Consideration of polarization rotation and connector offset in multimode fiber link simulation

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1. Mode coupling coefficient with an arbitrary polarized input beam

Under a weakly-guiding approximation for the electromagnetic filed in optical fibers, the longitudal components of the electric and magnetic fields are negligible and the fields are polarized in the fiber cross section plan. Due to the circular symmetry, the solution to the Maxwell equations is given in a separable form [1]:

$$F = \Psi(r)\exp(ilq)\exp(ib\,z - iw\,t) \tag{1.1}$$

where r, q, z are the triple of cylindrical coordinates and l is an integer describing the azimuthal dependence of the field and b is the propagation constant. The radial part $\Psi(r)$ is calculated from:

$$\frac{d^2\Psi}{dr^2} + \frac{1}{r}\frac{d\Psi}{dr} + \left[k^2 n^2(r) - b^2 - \frac{l^2}{r^2}\right]\Psi = 0 \tag{1.2}$$

Take into account two orthogonal polarization state, we construct a set of vector fields

$$\mathbf{E_{lm}} = \left\{\hat{\mathrm{x}}\Psi_{lm}(r)\exp(ilq),\ \hat{\mathrm{y}}\Psi_{lm}(r)\exp(ilq)\right\} \tag{1.3}$$

which we use for the decomposition of the input field.

The input field which has an arbitrary polarization can be written as

$$\mathbf{E_{in}} = f(r,q)\left(c_x\hat{\mathrm{x}} + c_y\hat{\mathrm{y}}\right) \tag{1.4}$$

where the parameter c_x and c_y are complex numbers and $c_x^2 + c_y^2 = 1$.

The mode coupling coefficient can be calculated by the overlap integrals of the input field and the modal field. We obtain the modal coupling coefficient as

$$\begin{aligned} a_{lm} &= \iint_{\substack{r\in(0,\infty)\\ q\in[0,2p]}} \mathbf{E_{in}}\cdot\mathbf{E_{lm}}\,rdrdq \\ &= \iint c_x f(r,q)\Psi_{lm}(r)\exp(ilq)rdrdq + \iint c_y f(r,q)\Psi_{lm}(r)\exp(ilq)rdrdq \end{aligned} \tag{1.5}$$

The two terms at the right hand side are corresponding to the mode coupling coefficient at two orthogonal polarization directions, respectively. To simplify the following discussion and to maintain the generality, we assume that $c_{\mathrm{x}} = \exp(if)$ and $c_{\mathrm{y}} = 0$, which case is equivalent as rotating a linearly polarized light . Then the mode coupling coefficient can be written as

$$\begin{aligned} a_{lm} &= \iint \exp(if)\, f(r,q)\Psi_{lm}(r)\exp(ilq)rdrdq \\ &= \iint f(r,q)\Psi_{lm}(r)\exp(ilq + f)rdrdq \end{aligned} \tag{1.6}$$

From (1. 6), we can see that rotating the polarization state of the input beam will change the relative location of the input beam and the modal field. If we use the input beam as the reference, it is equivalent as rotating the modal field by an angle of $-f$. Similarly, if we use the modal field as the reference, it is equivalent as rotating the input beam by an angle of f.

2. Modal power coupling coefficient and the power transfer between modes

Let's use the modal field as the reference. From (1.6), if the input beam is symmetric around the fiber core, the rotation of polarization will not change the mode coupling coefficient. Assume an off-center input Gaussian beam, linearly polarized and rotated by an angle of f, the electrical field of the input beam can be written in the cylindrical coordinate system as:

$$E_{\text{in}} = \exp\left(-\frac{(r\cos q - r_0\cos f)^2 + (r\sin q - r_0\sin f)^2}{2s^2}\right) \text{ and } q \in [0,2p) \qquad (2.\ 1)$$

where r_0 and f define the center of the Gaussian beam.

Let us consider one of the LP mode LP(l,m), l and m are angular index and radium index respectively. The modal field of this mode can be written as:

$$E_{\text{lm}} = \Psi(r)\exp(ilq) \text{ and } q \in [0,2p) \qquad (2.\ 2)$$

The mode coupling coefficient is the overlap integral of (1) and (2), which is

$$C = \iint E_{in}E_{lm}\,rdrdq \qquad (2.\ 3)$$

Substitute (1) and (2) to (3), we obtain

$$\begin{aligned} C &= \iint \exp\left(-\frac{(r\cos q - r_0\cos f)^2 + (r\sin q - r_0\sin f)^2}{2s^2}\right)\Psi(r)\exp(ilq)\,rdrdq \\ &= \iint \exp\left(-\frac{(r)^2 + (r_0)^2 - 2rr_0\cos(q-f)}{2s^2}\right)\Psi(r)\exp(ilq)\,rdrdq \qquad (2.\ 4) \\ &= \iint \exp\left(-\frac{(r)^2 + (r_0)^2}{2s^2}\right)\Psi(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\exp(ilq)\,rdrdq \end{aligned}$$

Let $A(r) = \exp\left(-\frac{(r)^2 + (r_0)^2}{2s^2}\right)\Psi(r)$, which is independent of q.

We can rewrite (2. 4) as:

$$C = \iint A(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\exp(ilq)rdrdq$$
$$= \iint A(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\cos(lq)rdrdq + i\iint A(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\sin(lq)rdrdq \tag{2. 5}$$

The first term in (2. 5) corresponds to the field coupling coefficient of the degenerated cos mode and the second term in (2. 5) corresponds to that of the degenerated sin mode. It is clear from (2. 5) that the electrical field coupling coefficients of both degenerated mode are function of f , given r_0 .

Now let's consider the intensity of this LP(l,m) mode.

$$P = \iint\left|A(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\cos(lq)\right|^2 rdrdq + \iint\left|A(r)\exp\left(\frac{rr_0\cos(q-f)}{s^2}\right)\sin(lq)\right|^2 rdrdq$$
$$= P_{\cos} + P_{\sin} \tag{2. 6}$$

Given r_0 and at a fixed radius *r*, the intensity coupling coefficient of cos mode $P_{\cos}(r)$ is a function of f , so does $P_{\sin}(r)$. This indicates the energy transfer between these two modes. However, the total intensity of these two modes is a constant.

In a real fiber link, modal selective loss may exist at the connectors or the fiber itself due to different reasons (this statement needs to be verified by more work and experiment, hopefully) [2]. For instance, the cos mode experiences more loss than the sin mode. In this case, the total intensity of this LP (l,m) mode is not a constant anymore and depends on the relative location of the input beam and the modal field. In addition, if the modal selective loss is fixed, lower order mode (smaller l) will suffer more variation.

3. Impact of connector and the simulation approach

It is known that a perfect multimode fiber transmits its guided modes without energy conversion to the other possible guided modes or continuous spectrum. However, the perturbation to the index profiles and imperfections introduce the power coupling among different modes. In the previous TIA work, one of the assumptions is the mode coupling between mode groups is completely absent and that coupling within a group is 100%. Therefore, the coupling amplitude from input mode $\boldsymbol{\Psi}_{l,m,v}$ (mode from the first fiber) to the output mode $\boldsymbol{\Psi}_{l',m',v'}$ (mode in the second fiber) is calculated as [3]:

$$a_{l',m',v'}^{l,m,v}(\mathbf{r}) = \int_A d^2\mathbf{x}\boldsymbol{\Psi}_{l',m',v'}^{*}(\mathbf{x})\boldsymbol{\Psi}_{l,m,v}(\mathbf{x}-\mathbf{r}) \tag{3. 1}$$

where **r** is the offset vector. The power coupled into output mode is calculated by:

$$w_{l',m',v'}^{out} = \sum_{l,m,v} w_{l,m,v}^{in}\left|a_{l',m',v'}^{l,m,v}(\mathbf{r})\right|^2 \tag{3. 2}$$

where $w^{in}_{l,m,v}$ is the power in mode *l,m,v*. Based on the assumption that the modes within one modal group have 100% mode mixing, the power in mode *l,m,v* can be written as:

$$w^{in}_{l,m,v} = \frac{w^{in}_u}{N_u} \tag{3. 3}$$

where N_u is the number of modes present in that modal group.

This method worked well in the study of 1Gb/s Ethernet. In previous TIA work, the above formulas were used in the simulation of long fiber and short fiber patch core as well. If fiber is not long enough, the equation (3. 3) is not valid any more, which causes energy redistribution among modal groups. However, in 1Gb/s Ethernet study, using off-set launch, the modal delay after propagation is relatively small compared to the bit period. Hence the potential power variation between different modal groups does not introduce dramatic pulse distortion in time domain. Therefore, it is reasonable to ignore the fact that in short fiber the modes within a modal group are not mixed completely.

In 10Gb/s system, modal delay of different modal group after propagation is comparable to the bit period or even larger, especially for center launch case. The variation of power coupling of different modes will introduce pulse distortion. Therefore the assumption of 100% mode mixing in one modal group needs to be reexamined, depending on the setup of MMF link. In the following paragraphs, we descript the improved method to simulate the impact of connector offset in a 10Gb/s MMF link.

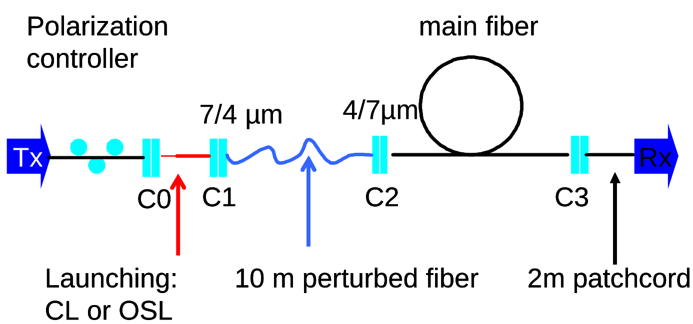

Fig. 1. Schematic diagram of the multimode link

The proposed MMF link is shown as Fig 1. The typical length scale of mode coupling within mode groups is hundreds meters. The first piece of multimode fiber in the proposed fiber link, as shown in Fig. 1, is only 10 meters, which is much shorter than the length scale that allows mode coupling with mode groups to happen. Therefore, every individual mode needs to be considered separately at the connector C2.

In addition, due to different propagation constant of modes, the arriving time at the end of the first MMF (C2) varies accordingly. The modal field (*l*, *m*) of the input fiber can be written as:

$$\begin{aligned} E_{l.m} &= \Psi_{l,m}(r)\exp(ilq)\exp(ib_{l,m}z - iwt) \\ &= \left[\Psi_{l,m}(r)\cos(lq) + i\Psi_{l,m}(r)\sin(lq)\right]\exp[-iw(t - t_{l,m})] \end{aligned} \tag{3. 4}$$

where $t_{l,m} = b_{l,m} z/w$, denoting the relative delay due to modal dispersion.

The input electrical field profile to the second fiber can be obtained by superposing the modal fields of the first fiber. We can write:

$$E_{1\to 2} = \sum_{l,m} c_{l,m} E_{l,m} \qquad (3.\ 5)$$

where $c_{l,m}$ is the mode coupling coefficient calculated from (2. 5), taking into account an off-centered launching condition. If substitute (3. 4) to (3. 5), one can easily find that the electrical field profile at the end of first fiber varies with time or in other words the composite electrical field of different modal group. For a given modal group, the field profile is not symmetric around the fiber core any more. An example of output field profile is given in Fig. 2.

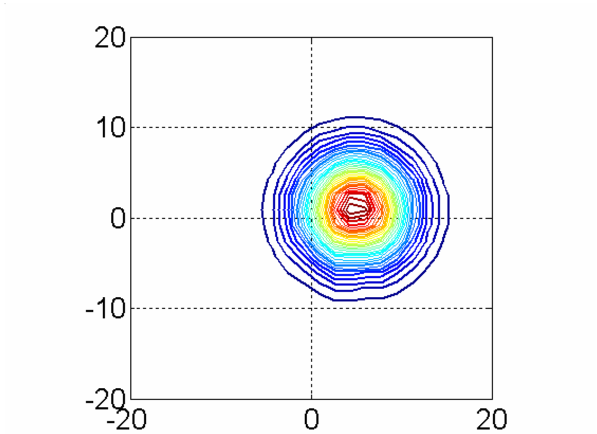

Fig. 2, an example of the output field profile after 10 m fiber with 5 µm offset at the input

For a given modal group ?, the mode coupling coefficient for mode (*l'*,*m'*) in the second fiber can be calculated from

$$c_{l',m'}(t_{l,m}) = \iint E_{1\to 2}^{(n)*} E_{l',m'} r dr dq \qquad (3.\ 6)$$

where $E_{1\to 2}^{(n)} = \sum c_{l,m}^{(v)} \Psi_{l,m}^{(n)}(r)\exp(ilq)$ is the composite modal field of modal group ? and $E_{l',m'}$ is the model field of mode (*l'*,*m'*) in the second fiber.

In considering the offset at C2, one can write $E_{l',m'} = G(r,q,r_0,j)$, where parameters r_0 and j define the offset center. For a given r_0, $c_{l',m'}(t_{l,m})$ is a function of j. The power coupling coefficient for mode (*l'*,*m'*) is calculated by $p_{l',m'}(j) = c_{l',m'}(j) c^{*}{}_{l',m'}(j)$. Therefore, to simulate possible impulse response due to an offset r_0, one needs to consider varying j from 0 to p, as illustrated in Fig. 3.

In Fig. 3, the dashed circle and the solid circle are represented two possible offset with magnitude of r_0. It is clear that the overlapping of the composite modal field from the first fiber to these of the second fiber depends on the offset center.

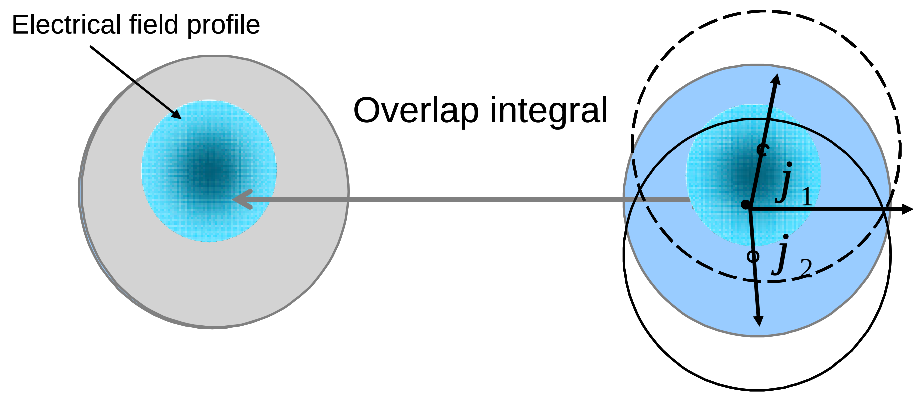

Fig. 3. Illustration of connect offset

4. Proposals for Simulationing 10Gb/s MM Link

In the simulation of proposed link:

1) It is possible to use the result of scalar wave equation to simulate the polarization rotation in a multimode fiber link.
2) Polarization rotation of the input beam is equivalent as rotating the modal field in the fiber. The power coupling coefficient of individual modes varies accordingly, causing the energy transfer among the two degenerated modes.
3) The pulse variation due to the change of polarization can be simulated if the mode mixing is not completed or there is a mode selective loss in the link.
4) Due to the short length of the first MMF in the proposed link, modes within one modal group need to be treated individually.
5) The modal field profile at the end of first MMF varies with time and is not symmetric around the fiber core with an off-centered launching condition.
6) The overlap of modal fields at the connector depends on the relative location of the offset center to the reference coordinates and the pulse response will change accordingly.

References:

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[3] P. Pepeljugoski, S. Golowich, A. J. Ritger, P. Kolesar and A. Risteski, “Modeling and simulation of next-generation multimode fiber links,” J. Lightwave Technol., vol 21, no. 5, pp. 1242–1255, 2003.