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Measuring channel variation in MMF

The formulation put forward in the initial note on estimating the impact of channel variation in MMF can be generalized to include the case of a mode selective loss (MSL) within the fiber which causes power in the modes to transfer to other modes and/or be lost at the MSL event.

From [1], the electric field $E(t)$ of an optical signal in a multimode fiber at the input of an MSL event is expressed as

$$\bar{E}_I(t) = \sum_{v=1}^N A_v \exp(i(\phi_v - \tau_v(\omega - \omega_o))) \exp(i\omega t) \bar{E}_{vI} \quad (1)$$

where A_v is the real amplitude of the v th mode, τ_v is the propagation delay of v th mode, ϕ_v is the phase at ω_o for the v th mode and E_{vI} represents the electric field of the v th mode. For the case of sinusoidal modulation, A_v is

$$A_v = a_v \sqrt{P_o + P_m \cos(\omega_m t)} \quad (2)$$

where P represents the optical power, a_v the portion of the total power in mode v and ω_m the frequency of the modulation. Assuming ϕ_v is equal to 0 and setting

$$\theta_v = \tau_v(\omega - \omega_o) \quad (3)$$

Equation 1 simplifies to

$$\bar{E}_I(t) = \sum_{v=1}^N A_v \exp(i\theta_v) \exp(i\omega t) \bar{E}_{vI} \quad (4)$$

If we limit this example to three dominant modes, this equation further simplifies to

$$E_I(t) = \sum_{v=1}^3 A_v \exp(i\theta_v) \exp(i\omega t) \quad (5)$$

At an MSL event, the power within each of the fiber modes before the MSL event will transfer to the available modes in the fiber after the MSL event with a certain coupling coefficient I_{vu} where v are the modes before the MSL and u are the modes after the MSL. The exact coupling coefficients can be calculated for the case of a fiber connector offset with the knowledge of the offset and the fiber index profiles and geometries [1]. For our simplified analysis, the coupling strength of the optical field from mode v to mode u will be captured within a single real value I_{vu} . Therefore, the equation for the electric field in the fiber after the MSL event can be expressed as

$$E_{II}(t) = \sum_{\mu=1}^3 \sum_{v=1}^3 I_{v\mu} A_v \exp(i\theta_v) \exp(i\omega t) \quad (6)$$

Where I_{vu} is the coupling co-efficient from mode v before the MSL event to mode u after the MSL event.

At the optical receiver, the optical power will be equal to the sum of the contributions of the various modes given by

$$P_{out}(t) = \left| \sum_{\mu=1}^3 \exp(i\sigma_{\mu}) \sum_{v=1}^3 I_{v\mu} A_v \exp(i\theta_v) \exp(i\omega t) \right|^2 \quad (7)$$

where σ_{μ} is due the modal group delay in the second fiber.

$$\sigma_{\mu} = \tau_{\mu} (\omega - \omega_o) \quad (8)$$

$$P_{out}(t) = \left| \sum_{\mu=1}^3 \exp(i\sigma_{\mu}) \exp(i\omega t) \sqrt{P_o + P_m \cos(\omega_m t)} \sum_{v=1}^3 I_{v\mu} a_v \exp(i\theta_v) \right|^2 \quad (9)$$

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left| \exp(i\omega t) \sum_{\mu=1}^3 \exp(i\sigma_{\mu}) \sum_{v=1}^3 I_{v\mu} a_v \exp(i\theta_v) \right|^2 \quad (10)$$

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left| \sum_{\mu=1}^3 \exp(i\sigma_{\mu}) \sum_{v=1}^3 I_{v\mu} a_v \exp(i\theta_v) \right|^2 \quad (11)$$

Expanding

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left| \sum_{\mu=1}^3 \exp(i\sigma_{\mu}) \{ I_{1\mu} a_1 \exp(i\theta_1) + I_{2\mu} a_2 \exp(i\theta_2) + I_{3\mu} a_3 \exp(i\theta_3) \} \right|^2 \quad (12)$$

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left| \sum_{\mu=1}^3 I_{1\mu} a_1 \exp(-i(\theta_1 + \sigma_{\mu})) + I_{2\mu} a_2 \exp(-i(\theta_2 + \sigma_{\mu})) + I_{3\mu} a_3 \exp(-i(\theta_3 + \sigma_{\mu})) \right|^2 \quad (13)$$

For the case of no MSL, σ_{μ} go to zero, the cross I terms go to zero, I_{11}, I_{22}, I_{33} go to 1 and you are left with

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left\{ a_1^2 + a_2^2 + a_3^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2) + 2a_1 a_3 \cos(\theta_1 - \theta_3) + 2a_2 a_3 \cos(\theta_2 - \theta_3) \right\} \quad (14)$$

To understand how the modal delays and time variance affect the channel bandwidth, assume no MSL event and only two dominant modes at the receiver. Equation 14, then simplifies to

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \{ a_1^2 + a_2^2 + 2a_1 a_2 \cos((\omega - \omega_o)(\tau_1 - \tau_2)) \} \quad (15)$$

which gives the familiar frequency dependent channel response which will vary with magnitude of the a_v and τ_v terms. The contributions of all the relevant modes will contribute to the frequency response of a real channel. Time variance due to mechanical vibrations would manifest as time variance in the a_v and/or θ_v terms of the individual modal components.

For the more general case in the presence of MSL, the final received power, for the case of sinusoidal optical modulation will take the form of

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left| \begin{aligned} &I_{11}a_1 \exp(-i(\theta_1 + \sigma_1)) + I_{21}a_2 \exp(-i(\theta_2 + \sigma_1)) + I_{31}a_3 \exp(-i(\theta_3 + \sigma_1)) + \\ &I_{12}a_1 \exp(-i(\theta_1 + \sigma_2)) + I_{22}a_2 \exp(-i(\theta_2 + \sigma_2)) + I_{32}a_3 \exp(-i(\theta_3 + \sigma_2)) + \\ &I_{13}a_1 \exp(-i(\theta_1 + \sigma_3)) + I_{23}a_2 \exp(-i(\theta_2 + \sigma_3)) + I_{33}a_3 \exp(-i(\theta_3 + \sigma_3)) \end{aligned} \right|^2 \quad (16)$$

$$P_{out}(t) = [P_o + P_m \cos(\omega_m t)] \left\{ \begin{aligned} &(I_{11}a_1)^2 + (I_{21}a_2)^2 + (I_{31}a_3)^2 + (I_{12}a_1)^2 + (I_{22}a_2)^2 + \dots \\ &2(I_{11}a_1 I_{21}a_2) \cos((\theta_1 + \sigma_1) - (\theta_2 + \sigma_1)) + \\ &2(I_{11}a_1 I_{31}a_3) \cos((\theta_1 + \sigma_1) - (\theta_3 + \sigma_1)) + \\ &2(I_{21}a_2 I_{31}a_3) \cos((\theta_2 + \sigma_1) - (\theta_3 + \sigma_1)) + \\ &2(I_{11}a_1 I_{12}a_1) \cos((\theta_1 + \sigma_1) - (\theta_1 + \sigma_2)) + \\ &2(I_{11}a_1 I_{13}a_1) \cos((\theta_1 + \sigma_1) - (\theta_1 + \sigma_3)) + \dots \end{aligned} \right\} \quad (17)$$

Mechanical vibrations may cause any of the terms; a_v , I_{vu} , θ_v , and σ_μ to vary in time. The time-varying phase terms (θ_v , and σ_μ) will cause the roll-off bandwidth of an individual modal contribution to the total bandwidth to vary in time and the magnitude terms (a_v and I_{vu}) will cause the magnitude of that bandwidth contribution to vary in time. For simple sinusoidal modulation, channel time-variance due to MSL and vibration will appear as sidebands upon the modulation signal.

Additional MSL events will create additional $(Ia)^2$ and $\cos()$ terms, with additional addition and subtraction terms due to more fiber delays, inside the \cos terms. However, the form of the equation will remain the same.

This formulation does not explicitly take into account the effect of a finite optical coherence time. The magnitudes of the time-varying terms will depend upon the distance of the MSL event from the source as related to the optical coherence time.

When viewed in the frequency domain, the received electrical signal should take the form shown in Figure 1.

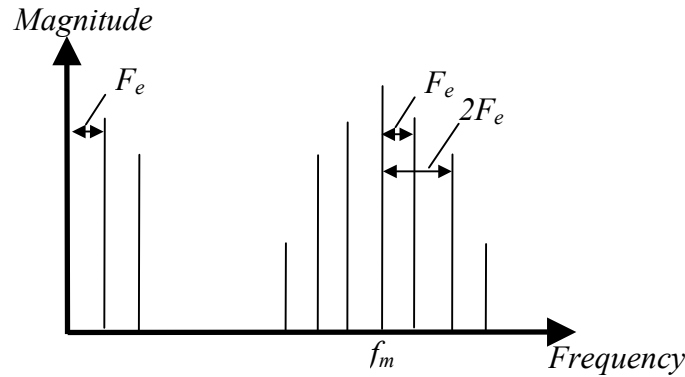


Figure 1. Received electrical signal for the case of sinusoidal modulation with mechanical vibrations at frequency f_e .

The maximum time variance in modal noise, channel bandwidth and impulse response will relate directly to the maximum sideband frequency. So the maximum rate of channel variations, nF_e will be bounded by a function of f_e , the channel mechanical excitation frequency.

The exact relationship between the frequency of an introduced mechanical vibration (and its resulting mechanical harmonic energies), and the equations for the a_v , I_{vu} , τ_v , t_u terms could be derived for the general case of multimode fiber including connector mismatch, fiber bends, etc but the large number of variables and physical mechanisms makes this a difficult exercise. A simpler approach to obtain the upper bound on channel variance frequency as a function of mechanical vibration frequency would be to mechanically perturb a link at a known frequency and measure the frequency components of the resulting time-variance in the channel.

Test set-up.

The frequency components of the channel time-variance due to mechanical vibration can be measured by passing a sinusoidal carrier signal through an MMF fiber link while introducing mechanical vibrations at a known frequency. The resulting frequency components of the channel time-variance of the received signal can be measured as the sidebands of the carrier signal using an electrical spectrum analyzer tuned to the carrier frequency. The presence and relative strength of higher sideband harmonics will indicate the upper bound of the frequency components of the time-varying channel.

Figure 2 shows the experimental test set-up.

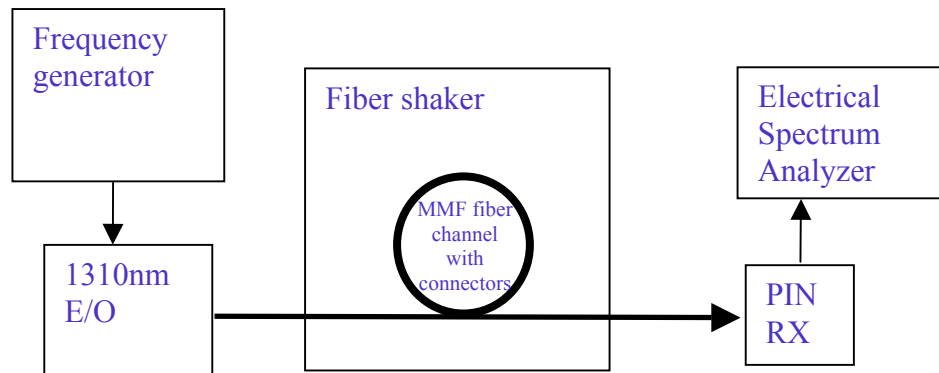


Figure 2. Test set-up for measuring the frequency of channel variations due to mechanical vibrations in an MMF link.

The frequency of the fiber shaker should be varied from DC (no significant sidebands should be present even though signal degradation may be evident) to $>10\text{kHz}$ at the same magnitude of mechanical stress to determine any effect the mechanical frequency has on sideband strength and harmonic generation. Several different examples of fiber channels should be tested to see if/how the frequency of time-varying channels varies across channels in response to mechanical vibrations.

REFERENCES

[1] – K. Petermann, “Nonlinear distortions and noise in optical communication systems due to fiber connectors”, IEEE J. Quantum Electronics, vol. QE-16, no. 7, July 1980, pp 761-770