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Abstract The EVM and MER measurement accuracy of m-QAM

Purpose To present a procedure to be used to determine the MER and EVM of m-QAM signals for the 802.15.3 draft standard.

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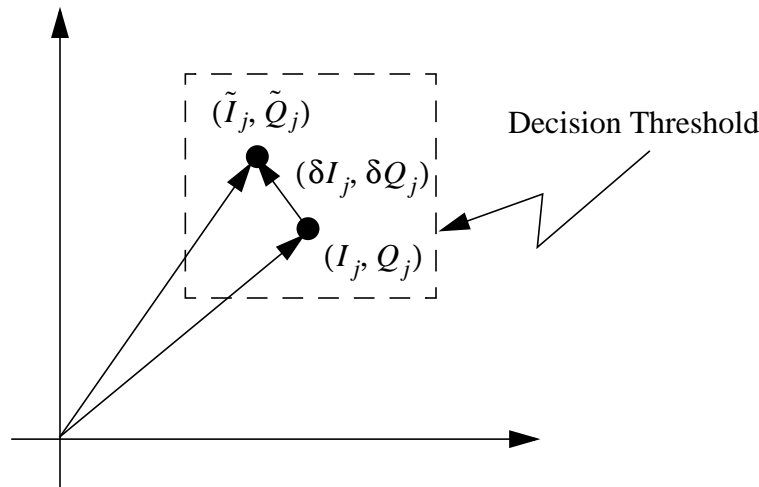
Statistical Analysis of Noise Measure Accuracy

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1.0 Notation

Modulation error ratio (MER) and error vector magnitude (EVM) are measures of the noise in the received signal vector.

In order to calculate these measures, a time record of N received signal co-ordinate pairs $(\tilde{I}_j, \tilde{Q}_j)$ is captured. For each received symbol, a decision is made as to which symbol was transmitted. The ideal position of the chosen symbol (the center of the decision box) is represented by the vector (I_j, Q_j) . The error vector $(\delta I_j, \delta Q_j)$ is defined as the distance from this ideal position to the actual position of the received symbol.



Thus, the received vector is the sum of the ideal vector and the error vector.

$$(\tilde{I}_j, \tilde{Q}_j) = (I_j, Q_j) + (\delta I_j, \delta Q_j) \quad (\text{EQ 1})$$

1.1 Modulation Error Ratio

1.1.1 Definition

Modulation error ratio is defined by the Digital Video Broadcast Measurements Group¹ as:

$$MER(dB) \equiv 10 \log \frac{\sum_{j=1}^N (I_j^2 + Q_j^2)}{\sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)} \quad (\text{EQ 2})$$

where (I_j, Q_j) represents the ideal received constellation point and $(\delta I_j, \delta Q_j)$ represents the received noise vector.

1.1.2 Statistical Distribution

If the received noise is additive white Gaussian noise with zero mean and variance σ_n^2 , then δI_j and δQ_j are iid Gaussian random variables with zero mean and variance $\sigma_n^2/2$, i.e., they are quadrature components of the channel noise.

The defining equation can be rewritten as:

$$MER(dB) \equiv 10 \log \frac{\frac{1}{N} \sum_{j=1}^N (I_j^2 + Q_j^2)}{\frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)} \quad (\text{EQ 3})$$

Here the numerator is simply the average power, which can be normalized to unity. The denominator represents the received noise power averaged over all received symbols, which are assumed to be random and equally likely.

The denominator is similar to a chi-squared distribution with $2N$ degrees of freedom [2] whose probability density function $p_y(y)$ is:

$$y = \chi^2 = x_1^2 + x_2^2 + \dots + x_n^2 \quad (\text{EQ 4})$$

$$p_y(y) = \frac{1}{2^{n/2} \sigma^n \Gamma(n/2)} y^{(n-2)/2} e^{-y/(2\sigma^2)}, \quad (y \geq 0) \quad (\text{EQ 5})$$

where x_j alternately represents the δI_j and δQ_j terms:

$$y = (\delta I_N^2 + \delta Q_N^2) + \dots + (\delta I_1^2 + \delta Q_1^2) = \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2) \quad (\text{EQ 6})$$

1. [1] sect. 6.9.2

1.1.3 Moments

The moments of the chi-squared distribution are [3]:

$$\begin{aligned} E\{y\} &= n\sigma^2 \\ E\{y^2\} &= 2n\sigma^4 + n^2\sigma^4 \\ \sigma_y^2 &= 2n\sigma^4 \end{aligned} \quad (\text{EQ 7})$$

We now define the modulation error (ME) as:

$$ME = \frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2) = \frac{1}{N} y \quad (\text{EQ 8})$$

Now the expected value becomes:

$$\begin{aligned} E\{ME\} &= \frac{1}{N} E\{y\} = \frac{n\sigma^2}{N} = 2 \frac{N\sigma_n^2/2}{N} = \sigma_n^2 \\ \sigma_{ME}^2 &= \frac{\sigma_y^2}{N^2} = \frac{2n\sigma^4}{N^2} = \frac{2 \cdot 2N(\sigma_n^4/4)}{N^2} = \frac{\sigma_n^4}{N} \end{aligned} \quad (\text{EQ 9})$$

since $\sigma^2 = \sigma_n^2/2$ and $n = 2N$

1.1.4 Accuracy

The Chebyshev inequality [2] can be used to place an upper bound on the probability of the measured noise power deviating from its expected value by more than ε .

$$\begin{aligned} P &= P\{|x - e\{x\}|\} \geq \varepsilon \leq \sigma_x^2 \\ \varepsilon &= \frac{\sigma_{ME}}{\sqrt{P}} = \frac{\sigma_n^2/(\sqrt{N})}{\sqrt{P}} = \frac{\sigma_n^2}{\sqrt{P \cdot N}} \end{aligned} \quad (\text{EQ 10})$$

This probability lends itself to defining a confidence interval of $(1 - P)$. This is the probability that the measured noise power is within some ε of the true noise power. The Chebyshev bound is rather loose unless $\sigma \ll \varepsilon$. If we define a confidence interval of 99%, then $P = 0.01$ and

$$\frac{\sigma_{ME}}{\varepsilon} = \sqrt{P} = 0.1 \quad (\text{EQ 11})$$

Now, rewriting the equation for MER

$$MER(dB) \equiv 10 \log \frac{\sum_{j=1}^N (I_j^2 + Q_j^2)}{N \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)} = 10 \log \frac{P_{avg}}{ME} \quad (\text{EQ 12})$$

If we normalize P_{avg} to unity

$$MER(dB) = -10 \log ME \quad (\text{EQ 13})$$

In the defined confidence interval $(1 - P)$, we know that ME is within ϵ of its mean

$$\begin{aligned} ME &= E\{ME\} \pm \epsilon \\ MER &= -10 \log(E\{ME\} \pm \epsilon) = -10 \log \left(\sigma_n^2 \pm \frac{\sigma_n^2}{\sqrt{P \cdot N}} \right) \\ &= -10 \log \sigma_n^2 \left(1 \pm \frac{1}{\sqrt{P \cdot N}} \right) = -10 \log \sigma_n^2 - 10 \log \left(1 \pm \frac{1}{\sqrt{P \cdot N}} \right) \end{aligned} \quad (\text{EQ 14})$$

the first term represents the desired measurement, while the second represents the absolute accuracy due to statistical sampling.

We can now plot the accuracy of the desired measurement (for a given confidence interval) as a function of the number of symbols averaged. Since $|\log(1 - x)| > |\log(1 + x)|$ for small positive x , we choose to plot

$$MER_{accuracy}(dB) = -10 \log \left(1 - \frac{1}{\sqrt{P \cdot N}} \right). \quad (\text{EQ 15})$$

1.1.5 Using Central Limit Theorem

The Central Limit Theorem states that as random variable are added together, their distribution approaches a Gaussian distribution. Gaussian distributions have the property that 99.6% of instances fall within three standard deviations (3σ).

$$\begin{aligned} \epsilon &= 3\sigma_{ME} = (3\sigma_n^2)/(\sqrt{N}) \\ MER_{accuracy}(dB) &= -10 \log \left(1 - \frac{3}{\sqrt{N}} \right) \end{aligned} \quad (\text{EQ 16})$$

This MER accuracy is seen to be much tighter bound than the Chebyshev Inequality for an equivalent confidence interval (99.6%). This bound holds if the mean is much less than the standard deviation, $E\{ME\} \ll \sigma_{ME}/$

$$\frac{E\{ME\}}{\sigma_{ME}} = \frac{\sigma_n^2}{\sigma_n^2/(\sqrt{N})} = \sqrt{N} \quad (\text{EQ 17})$$

in our case, we expect $N > 1000$, and $E\{ME\}/\sigma_{ME} \ll 31$.

1.2 Error Vector Magnitude

1.2.1 Definition

Error vector magnitude is defined by the DVB Measurements Group as:

$$EVM \equiv \sqrt{\frac{\frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)}{S_{max}^2}} \times 100\%, \quad (\text{EQ 18})$$

where S_{max} is the magnitude of the vector to the outermost constellation point and $(\delta I_j, \delta Q_j)$ represents the received noise vector.

1.2.2 Statistical Distribution

If the received noise is additive white Gaussian noise with zero mean and variance σ_n^2 , then δI_j and δQ_j are iid Gaussian random variables with zero mean and variance $\sigma_n^2/2$, i.e., they are the quadrature components of the channel noise. The numerator is similar to a generalized Rayleigh distribution with $2N$ degrees of freedom [3] whose probability density function $p_y(y)$ is:

$$\chi = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$p_\chi(\chi) = \frac{2}{2^{n/2} \sigma^n \Gamma(n/2)} \chi^{(n-1)} e^{-\chi^2/(2\sigma^2)}, \quad \chi > 0, \quad (\text{EQ 19})$$

where x_j alternately represents the δI_j and δQ_j terms:

$$\chi = \sqrt{(\delta I_1^2 + \delta Q_1^2) + \dots + (\delta I_N^2 + \delta Q_N^2)} = \sum_{j=1}^N \sqrt{(\delta I_j^2 + \delta Q_j^2)}. \quad (\text{EQ 20})$$

1.2.3 Moments

The moments of the generalized Rayleigh distribution are [3]:

$$E\{\chi^k\} = (2\sigma^2)^{k/2} \frac{\Gamma\left(\frac{1}{2}(n+k)\right)}{\Gamma\left(\frac{n}{2}\right)}, k > 0, \quad (EQ 21)$$

$$E\{\chi^2\} = 2\sigma^2 \frac{\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n}{2}\right)} = 2\sigma^2 \frac{\frac{n}{2}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} = n\sigma^2$$

since

$$\Gamma(m+1) = m\Gamma(m) \quad (EQ 22)$$

The expectation and therefore variance are not calculated as easily. It can shown that:

$$\sigma_\chi^2 = \sigma^2(2 - \pi/2) \quad (EQ 23)$$

$$E\{\chi\} = \sigma\sqrt{n-2+\pi/2}$$

It is interesting to note that the variance of the chi statistic does not increase with n.

We now define the averaged error vector magnitude as EV :

$$EV = \sqrt{\frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)} = \sqrt{\frac{1}{N}} \chi. \quad (EQ 24)$$

Now the expected value becomes:

$$E\{EV\} = \sqrt{\frac{E\{\chi\}}{N}} = \frac{\sigma\sqrt{n-2+\pi/2}}{\sqrt{N}} = \frac{\sigma_n\sqrt{2N-2+\pi/2}}{\sqrt{2N}}, \quad (EQ 25)$$

$$\sigma_{EV}^2 = \frac{\sigma_\chi^2}{N} = \frac{\sigma^2(2-\pi/2)}{N} = \frac{\sigma_n^2\sqrt{2N-2+\pi/2}}{\sqrt{2N}}$$

since $\sigma^2 = \sigma_n^2/2$ and $n = 2N$. Intuitively, we expect $E\{EV\}$ to be σ_n . Note that it is a biased estimate that approaches the desired mean as $N \rightarrow \infty$.

1.2.4 Accuracy

The Chebyshev Inequality can be used to place an upper bound on the probability of the measured noise power deviating from its expected value by more than ε .

$$P = P\{|x - e(x)| \geq \varepsilon\} \leq \frac{\sigma_x^2}{\varepsilon^2} \quad (\text{EQ 26})$$

$$\varepsilon = \frac{\sigma_{EV}}{\sqrt{P}} = \frac{\sigma_n \sqrt{(2 - \pi/2)/(2N)}}{\sqrt{P}} = \frac{\sigma_n \sqrt{2 - \pi/2}}{2PN}$$

This probability lends itself to defining a confidence interval of $(1 - P)$. This is the probability that the measured noise power is within some ε of the true noise power. The Chebyshev bound is rather loose unless $\sigma \ll \varepsilon$. If we define a confidence interval of 99% then $P = 0.01$ and

$$\frac{\sigma_{EV}}{\varepsilon} = \frac{\sigma_{EV}}{\sigma_{EV}/(\sqrt{P})} = \sqrt{P} = 0.1 \quad (\text{EQ 27})$$

Now, rewriting the equation for EVM

$$EVM \equiv \sqrt{\frac{\frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)}{S_{max}^2}} \cdot 100\% = \frac{EV}{S_{max}} \cdot 100\% \quad (\text{EQ 28})$$

In the defined confidence interval $(1 - P)$ we know that EV is within a ε of its mean

$$EV = E\{EV\} \pm \varepsilon$$

$$EVM = \frac{E\{EV\} \pm \varepsilon}{S_{max}} \cdot 100\% = \frac{E\{EV\}}{S_{max}} \cdot 100\% \pm \frac{\varepsilon}{S_{max}} \cdot 100\% \quad (\text{EQ 29})$$

the first term represents the desired measurement, while the second represents the absolute accuracy due to statistical sampling.

$$EVM_{accuracy} = \pm \frac{\varepsilon}{S_{max}} \cdot 100\% = \pm \frac{\sigma_n \sqrt{2 - \pi/2}}{S_{max} \sqrt{2PN}} \times 100\% = \frac{100\% \times \sigma_n \sqrt{2 - \pi/2}}{1.527 S_{avg} \sqrt{2PN}} \quad (\text{EQ 30})$$

since $S_{max} = 1.527 S_{avg}$ for 64-QAM.

Now,

$$C/N = 20 \left(\log \frac{S_{avg}}{\sigma_n} \right) \quad \text{or} \quad \frac{\sigma_n}{S_{avg}} = 10^{-\frac{(C/N)}{20}} \quad (\text{EQ 31})$$

and

$$EVM_{accuracy} = \pm \frac{10^{-\frac{(C/N)}{20}} \sqrt{2 - \frac{\pi}{2}}}{1.527 \sqrt{2PN}} \times 100\% \quad (\text{EQ 32})$$

We can now plot the accuracy of the desired measurement (for a given confidence interval) as a function of the number of symbols averaged. We choose a confidence interval of 0.99.

1.2.5 Using Central Limit Theorem

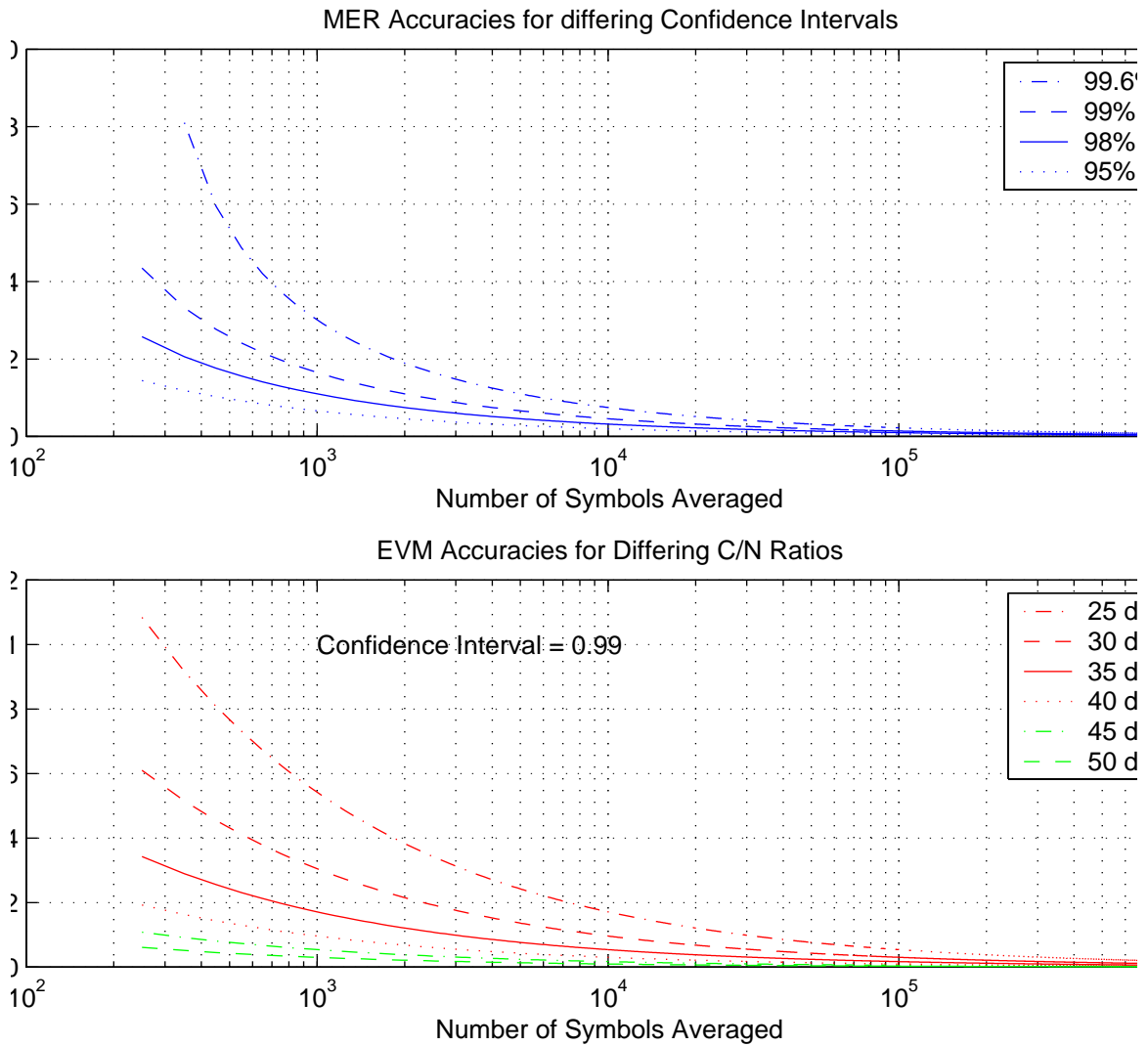
The Central Limit Theorem states that as random variable are added together, their distribution approaches a Gaussian distribution. Gaussian distributions have the property that 99.6% of instances fall within three standard deviations (3σ).

$$\begin{aligned} \varepsilon &= 3\sigma_{EV} = 3 \frac{\sigma_n \sqrt{2 - \pi/2}}{\sqrt{2N}} \\ EVM_{accuracy} &= \pm \frac{\varepsilon}{S_{max}} \times 100\% = \frac{3\sigma_n \sqrt{2 - \pi/2}}{1.527 S_{avg} \sqrt{2N}} \times 100\% \\ &= \pm \frac{3 \cdot 10^{-\frac{(C/N)}{20}} \sqrt{2 - \pi/2}}{1.527 \sqrt{2N}} \times 100\% \end{aligned} \quad (\text{EQ 33})$$

This EVM accuracy is plotted below for different C/N ratios. It is seen that this is a much tighter bound than the Chebyshev Inequality for an equivalent confidence interval (99.6%). This bound holds if the mean is much less than the standard deviation, $E\{EV\} \ll \sigma_{EV}$.

$$\frac{E\{EV\}}{\sigma_{EV}} = \frac{\sigma_n \sqrt{2N - 2 + \frac{\pi}{2}}}{\sqrt{2N}} = \frac{\sqrt{2N}}{\sigma_n \sqrt{2 - \frac{\pi}{2}}} = \frac{\sqrt{2N - 2 + \pi/2}}{\sqrt{2 - \pi/2}} \quad (\text{EQ 34})$$

in this case, we expect $N > 1000$, and $(E\{EV\})/\sigma_{EV} > 68$.



1.3 MER Measurement Procedure

- Acquire N received symbols (with coordinates (I_j, Q_j) in the constellation diagram). For each received symbol, decide which symbol was transmitted based on ML criterion and measure the error vector from the received vector $(I_j + jQ_j)$ to the ideal constella-

tion position ($I_o + jQ_o$) (the center of the decision box with no distortion present).

Calculate MER based on (EQ 3) using EQ 35 below for a data take. The recommended number of symbols taken as data for this measurement is 2000 or more.

$$\begin{aligned}
 I_{oj} &= \operatorname{argmin}[I_j - I_o(k)]^2, \quad k=1, \dots, M \\
 \delta I_j &= I_j - I_{oj} \\
 Q_{oj} &= \operatorname{argmin}[Q_j - Q_o(k)], \quad k=1, \dots, M \\
 \delta Q_j &= Q_j - Q_{oj}
 \end{aligned}
 \tag{EQ 35}$$

1.4 EVM Measurement Procedure

- Similar procedure as for the MER measurement with EQ 18 being implemented and an assigned value to S_{max} .

1.5 Reference Receiver Calibration

If the the reference receiver used within the test is not calibrated to correct for its front-end induced errors, then a recommended calibration filter design is as follows.

- The ripple of amplitude response (TBD %).
- The ripple of group delay (TBD ns).
- Given:
 - a desired frequency response $H_d(\omega_i)$
 - a received frequency response $H_a(\omega_i)$, where $\{\omega_1, \dots, \omega_N\}$ are evenly spaced in $[0, \pi]$.
- Design:
 - a filter with impulse response $h(n)$, $n = 1, 2, \dots, M$.
- Procedure:
 - 1) extend $H_d(\omega_i)$ and $H_a(\omega_i)$ to $[0, 2\pi]$ so they are conjugate symmetric with respect to π .
 - 2) derive $h(n)$ using the following equations, where $W(\omega)$ is a weighting function.

$$\sum_{n=1}^M h(n)e^{-j\omega n} = \frac{H_d(\omega)}{H_a(\omega)} = H(\omega)
 \tag{EQ 36}$$

(EQ 37)

Or,

$$\begin{bmatrix} W(\omega_1)H(\omega_1) \\ W(\omega_2)H(\omega_2) \\ \dots \\ W(\omega_{2N})H(\omega_{2N}) \end{bmatrix} = \begin{bmatrix} W(\omega_1)e^{-j\omega_1} & W(\omega_1)e^{-j2\omega_1} & \dots & W(\omega_1)e^{-jM\omega_1} \\ W(\omega_2)e^{-j\omega_2} & W(\omega_2)e^{-j2\omega_2} & \dots & W(\omega_2)e^{-jM\omega_2} \\ \dots & \dots & \dots & \dots \\ W(\omega_{2N})e^{-j\omega_{2N}} & W(\omega_{2N})e^{-j2\omega_{2N}} & \dots & W(\omega_{2N})e^{-jM\omega_{2N}} \end{bmatrix} \begin{bmatrix} h(1) \\ h(2) \\ \dots \\ h(M) \end{bmatrix}$$

$$H = Ah \quad (\text{EQ 38})$$

and,

$$h = (A'A)^{-1} A'H \quad (\text{EQ 39})$$

$W(\omega_i)$, the weight, is to be determined by the designer to meet the ripple criteria.

[1] Telecommunications Standards Institute, "Digital Video Broadcasting (DVB), Measurement Guidelines for DVB Systems." 1 February 1997, ETR 290.

[2] Papoulis, *Probability, Random Variables and Stochastic Processes*, 1965 McGraw-Hill.

[3] Proakis, *Digital Communications*, Third Edition., 1995 McGraw-Hill.