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Through-The-Air Token Bus Physical Layer

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SUBJECT: DQPSK Spread-Spectrum Modulation/Demodulation

# 1. INTRODUCTION

The mathematical structure of our proposed modulation/demodulation scheme is shown. A general equation for the spectral density of a Direct Sequence Spread Spectrum Signal is derived. The Spectral Density of the 11 chip Barker sequence is calculated and plotted. An expression for the demodulated decision variable is given which can be used for further system performance calculations.

# 2. DIFFERENTIAL QUADRATURE PHASE MODULATION

## 2.1. Transmission

Any real valued signal who's frequency content is concentrated around a carrier, $f_c$, can be expressed:

$$S(t) = a(t)\cdot \mathrm{Cos}(2\pi f_c t + \Omega(t)) \qquad [1]$$

For ease of mathematical manipulation this can be expressed in complex notation:

$$S(t) = \mathrm{RE}[U(t)\cdot \exp(j2\pi f_c t)] \qquad [2]$$

Where:

$U(t) =: a(t)\exp(j\Omega)$ is the Lowpass representation of the signal.

With Digital Phase Modulation the Lowpass function is given by:

$$U(t) = \sum_{n=-\infty}^{\infty} \exp(j\Omega_n)\cdot g(t-nT) \qquad [3]$$

Where:

T is the symbol period of the transmitted information.

n is an index representing the nth transmitted symbol.

g(t) is the baseband modulating pulse with a duration T.

$\exp(j\Omega_n)$ is the Information Vector, $I_n$. With Quadrature modulation four phase positions are transmitted $\epsilon\{\ \pi/4,\ -\pi/4,\ 3\pi/4,\ -3\pi/4\ \}$.

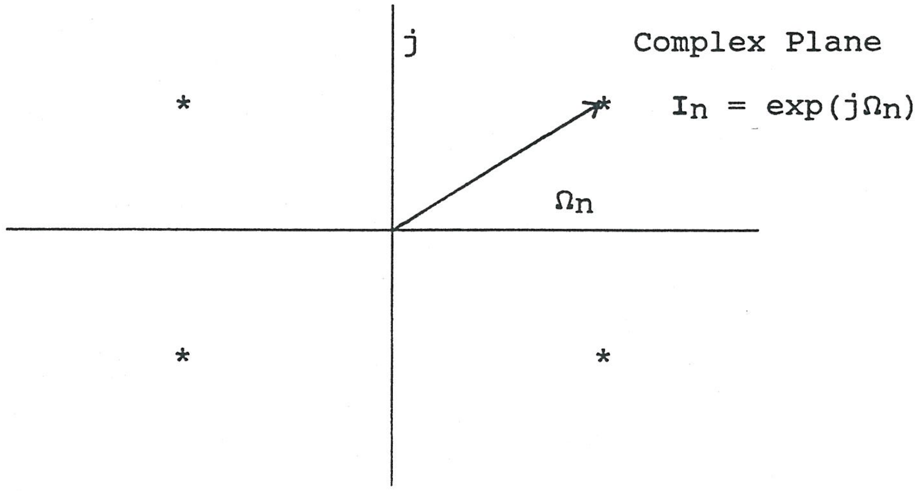



Figure 1 Information Vector in the Complex Plane

To give more insight into the transmitter's functional block diagram, equation [3] can be written as:

$$U(t) = \sum_{n=-\infty}^{\infty} (Ir_n + jIi_n) \cdot g(t-nT) \qquad [4]$$

Where:

$Ir_n \in \{1,-1\}$ is the real axis projection of the Vector $\mathbf{I_n}$.
$Ij_n \in \{1,-1\}$ is the imaginary axis projection of the Vector $\mathbf{I_n}$.

Substitution of [4] into [2] gives the following:

$$S(t) = \sum_{n=-\infty}^{\infty} [Ir_n \cdot g(t-nT) \mathrm{Cos}(2\pi f_c t) - Ii_n \cdot g(t-nT) \mathrm{Sin}(2\pi f_c t)] \qquad [5]$$

Equation [5] lends itself to an easy interpretation. The transmitted phase modulated digital signal can be seen as the sum of two carriers, with a 90 degree offset. The amplitude of each carrier is multiplied by the "wave shaping pulse" g(t), which determines the signal's spectral characteristics, and the Information symbol, $Ir_n$ and $Ii_n \in \{1,-1\}$, which determines the symbol phase state of the transmitted Information Vector. It is noted that due to g(t) the phase of the transmitted signal could change often during a symbol period, for example using Spread-Spectrum Modulation, but this is not dependent upon the information vector and therefore does not determine the symbol phase state.

In order to ease the receiver implementation, within a quasi-stationary channel, Differential Phase Modulation is implemented. In this case the absolute transmitted symbol phase is a function of the previous symbol phase state. The symbol phase encoding is as follows:

$$\Omega_n = d\Omega_n + \Omega_{n-1} \qquad [6]$$

Where:

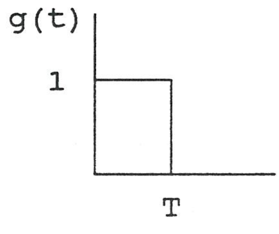
$d\Omega_n$ is the differential symbol phase shift, defined by table I.
$\Omega_n$ is the transmitted symbol phase.
$\Omega_{n-1}$ is the previous symbol (T delayed) transmitted symbol phase.

The differential symbol phase shift transmitted is derived from the data source following the data mapping of table I.

TABLE I: Data dibit to Differential Phase Mapping

| dibit pattern (Left Sent First) | $d\Omega$ |
|---|---|
| 0 0 | 0 |
| 0 1 | $\pi/2$ |
| 1 1 | $\pi$ |
| 1 0 | $-\pi/2$ |

For clarity an example of the various signals for the transmission of Differential Quadrature Phase Modulation is shown. It is noted, in this example, that the "wave shaping pulse" g(t) is given as:



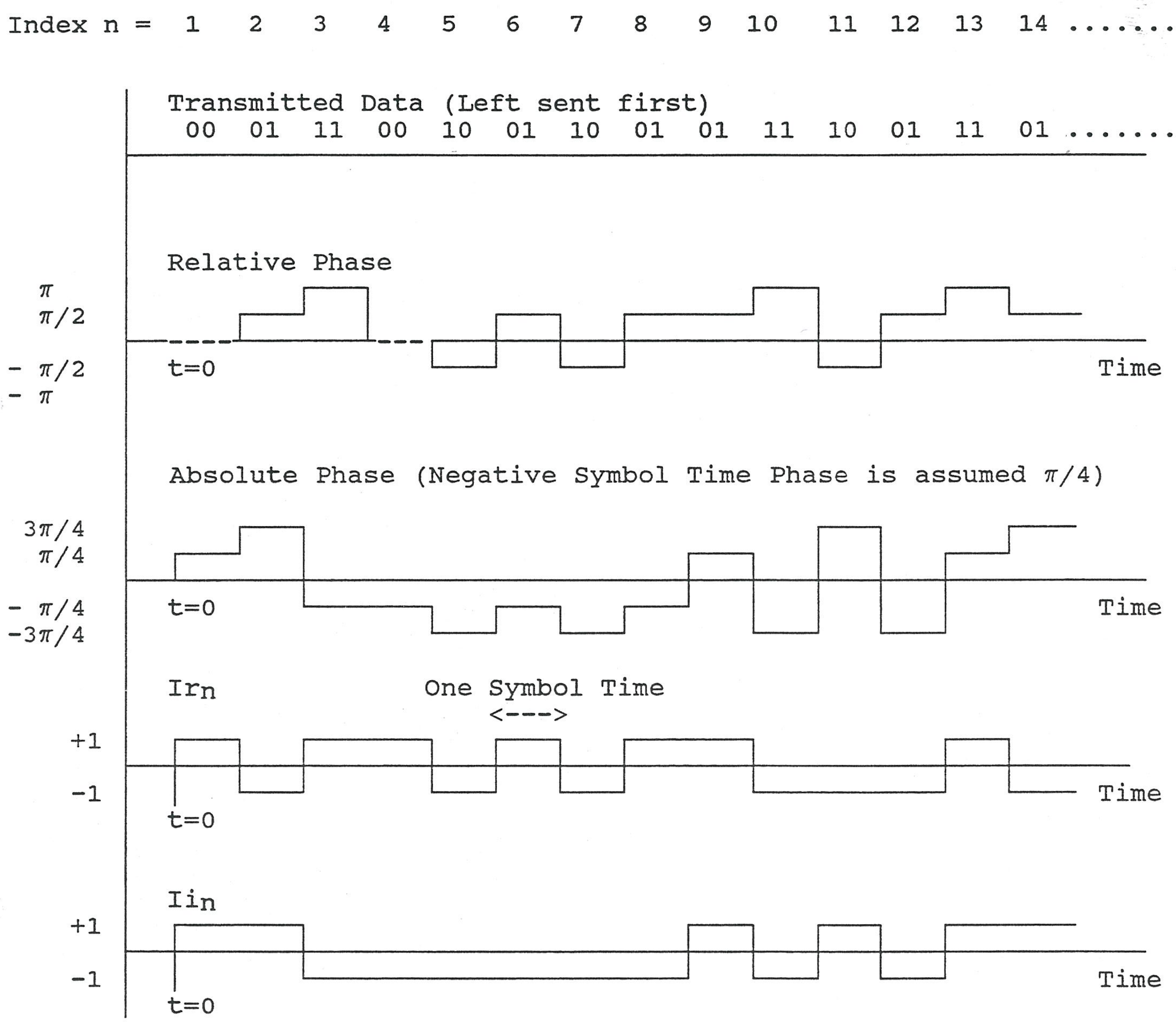



Figure 2 Differential Quadrature Modulation Waveforms

Of course the output signal will be band-limited, which is not shown in figure 2. In figure 3 a functional block diagram can be seen. Note that eventual band limiting of the output signal is shown as a low pass filter F(f) in each real and imaginary axis.

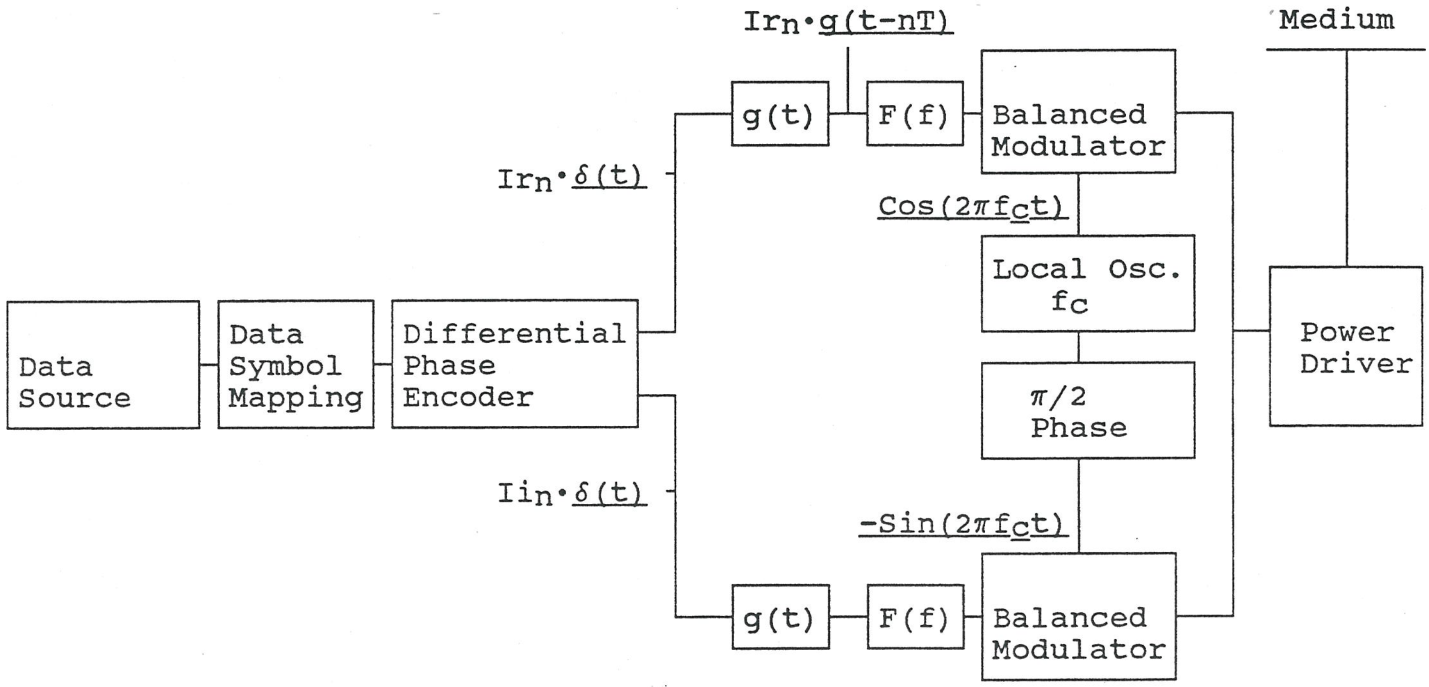



Figure 3 The DQPSK Block Diagram

Two bits from the Data Source, dibit n, are mapped by the Data Symbol Mapping Block according to Table I. The differential phase, $d\Omega_n$, and the previous absolute phase, $\Omega_{n-1}$, are used to calculate the present transmitted phase, $\Omega_n$, by the Differential Phase Encoder following equation [6]. The output of the Differential Phase Encoder are two dirac impulses, $\delta(t)$, weighted by the real and imaginary components of the information Vector $I_n$.

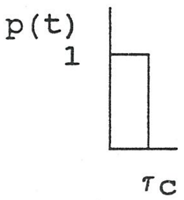
### 2.1.1. Direct Sequence Spread Spectrum

At this point we have DQPSK transmission in which the spectral bandwidth is determined by g(t) and the spectral limiting function F(f). In the example of figure 2, g(t) is a unit pulse with a duration of one dibit period T. This would be the case with "normal" DQPSK modulation. In Direct Sequence Spread Spectrum, DSSS, the transmitted bandwidth is larger than the "information bandwidth". This is accomplished by having transitions in the output variable, the phase in this case, which are shorter in duration than the information symbol time. This can be accomplished, without any changes with the block diagram of figure 3, by defining g(t) as the spread spectrum sequence:

$$g(t) = \sum_{k=1}^{N} X_k \cdot p(t-k\tau_c) \qquad [7]$$

Where:

g(t) is the spread spectrum sequence.
p(t) is a "chip" pulse:




$\tau_c$ is the chip duration such that $N = T/\tau_c$ is an integer.
$X \in \{1,-1\}$ and is the kth chip's coefficient.

The spread spectrum sequence, g(t), is defined by the coefficient vector $X_k$. In our case the N = 11 chip Barker sequence is used:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Index k = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| $X_k$ = | [1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1] |

The spectrum spreading is characterized by the parameter N. This gives the number of chips within one symbol period. This is also how much the spectrum bandwidth is increased relative to the no spreading case N=1 (chip = symbol).

### 2.1.2. Output Spectral Density

The output spectral density of a process with a lowpass representation, U(t), is given by the Fourier Transformation of the autocorrelation function:

$$\phi uu(\tau) \text{ <---> } \Phi uu(f) = \int_{-\infty}^{\infty} \phi uu(\tau)\cdot \exp(-j2\pi\tau)d\tau \qquad [8]$$

Where:

$\phi uu(\tau) = \frac{1}{2}E[U(t)\cdot U^*(t)]$ is the complex autocorrelation of the process U(t) and * signifies the complex conjugate.

U(t) is a cyclostationary process, its statistics are periodic in T, and therefore, as shown by Proakis [ref 1], the average autocorrelation function, $\phi uu(\tau)$, of the process U(t) given by equation [3] is:

$$\phi uu(\tau) = 1/T\cdot \sum_{m=-\infty}^{\infty} \phi ss(m)\cdot \phi gg(\tau - mT) \qquad [9]$$

Where:

$\phi ss(m)$ is the complex autocorrelation of the Information Symbol Vector, $\mathbf{I}_n$, defined as:

$$\phi ss(m) = \frac{1}{2}E[\mathbf{I}^*_n \cdot \mathbf{I}_{n+m}] \qquad [10]$$

(* signifies complex conjugation)

$\phi gg(\tau)$ is the time autocorrelation of the function g(t).

Substitution of [9] into [8] gives the output spectral density as:

$$\Phi uu(f) = 1/T\cdot |G(f)|^2 \cdot \Phi ss(f) \qquad [11]$$

Where:

$|G(f)|$ is the magnitude of the Fourier Transform of g(t).
$\Phi ss(f)$ is the Spectral Density of the Information Vector given by:

$$\Phi ss(f) = \sum_{m=-\infty}^{\infty} \phi ss(m)\cdot \exp(-j2\pi f\cdot mT) \qquad [12]$$

Expressing In in real and imaginary components and substitution into [10] gives:

$$\phi ss(m) = \frac{1}{2}[\phi rr(m) + \phi ii(m) - j\phi ri(m) + j\phi ir(m)] \qquad [13]$$

Since the data source is random, as are the dibits which determine the sign of the real and imaginary Information Vector components, these crosscorrelation products are zero:

$$\phi ri(m) = 0 \quad \text{for all m}$$
$$\phi ir(m) = 0$$

Also real and imaginary axis autocorrelation functions are equal, since the statistics are the same for each axis:

$$\phi rr(m) = \phi ii(m) \text{ for all m}$$

Since the data sources random binary bits, previous and present dibits are random. This means that the autocorrelation is:

$$\phi rr(m) = \delta_{mo}$$

Where $\delta_{jk}$ is the Kronecker delta such that:

$$\delta_{jk} = \begin{matrix} 1 & j = k \\ 0 & j \neq k \end{matrix}$$

Substitution of the correlation statistics given into [13] gives:

$$\phi ss(m) = \phi rr(m) = \delta_{mo} \qquad [14]$$

It is noted that due to [14] the autocorrelation of the output given in [9], $\phi uu(\tau)$, is only a function of the real (or imaginary) component of the Information Vector and the pulse g(t).

Substitution of [14] into [12] gives the Spectral Density of the Information Vector as:

$$\Phi ss(f) = 1$$

From [11] and $\Phi ss(f) = 1$ the Spectral Density of the Output Process is found:

$$\Phi uu(f) = 1/T \cdot |G(f)|^2 \qquad [15]$$

The next step is to find the function $|G(f)|$ which uniquely determines the Output Spectral Density. The function g(t) given in [7] can be written in another form:

$$g(t) = \sum_{k=1}^{N} X_k \cdot p(t-k\tau_c) = p(t) * \sum_{k=1}^{N} X_k \cdot \delta(t-k\tau_c) \qquad [16]$$

Where:

$*$ denotes the convolution operation:

$$z(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\cdot z(t-\tau)d\tau$$

Looking at [16], imposing the relationship between time domain convolution and frequency domain multiplication, $G|(f)|^2$ can be expressed as the multiplication of two terms:

$$|G(f)|^2 = |P(f)|^2 \cdot |X(f)|^2 \qquad [17]$$

Where:

$|P(f)|$ is the magnitude of the Fourier Transformation of the chip pulse p(t).

$|X(f)|$ is the magnitude of the Fourier Transformation of x(t) which is given by:

$$x(t) = \sum_{k=1}^{N} X_k \cdot \delta(t-k\tau_c) \qquad [18]$$

Noting from the Fourier Transformation that the relationship holds:

$$x(-t) \;\text{<--->}\; X^*(f) \qquad [20]$$

Since $|X(f)|^2 = X(f)\cdot X^*(f)$, using [20] we have:

$$x(t) * x(-t) \;\text{<---->}\; X|(f)|^2 \qquad [21]$$

Substitution of [18] into [21] gives:

$$x(t) * x(-t) = \sum_{k=1}^{K} \sum_{i=1}^{K} X_k \cdot X_i \cdot \int_{-\infty}^{\infty} \delta(t+\alpha - k\tau_c)\cdot\delta(\alpha - i\tau_c)d\alpha \qquad [22]$$

Using the fact that $\delta(\alpha - i\tau_C)$ has unity weight only for $\alpha = i\tau_C$, else zero, equation [22] can be simplified:

$$x(t) * x(-t) = \sum_{k=1}^{K} \sum_{i=1}^{K} X_k \cdot X_i \cdot \delta(t-(k-i)\tau_C) \qquad [23]$$

Taking the Fourier Transformation of [23] gives the Spectral Density of the Spread-Spectrum Code Sequence:

$$|X(f)|^2 = \sum_{k=1}^{K} \sum_{i=1}^{K} X_k \cdot X_i \cdot \exp(-j2\pi f(k-i)\tau_C) \qquad [24]$$

The first term in the double addition of [24], k=i=1, is a DC component which can be separated, combining negative and positive exponential terms gives:

$$|X(f)|^2 = \sum_{k=1}^{K} X^2{}_k + 2 \cdot \sum_{k=2}^{K} \sum_{i=1}^{k-1} X_k \cdot X_i \cdot \mathrm{Cos}(4\pi f(k-i)\tau_C) \qquad [25]$$

With the 11 chip Barker Code, $X_k$ = [1 -1 1 1 -1 1 1 1 -1 -1 -1], substitution into [25] gives a further simplification:

$$|X(f)|^2 = 11 - 2 \cdot \sum_{k=1}^{5} \mathrm{Cos}(4\pi f \times k\tau_C) \qquad [26]$$

Using the fact the chip pulse, given in [7], p(t) <----> P(f) is given by:

$$|P(f)| = T \cdot \mathrm{Sinc}(\pi f \tau_C)$$

The Output Spectral Density, from [15], is:

$$\Phi uu(f) = T \cdot \mathrm{Sinc}^2(\pi f \tau_C) \cdot [11 - 2 \cdot \sum_{k=1}^{5} \mathrm{Cos}(4\pi f \cdot k\tau_C)] \qquad [27]$$

In figure 4 a plot of the Output Spectral Density is shown. It is noted that the Spectral "Side-Lobes" of the Sinc function are quite significant. Spectral side-lobe suppression will be necessary using the Low Pass filters F(f) shown in the figure 3. It is interesting to note that the spectrum is "smooth". This is due to the Barker sequence and is coupled with its unity bounded non-periodic and odd-periodic autocorrelation sidelobe properties.

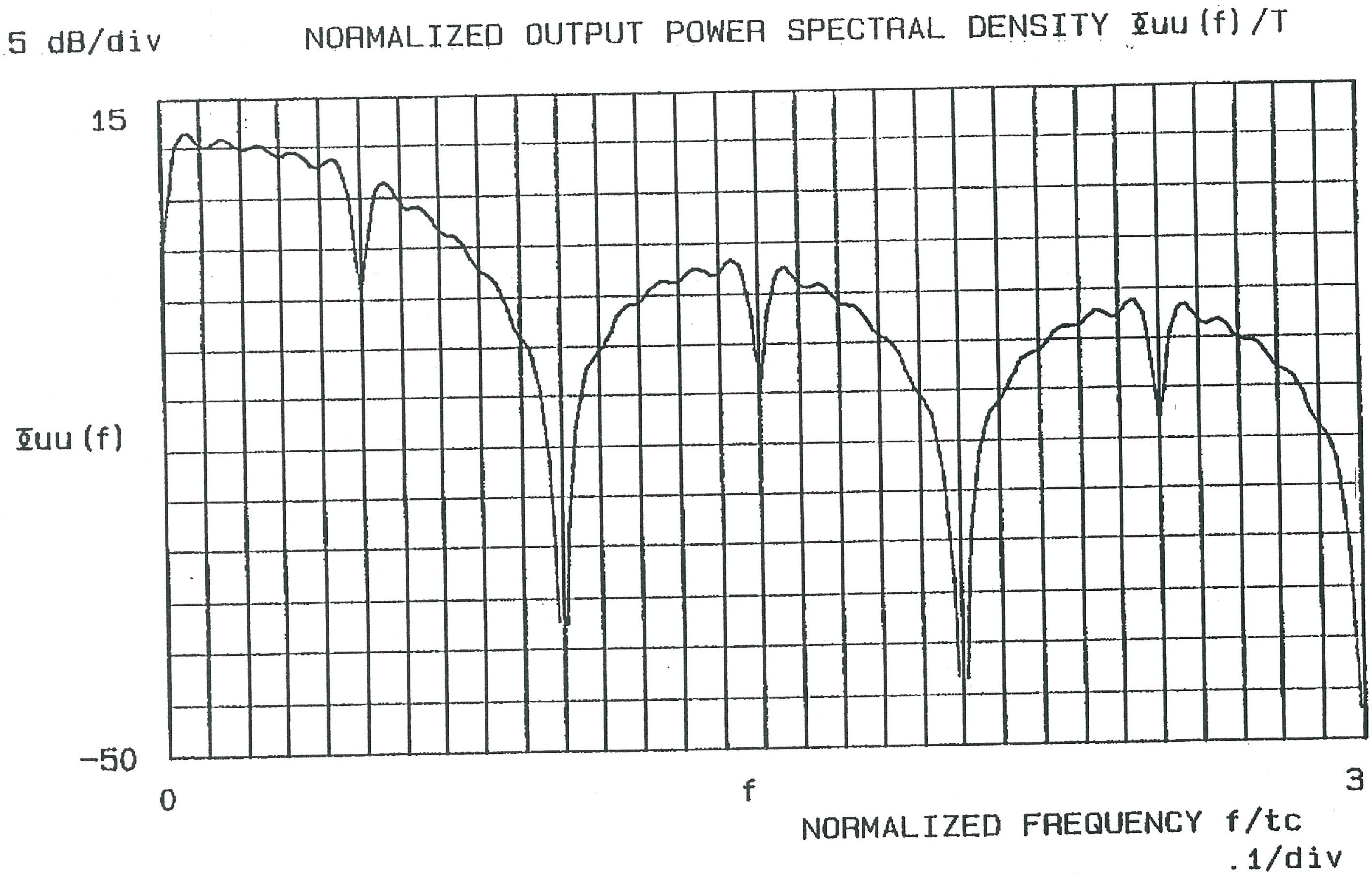

5 dB/div
NORMALIZED OUTPUT POWER SPECTRAL DENSITY Φuu (f) /T
15
Φuu (f)
-50
0
f
3
NORMALIZED FREQUENCY f/tc
.1/div

FIGURE 4

## 2.2. Reception

The transmitted signal has been given in [3] and is repeated here:

$$U(t) = \sum_{n=-\infty}^{\infty} I_n \cdot g(t-nT) \qquad [28]$$

The time-invariant channel (between two symbol intervals) is modeled as:

$$h(\tau) = \sum_{i=1}^{L} \beta_i \cdot \delta(\tau-\tau_i) \cdot \exp(-j\alpha_i) \qquad [29]$$

Where:

$\beta_i$ is the ith path gain, with probability density function, pdf, which is Rayleigh distributed.
L is the number of paths.
$\alpha_i$ is the ith path phase, with a unit pdf within $[0,2\pi]$.

The received signal is the convolution of [28] with [29] giving:

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{i=1}^{L} I_n \cdot \beta_i \cdot \exp(-j\alpha_i) \cdot g(t-nT-\tau_i) \qquad [30]$$

Let the receiver consist of a "Matched Filter" and a symbol delay T. The impulse response of the Matched Filter is given by:

$$hfilter(t) = g(T-t) \qquad [31]$$

From the received signal r(t) given in [30] the present, n=0, and previous symbol, n=-1, shall be used in the calculations for intersymbol interference effects. (It is assumed that the delay of the channel is much less than two symbol periods and multiple symbol interference can be neglected).

For convenience the first path, which is chosen for detection by the receiver, of the channel is set with $\tau_c = 0$. The complex signal under investigation is:

$$r'(t) = \sum_{i=1}^{L} I_0 \cdot \beta_i \cdot \exp(-j\alpha_i) \cdot g(t-\tau_i) + \sum_{i=1}^{L} I_{-1} \cdot \beta_i \cdot \exp(-j\alpha_i) \cdot g(t+T-\tau_i) \qquad [32]$$

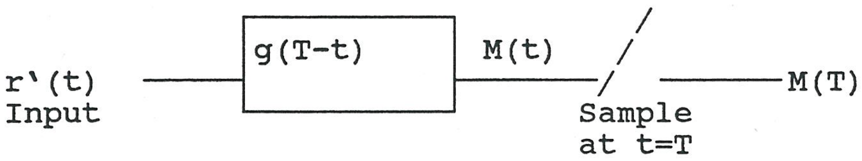



Figure 5 Matched Filter (Correlation) Receiver

In order to calculate M(t) it is noted that the receiver forms a linear system and superposition applies. Therefore each summation term in equation [32] can be applied to the system separately, adding each resulting output to give the total response. Applying the terms of r'(t) containing the $I_o$ complex Information Vector gives:

$$M_o(t) = g(T-t) * g(t)\cdot I_o\cdot\beta_1\cdot\exp(-j\alpha_1) + \sum_{i=2}^{L} I_o\cdot\beta_i\cdot\exp(-j\alpha_i)\cdot g(t-\tau_i) * g(T-t)$$

Taking the value of Mo(t) at the sampling moment T gives:

$$M_o(T) = T\beta_1\cdot\exp(-j\alpha_1) + \sum_{k=2}^{L} I_o\cdot\beta_i\cdot\exp(-j\alpha_i)\cdot\int_{-\infty}^{\infty} g(\alpha-\tau_i)\cdot g(\alpha)\,d\alpha \qquad [33]$$

Where the first chosen path, with zero delay, has been separated from the expression. The integral in [33] is the time autocorrelation of the spread spectrum signal. From the expression for g(t) in [7] for $\tau_1 = 0$ the autocorrelation equals T which is the factor of the first term.
Since g(t) has a duration of T, as shown in figure, 6 the autocorrelation function can be expressed as:

$$R''(\tau) = \int_{-\infty}^{\infty} g(\alpha-\tau)\cdot g(\alpha)\,d\alpha = \int_{\tau}^{T} g(\alpha-\tau)\cdot g(\alpha)\,d\alpha \qquad [34]$$

Only the cross areas contribute to the integral.

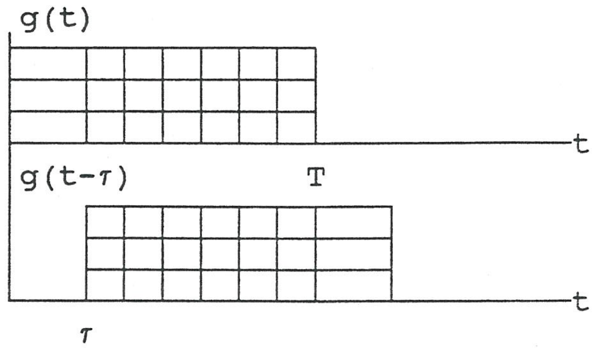



Figure 6 Spread-Spectrum Sequence Autocorrelation

$M_o(T)$ can now be expressed as:

$$M_o(T) = T \cdot \beta_1 \cdot I_o \cdot \exp(-j\alpha_1) + I_o \cdot \sum_{i=2}^{L} \beta_i \cdot \exp(-j\alpha_i) \cdot R''(\tau_i) \qquad [35]$$

Using the same arguments as given above, the sampled output due to the terms in r'(t) containing the Complex Information Vector $I_{-1}$ gives:

$$M_1(T) = I_{-1} \cdot \sum_{i=1}^{L} \beta_i \cdot R(\tau_i) \qquad [36]$$

Where:

$$R(\tau) = \int_{-\infty}^{\infty} g(\alpha+T-\tau) \cdot g(\alpha)\, d\alpha = \int_{0}^{\tau} g(\alpha-\tau) \cdot g(\alpha)\, d\alpha$$

Summation of [35] and [36] gives the total output as sample moment T:

$$M(T) = I_o \cdot [T \cdot \beta_1 \cdot \exp(-j\alpha_1) + \sum_{i=2}^{L} \beta_i \cdot \exp(-j\alpha_i) \cdot R''(\tau_i)] + I_{-1} \cdot \sum_{i=2}^{L} \beta_i \cdot \exp(-j\alpha_i) \cdot R(\tau_i) \qquad [37]$$

Due to the choice of the 11 chip Barker sequence and its unity bounded odd-periodic correlation function, the autocorrelation terms in [37] are bounded by:

$$R''(\tau) = R(\tau) < \tau_c \qquad [38]$$

Which is the area under one chip pulse.

Substitution of the upper bound of [38] into [37] gives:

$$M(T) = \mathbf{I_0} \cdot [T \cdot \beta_1 \cdot \exp(-j\alpha_1) + \mathbf{B} \cdot \tau_c] + \mathbf{I_{-1}} \cdot \mathbf{B} \cdot \tau_c \qquad [38]$$

Where:

$$\mathbf{B} = |\mathbf{B}| \exp(-j\Phi) = \sum_{i=2}^{L} \beta_i \cdot \exp(-j\alpha_i)$$

It is noted that the second term of equation [38] is the intersymbol interference term of the previous ($\mathbf{I_{-1}}$) information symbol with the present ($\mathbf{I_0}$). One can see how Spread-Spectrum Modulation is robust with respect to Multipath Delay. The received path's strength is multiplied by a factor T while the interference terms by $\tau_c$. The ratio, $T/\tau = N$, being the "Processing Gain" of the system.

A symbol delay, T, is now added at the output of the matched filter receiver. At the present sample moment M(T) the previous sample output, M(0), is available. Note that M(0) has is the same as [38] except that the Information Vectors are delayed one symbol:

$$M(0) = \mathbf{I_{-1}} \cdot [T \cdot \beta_1 \cdot \exp(-j\alpha_1) + \mathbf{B} \cdot \tau_c] + \mathbf{I_{-2}} \cdot \mathbf{B} \cdot \tau_c \qquad [39]$$

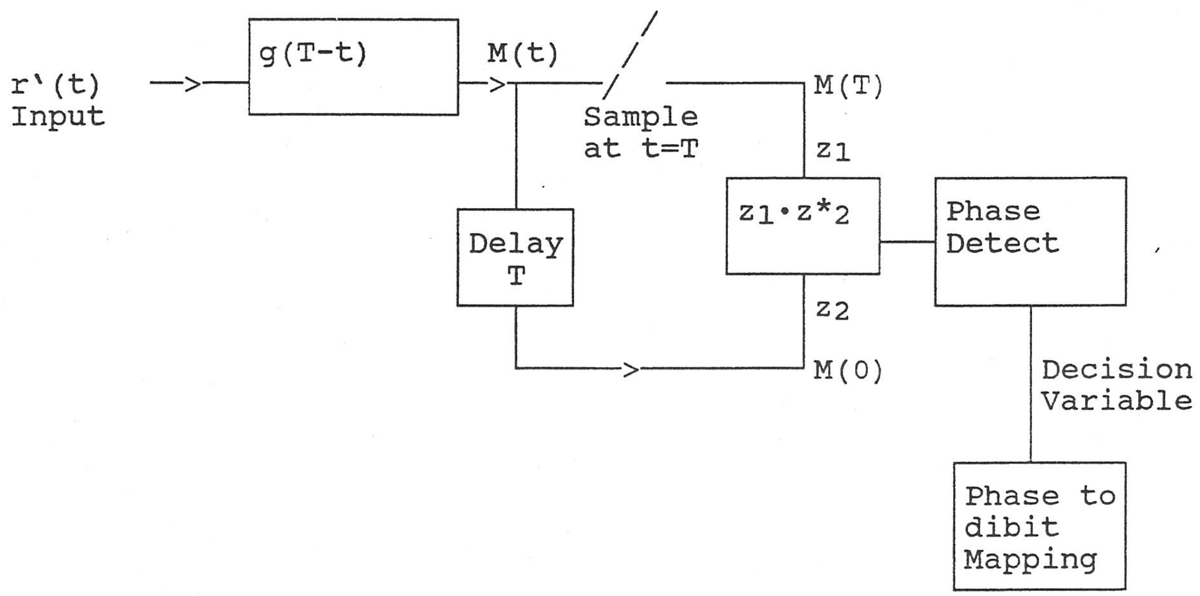



Figure 7 Differential Phase Demodulation Receiver

Performing the complex conjugate multiplication shown in Figure 7 gives:

$$M(T) \cdot M^*(0) = (\beta_1 \cdot T)^2 \cdot \exp[j(\Omega_0 - \Omega_{-1})] + \text{Intersymbol Interference} \quad [40]$$

Taking the phase of [40] gives the decision variable, the differential phase output:

$$d\Omega_0 = \Omega_0 - \Omega_{-1} + \text{Intersymbol Phase Interference}$$

Which gives the output variable needed in order to determine, following the $d\Omega$ to dibit mapping of Table I, the transmitted data dibit.

## 3. REFERENCES

1. Digital Communications, McGraw-Hill ISBN 0-07-Y66490-0
John G. Proakis

2. Power Spectrum, Internal NCR Report
Kiwi Smit