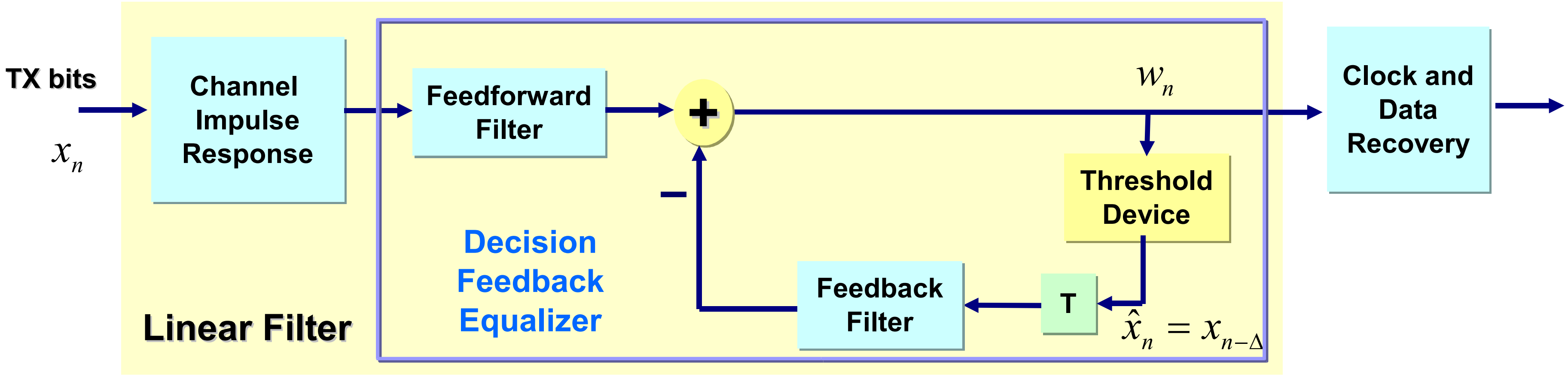


- For each transmitted bit find the ISI pattern (or Gaussian) to which it belongs
- Estimate means and standard deviations of each Gaussian
- Bit Error Rate at a threshold D and channel span L is given by:

$$P_e = \frac{1}{2^L}\sum_{i\in S_0}\operatorname{erfc}\left(\frac{D-m_i}{\sqrt{2}\sigma_i}\right) + \frac{1}{2^L}\sum_{i\in S_1}\operatorname{erfc}\left(\frac{m_i-D}{\sqrt{2}\sigma_i}\right)$$

- **Complexity of Method 2 results in Long simulation times**
  - **increases exponentially with channel memory**
  - Also requires large number of bits to reliably estimate mean & std. deviation

Proposed Method: ISI Statistics Analytically



- Assume that there is no error propagation in the DFE
  - Usually valid at BERs lower than 10^{-5};we are operating at even lower BERs
  - Then $\hat{x}_n = x_{n-\Delta}$
    - x_n denotes the transmitted bit for the nth bit period and
    - Δ is an appropriately chosen delay
- **Entire system up to CDR is a linear filter with known coefficients!!**
  - System is from x_n to w_n
  - Ideal equalizer coefficients are determined based on channel response

Proposed Method contd.

- Thus the Input to the threshold device is:

$$w_n = d_\Delta x_{n-\Delta} + r_n + \nu_n; \qquad r_n = \sum_{i \neq \Delta} d_i x_{n-i}$$

  - Where r_n corresponds to the residual ISI and ν_n is the additive noise

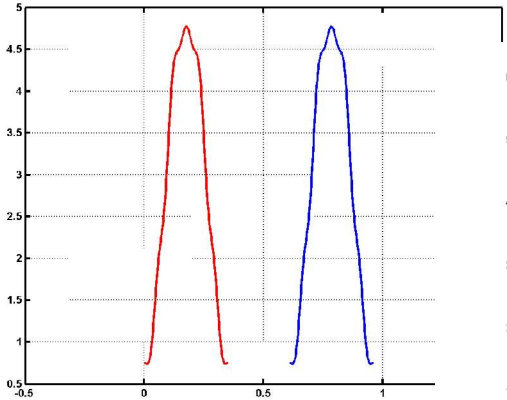
- Probability Density Function, $\hat{p}(r_n)$, of ISI can be computed analytically
  - Since its Characteristic Function is a direct function of above coefficients
  - And the input alphabet statistics

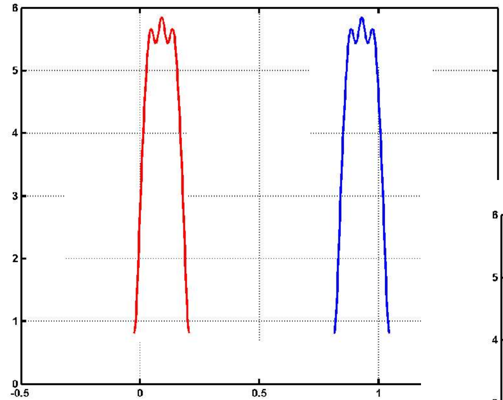
- And so the minimum BER and optimum Threshold are given by:

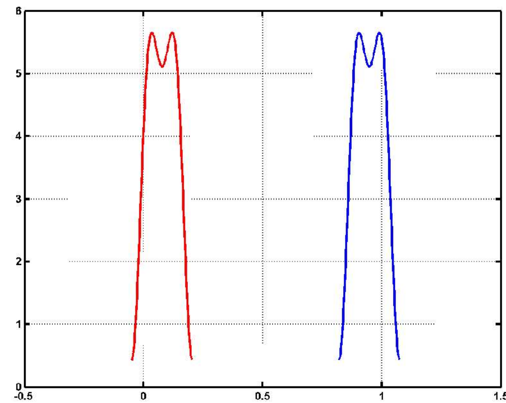
$$P_e = \frac{1}{2} \int_{-\delta/2}^{\delta/2} \mathrm{p(y)erfc}\left(\frac{d_\Delta / 2 - y}{\sqrt{2}\sigma} \right) \mathrm{d}y; \qquad D = \frac{d_\Delta + \sum_{i \neq \Delta} d_i}{2}$$

- Where $\delta = \sum_{i \neq \Delta} |d_i|$

Results: ISI Statistics



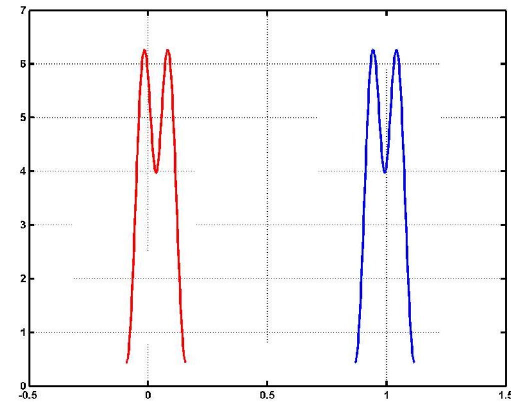




Red for zeros

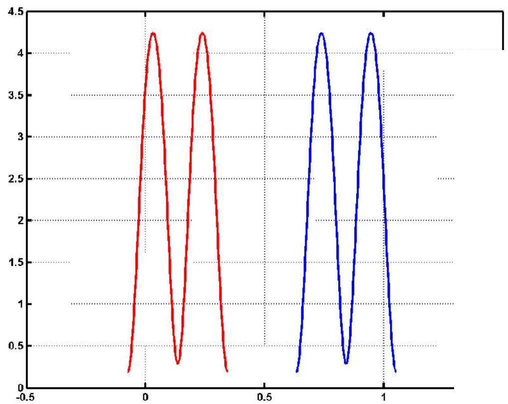
Blue for ones

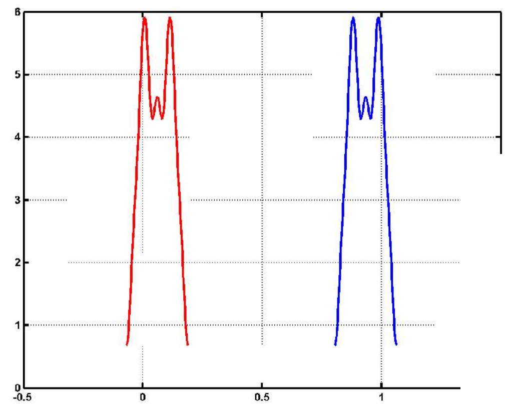
Increasing SNR



- **Two feed-forward taps and One feedback tap**
- Probability Density Function of ISI at input to slicer
- Non-Impulsive nature of ISI PDF indicates significant residual ISI
- Non-Gaussian nature of ISI evident

Results: ISI Statistics





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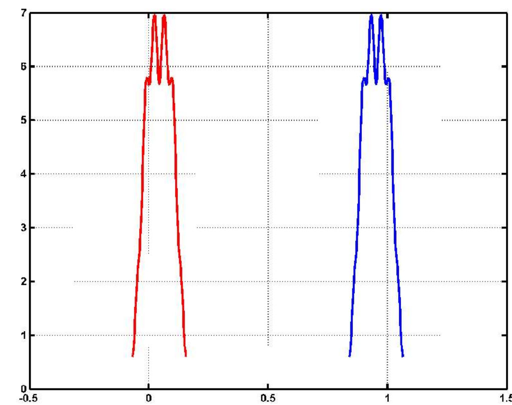
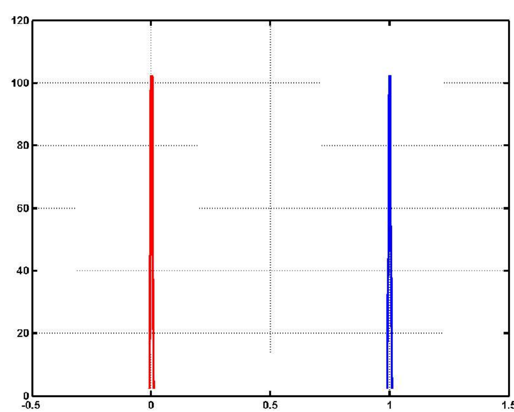
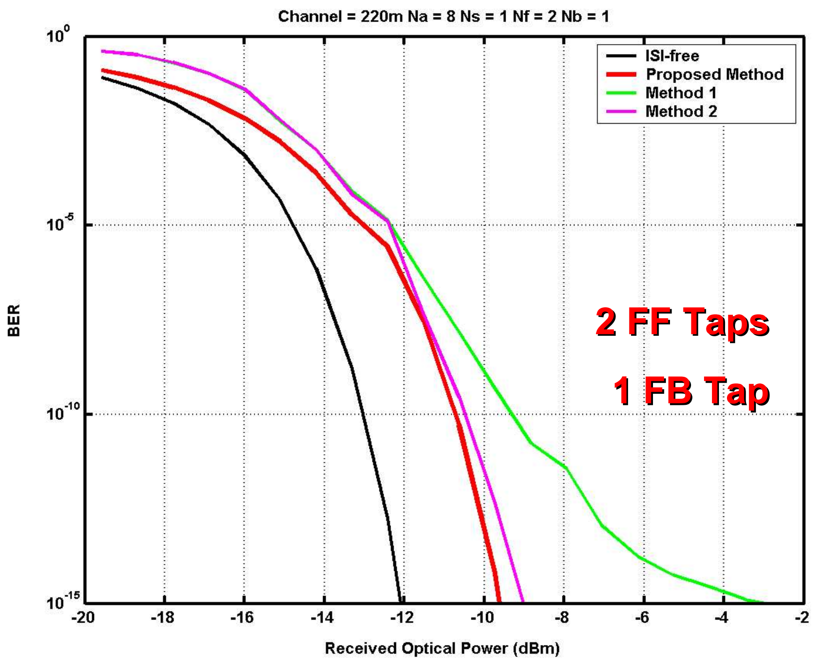


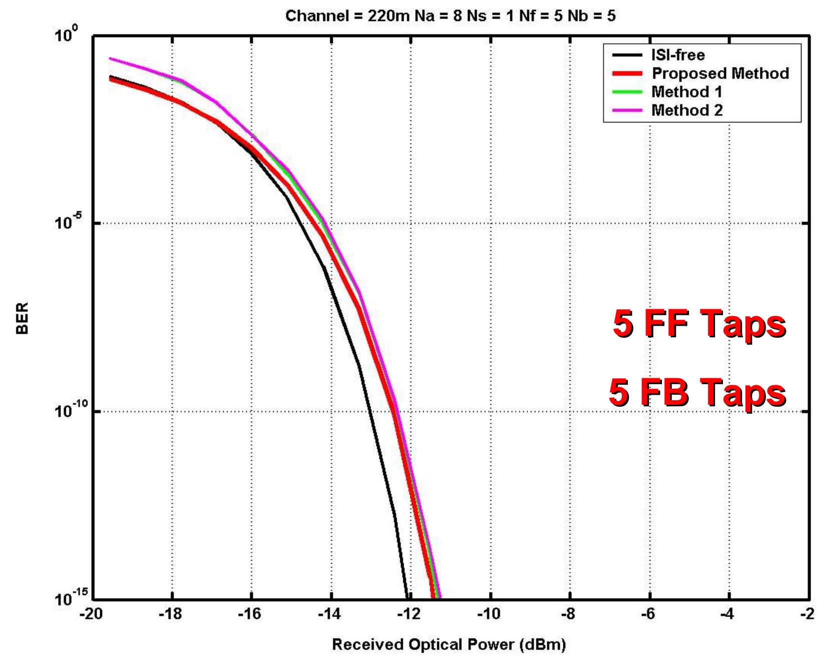
Increasing SNR



- **Five feed-forward taps and Five feedback taps**
- Non-Gaussian ISI at low TX powers
- Negligible ISI at high TX powers

Results: BER Comparison

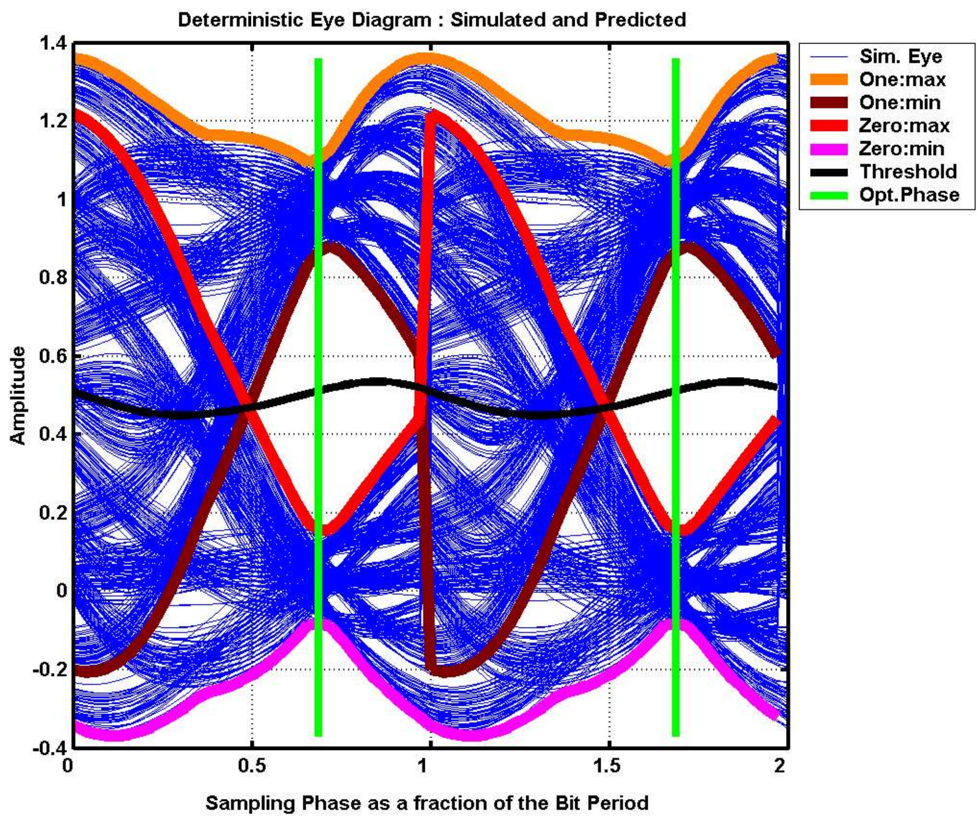




- 2 FF Taps + 1 FB Tap case:
  - Single Gaussian Approx. (Method 1) deviates significantly with fewer equalizer coefficients
  - Multiple Gaussian Approach (Method 2) better but still deviates
    - Since ISI is assumed to have contributions from only two adjacent bits
- 5 FF Taps + 5 FB Taps case:
  - Estimates from all three approaches agree at high sensitivities
  - Methods 1 & 2 still deviate at low sensitivities : ISI is not Gaussian in this regime
- **Proposed method is about 1000x times faster than Method 2 for all transmit powers!!**

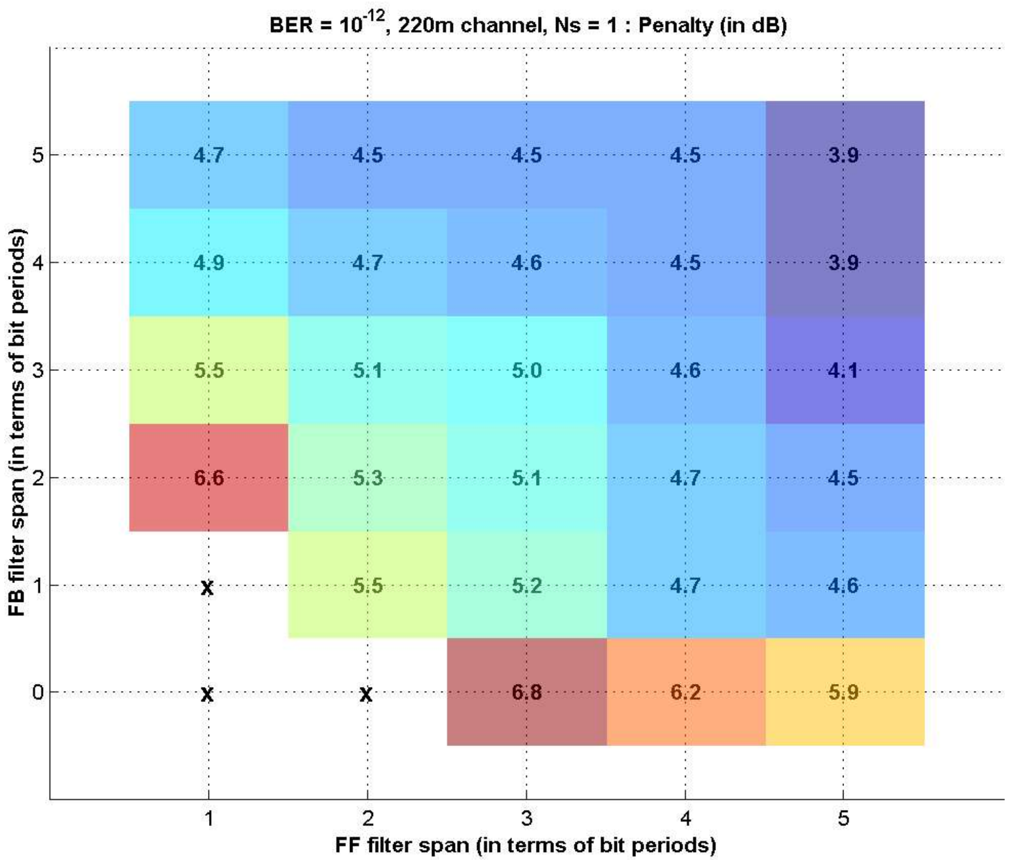
Estimation of Deterministic Eye Diagram

- For each sampling phase, noise-free input to slicer: $w_n = d_\Delta x_{n-\Delta} + \sum_{i \neq \Delta} d_i x_{n-i}$
- For each value of $x_{n-\Delta}$ (zero)
  - Maximum value of ISI = sum of all positive d_i
  - Minimum value of ISI = sum of all negative d_i
  - When $x_{n-\Delta}$= one, we need to add d_Δ to each max/min value
  - Both can be exactly computed
- Can be used to find optimum sampling instant and threshold also



Contour Plots

- Proposed technique can be used to quickly explore the equalizer design space via Contour Plots
- ISI Penalty = additional RX sensitivity required to achieve BER of 10^{-12}



Conclusions

- **An efficient BER estimation method has been proposed**
- Advantages:
  - **Accurate:**
    - More accurate than other methods when significant ISI is present
    - At least as accurate as other methods when ISI is negligible
    - Can even be applied at the input to the equalizer with accurate results
  - **complexity that increases linearly with channel memory**
    - As opposed to exponential complexity of Method 2
  - **about 1000x faster than other techniques**
  - **Independent of the equalizer adaptation technique**
- Permits easy estimation of the Deterministic Eye
  - Can also find optimum sampling instant and threshold
- Permits quick exploration of the equalizer design space