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Re:	IEEE 802.16e D4 Draft
Abstract	To improve the closed loop MIMO
Purpose	To incorporate the changes here proposed into the 802.16e D5 draft.
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Closed Loop MIMO Precoding

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1 Background

Precoding for MIMO was introduced in IEEE802.16eD4. We here specify a framework for feedback of precoding information from the receiver to the transmitter and the precoder design algorithm.

Motivation:

Precoding is aimed to exploit some channel information in order to improve performance. This channel information can be in various forms, and we focus on two types of information: long-term channel statistics and instantaneous channel estimates.

The real world communication channels are usually not i.i.d. but have some underlying statistics, such as transmit antenna correlation and a non-zero mean. This information can be exploited designing a precoder to enhance the system performance when using together with space-time block codes, which are designed based on i.i.d channel assumptions.

Another form of channel information is the instantaneous channel estimate. If the channel estimates were accurate at the time of precoder use, this would be the only information needed for precoder design. However, there is usually a unspecified delay between the time when the channel is estimated and the time when the precoder based on this channel information is used. Hence the channel estimates need to have some aging factors to reflect this time lag.

Here we propose a precoding scheme based on a combination of both long-term channel statistics and aged channel estimates. The scheme transits smoothly from precoding based on instantaneous channel estimate when the estimate quality is good (short time lag) to precoding based on long-term statistics only when the channel estimate has little correlation to the true channel (long time lag).

Rough outline:

- Feedback mechanism
- Codebook design
- Precoding design algorithm
- Simulation results

2 Specific text changes

`---------Start text proposal-------`

[Modify the following Table 298a in section 8.4.5.4.12.1]

Table 298a. CQICH Enhanced allocation IE format

Syntax	Size (bits)	Notes
CQICH_Enhanced_Alloc_IE() {		
Extended DIUC	4	0x09
Length	4	Length in bytes of following fields
CQICH_ID	variable	Index to uniquely identify the CQICH resource assigned to the MSS
Period (=p)	2	A CQI feedback is transmitted on the CQICH every 2^p frames
Frame offset	3	The MSS starts reporting at the frame of which the number has the same 3 LSB as the specified frame offset. If the current frame is specified, the MSS should start reporting in 8 frames
Duration (=d)	3	A CQI feedback is transmitted on the CQI channels indexed by the CQICH_ID for 10 x 2^d frames. If d== 0, the CQICH is de-allocated. If d == 111, the MSS should report until the BS command for the MSS to stop.
N_T actual BS antennas	3	001 = Reserved 010 = 2 actual antennas 011 = 3 actual antennas 100 = 4 actual antennas 101 = 5 actual antennas 110 = 6 actual antennas 111 = ~~7~~ 8 actual antennas 000 = ~~8~~ 12 actual antennas
Feedback_type	4	0000 = Open loop precoding. Pilots in burst to be precoded with *W*. SS to rely only on pilots in burst for channel estimation. 0001 = ~~Complex weight of specific element of *W*~~ *Quantized value of W.* 0010 = Fast DL measurement 0011 = Layer specific channel strengths 0100 = MIMO mode and permutation zone feedback 0101 = ~~Feedback of subset of antennas to use.~~ Channel and

		Long-term Statistics based precoding 0110 ~ 1111 reserved
CQICH_Num	4	Number of CQICHs assigned to this CQICH_ID is (CQICH_Num +1)
for (i=0;i<CQICH_Num;i++) {		
Allocation index	6	Index to the fast feedback channel region marked by UIUC=0
}		
if (Feedback_type != 0100) { MIMO_permutation_feedback cycle }	2	00 = No MIMO and permutation mode feedback 01 = the MIMO and permutation mode indication shall be transmitted on the CQICH indexed by the CQICH_ID every 4 frames. The first indication is sent on the 8th CQICH frame. 10 = the MIMO mode and permutation mode indication shall be transmitted on the CQICH indexed by the CQICH_ID every 8 frames. The first indication is sent on the 8th CQICH frame. 11 = the MIMO mode and permutation mode indication shall be transmitted on the CQICH indexed by the CQICH_ID every 16 frames. The first indication is sent on the 16th CQICH frame.
if (Feedback_type != 0101) { K - Nr CQICH for instantaneous channel feedback }	4	K - Nr CQICH for instantaneous channel feedback
Padding	*variable*	The padding bits are used to ensure the IE size is integer number of bytes.

[Add the following text into section 8.4.8.3.6]

Channel and Long-term Statistics based Precoding

A. Feedback mechanism

The SS feeds back information about the instantaneous channel, H_0, the long term mean of the channel, H_m, the transmit covariance (transmit antenna correlation) of the channel, R_t, the signal to noise ratio at the receiver, MIMO SNR, and a measure of the fading rate, f_d. From this information, given the STC scheme being used, the base station computes the precoding matrix W.

The instantaneous channel estimates is fed back using the fast-feedback channel. The channel mean $H_{m,}$ transmit correlation $R_{t\,,}$ the signal to noise ratio and fading rate measure are fed back on a slow bases as needed, using the PRC-CHA-FBCK MAC management message.

The instantaneous channel estimate, the channel mean and the transmit covariance matrix are fed back in terms of a code book quantization of its right singular vector matrices and its non-zero singular values. That is, define the singular value decompositions:

$$H_0 = U_0 S_0 V_0^H ,$$

$$H_m = U_m S_m V_m^H$$

and

$$R_T = V_R S_R V_R^H,$$

The instantaneous channel, the channel mean and the transmit covariance matrix of the channel is fed back in terms of the columns in V_0, V_m and V_R that corresponds to the non-zero singular values in S_0, S_m and S_R respectively, and the non-zero singular values.

The unitary matrices are quantized using the codebook described in the following.

B. Codebook design for pre-coding

The unitary matrices are quantized according to a code book and fed back either on the fast feedback channel or in a MAC management message.

Let L denote the total number of entries in the codebook. Similar to [Hochwald et al], given a 2 x M_t matrix

$$U = [I\ U'],$$

M_t x M_t diagonal matrices

$$[C_k]_{m,m} = e^{\frac{j2\pi}{\sqrt{L}}[U]_{k,m}}, k = 1,2; m = 1,\ldots,M_t; C_k^{\sqrt{L}} = I$$

and M_t x B matrix Y ($B<=M_t$), the entries in the codebook are given as

$$W_l = C_1^{l_1} C_2^{l_2} Y,$$

with $l = [l_1 \quad l_2]$, and l_i are elements in the ring of integers mod $\sqrt{L}$. For simplicity, the basis matrix Y can be chosen as selection of total of B columns (set of indexes $\boldsymbol{B}_c$) of the DFT matrix

$$[DFT]_{m,b} = e^{j\frac{2\pi}{M_t}(m-1)(b-1)}, \qquad m,b = 1,\ldots,M_t.$$

This approach brings set partitioning into the codebook approach which would enable more degrees of freedom for variety of feedback rates (e.g., in general, sending back l_1 with one rate and l_2 with another) and better tuning of pre-coding to instantaneous and statistical channel information.

The elements of the matrix U' and the column subset selection $\boldsymbol{B}_c$ for Y will be further specified. The criteria for the selection of a particular W_l (l_1 or l_2 or both) at the receiver will depend on the given transmission scheme (A, B, C) and the type of detection algorithm.

The slowly varying quantities H_m and R_T, are fed back in two parts: the singular vector matrices and the non-zero singular values. The right singular vector matrices, V_m, V_R, specified by indices in their code books of quantized unitary matrices, together with the corresponding quantized non-zero singular values, are fed back using the PRE-CHA-FBCK MAC management message. The slowly varying quantities, the quantized MIMO SNR value and the quantized fading rate measure f_d are also fed back here.

The unitary matrix V_0 of the instantaneous channel estimate, H_0, is fed back on K CQICH channels in its code book quantized form together with the quantized forms of its non-zero singular values.

Quantization of other parameters

The MIMO SNR is defined as the ratio of the sum of the diagonal elements of R_T to the receiver noise variance at one antenna measured in dB and quantized into 5 bits according Table 315a.

MIMO SNR	**Payload bits**
< 3 dB	00000
n+3 dB < MIMO SNR < n+4 dB	00001 to 11110
> 27 dB	11111

Table 315a: Quantization of the MIMO SNR.

The fading rate measure, f_d, is defined as the variance of the Doppler spectrum and is quantized into 5 bits according Table 315b.

Fading rate measure, f_d, in Hz	**Payload bits**
$f_d < 1$	00000
$2^{(n-1)} < f_d \le 2^n$	00001-00011
$8+(n-4)(200-8)/28 < f_d \le 8+(n-3)(200-8)/28$	00011-11110
f_d>200	11111

Table 315b: Quantization of the fading rate measure, f_d.

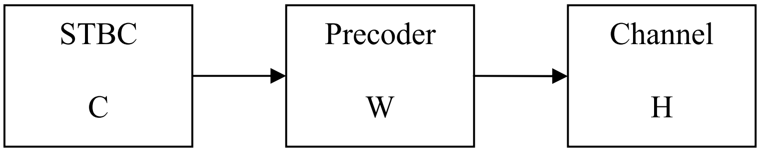
C. Precoder design algorithm

Given the above feedback the BS computes the precoding matrix W according to the method described below.

C.1. Computation of the precoding matrix W given the channel mean and the channel transmit covariance matrix

The computation of the precoding matrix here is based on the derivation of the optimal linear precoder for orthogonal space-time block codes given the mean value of the channel and its transmit covariance matrix [1].

The precoder design is applicable for all Matrices A for the case of 2, 3 or 4 transmit antennas and Matrices B for the case of 3 or 4 transmit antennas. In all of these cases the space-time code can be divided up into a *precoding part* followed by a space-time code. We can therefore use the derivation in [1] to compute the optimal precoding matrix.



Precoding matrix formula

Given a channel mean H_m and a transmit correlation matrix R_t, the precoder W has the form

$$W = V_A S_A^{1/2} P$$

where V_A and S_A are the eigenvectors and eigenvalues of a matrix $A = V_A S_A V_A^H$, and P is an orthogonal matrix specified per number of transmit antennas and space-time block code matrix configuration. The matrix P is given in a later section.

The matrix A is given as

$$A = \frac{1}{2\upsilon}(MI_N + \Psi^{\frac{1}{2}}) - R_t ,$$

with

$$\Psi = M^2 I_N + 4\upsilon R_t^{-1} H_m^* H_m R_t^{-1} ,$$

where M is the number of receive antennas, and N is the number of transmit antennas. In order to form Ψ and A, we need to calculate υ. The procedure for calculating υ is outlined as follows.

Procedure for calculating υ

The precoder design is based on the minimum distance between the space-time block codewords. For orthogonal space-time block codes such as matrix A for 2 and 4 antennas, we can express the product distance between two code-words C and Ĉ as:

$$(C-\hat{C})(C-\hat{C})^{*}=\mu PI,$$

where P is the average total transmit power and μ represents the distance between the two code words. Let μ_0 be the minimum code word distance, considering all space-time block code words. For non-orthogonal space-time block code such as matrices B for 3 and 4 transmit antennas, and matrix A for 3 transmit antenna case, μ_0 is taken to be the smallest non-zero eigenvalue amongst all the product distance matrices between two space-time block code words (C-Ĉ)(C-$\hat{C}$)*. The initial rank of the precoding matrix is equal to the number of transmit antennas.

Solve the equation for υ:

$$MN+\sum_{i=1}^{N}\sqrt{M^2+4\upsilon\lambda_i}=\beta\upsilon \qquad (*)$$

where λ_i are the eigenvalues of

$$R_t^{-1}H_m^{*}H_mR_t^{-1}$$

and

$$\beta=2[tr(R_t^{-1})+\mu_0\rho/4],$$

where ρ is the MIMO SNR, i.e. total transmit power P over the receive noise variance per antenna (with proper normalization of the channel).

In the general case, this equation does not appear to have closed form solution. However, solving for υ can be done efficiently using binary search, which we call the inner algorithm.

Inner algorithm: The following lower and upper bounds on υ are established

$$v_{lower}=\frac{MN}{\beta}$$

$$v_{upper}=\frac{1}{\beta^2}(4N\sum_{i=1}^{N}\lambda_i+2\beta MN)$$

Binary numerical search can then be carried out to find solution for (*) between these bounds up to a desired precision.

Given υ from the equation above we now compute the matrix A as specified earlier.

If A is positive semi-definite we can obtain the precoder matrix W. If not, we need to solve the equation

$$\sum_{i=2}^{N} \lambda_i \left(\frac{1}{2\upsilon}(MI_N + \Psi^{1/2}) - R_t^{-1}\right) = \frac{1}{4}\mu_0\rho \qquad (**)$$

Here we are starting the sum at 2, i.e. we dropped the term corresponding to the smallest eigenvalue λ_i. If the new A becomes positive semi-definite we go onto to calculate W, otherwise we go back again and drop another term in (**) from the sum corresponding to the next smallest eigenvalue λ_i, etc…

To solve equation (**), we use the outer algorithm as outlined below:

> Outer algorithm: There is no explicit function that relates the individual eigenvalues in the expression in (**) to υ. Fortunately, again we can derive upper bound and lower bound values on υ and then use binary search to find the solution efficiently. Using inequalities on eigenvalues of a sum of Hermitian matrices, we obtain the following bounds on the left-hand-side expression of (**):
>
> $$f_{upper} = \frac{M(N-k)}{2\upsilon} + \sum_{i=k+1}^{N} \frac{\sqrt{M^2 + 4\nu\lambda_i}}{2\nu} - \sum_{i=1}^{N-k} \lambda i(R_t^{-1})$$
>
> $$f_{lower} = \frac{M(N-k)}{2\upsilon} + \sum_{i=k+1}^{N} \frac{\sqrt{M^2 + 4\nu\lambda_i}}{2\nu} + k\lambda_1(R_t^{-1}) - tr(R_t^{-1})$$
>
> Here the bounds are given for the general case when k modes are dropped. Equating each expression to $\rho\mu_0 / 4$ and solve for ν, using the inner algorithm mentioned in the previous section. The solutions of these two equations then become the upper bound and lower bound on the solution for ν in (**). Noting that the sum of eigenvalues is monotonous in ν, a binary search can then be carried out to solve equation (**) efficiently up to a desired numerical precision.

C.2. Computation of the precoding matrix W given an aged channel estimate

Given the channel estimate H_0 together with the channel mean H_m and transmit correlation R_t, we can use the above method to compute the precoding matrix W after a transformation of the channel.

Both the channel estimate H_0 and the true channel H are drawn from the same statistics with transmit covariance R_t. They can be related to each other via a time correlation function ρ

$$E[H^* H_0] = E[H_0^* H] = \rho R_t$$

with

$$0 \le \rho \le 1.$$

If ρ=1, we have an exact estimate of the channel, and if ρ=0, we do not have a useful estimate of the channel but can only use the channel statistics.

From this, based on MMSE estimation theory, an effective channel with mean and transmit covariance are defined as

$$
\begin{aligned}
H_{m,eff} &= \rho H_0 + (1-\rho)H_m \\
R_{t,eff} &= (1-\rho^2)R_t
\end{aligned}
$$

The precoder W can then be designed as outlined in the previous section, using $H_{m,eff}$ and $R_{t,eff}$ instead of H_m and R_t.

C.3. Computation of the channel time correlation function ρ

ρ represents the time correlation of the channel and is a function of the Doppler spread f_d and the time lag between when the channel is estimated and when is it used for precoder design. Here we propose using the standard Bessel function based on Jake spectrum for ρ

$$
\rho = \begin{cases} J_0(2\pi f_d \Delta t) & \text{if} \quad J_0(2\pi f_d \Delta t) \geq 0.6 \\ 0 & \phantom{\text{if}} \quad J_0(2\pi f_d \Delta t) < 0.6 \end{cases}
$$

where Δt is the time difference between the estimation of the channel and the time at which the transmission occurs.

C.4. Right singular matrix P

For 4 transmit antennas:
For matrix A, P is given as

$$
P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}
$$

For matrix B, if W has rank 4 or 3, P is not needed. If W has rank 2, P is a permutation matrix that feeds row 1 and row 4 of matrix B to the two non-zero eigenmodes of W. If W has rank 1, P is a matrix that feeds the sum of row 1 and row 4 to the non-zero eigenmode of W. For example, if the non-zero eigenmode of W is the first one, P is given as

$$P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

For 3 transmit antennas:
For matrix A, if W has rank 3, P is given as

$$P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

If W has rank 2, P is a permutation matrix that feeds the data stream in each half of the STC matrix to the two non-zero eigenmodes of W.
If W has rank 1, P is a permutation matrix that feeds the second row of the STC matrix to the non-zero eigenmode of W.

For matrix B, if W has rank 3, P is not needed. If W has rank 2, P is a permutation matrix that feeds row 1 and row 3 of the STC matrix to the two non-zero eigenmodes of W. If W has rank 1, P is a matrix that feeds the sum of the row 1 and row 3 to the non-zero eigenmodes of W. For example, if the non-zero eigenmode of W is the first one, then P is as specified earlier for matrix A.

For 2 transmit antennas:
Here P is not needed.

[Add section 6.3.2.3.57]

6.3.2.3 MIMO precoding management

Table 106n – Feedback of long-term MIMO channel statistics (PRC-CHA-FBCK) message format

Syntax	**Size**	**Notes**
DREG-REQ format(){		
Management message type = 61	**8 bits**	
Message body	**variable**	**See 8.4.8.3.6**
}		

```
---------End text proposal-------
```

3 Simulation results

We present three simulation results when applying the proposed precoding scheme to a 4x1 system (4 transmit and 1 receive antenna) which uses space-time code block matrix A as specified in the current standard.

3.1. Precoding on long-term statistics only

Figure 1 shows the gain based on long-term statistics precoding only. In this simulation, the underline channel has arbitrarily chosen non-zero mean and transmit correlation given as

$$\mathrm{H}_{\mathrm{m}} = \begin{bmatrix} -0.15-0.12\mathrm{i} & -0.326-0.17\mathrm{i} & -0.42+0.07\mathrm{i} & 0.20-0.033\mathrm{i} \end{bmatrix}$$

and

$$R_t = \begin{bmatrix} 1.32 & -0.55-0.23\mathrm{i} & 0.49-0.46\mathrm{i} & 0.103-0.41\mathrm{i} \\ -0.55-0.23\mathrm{i} & 0.43 & 0.013+0.31\mathrm{i} & 0.068-0.074\mathrm{i} \\ 0.49-0.46\mathrm{i} & 0.013+0.31\mathrm{i} & 0.68 & 0.0003-0.56\mathrm{i} \\ 0.103-0.41\mathrm{i} & 0.068-0.074\mathrm{i} & 0.0003-0.56\mathrm{i} & 1.56 \end{bmatrix}$$

This channel has a K factor of 0.1.

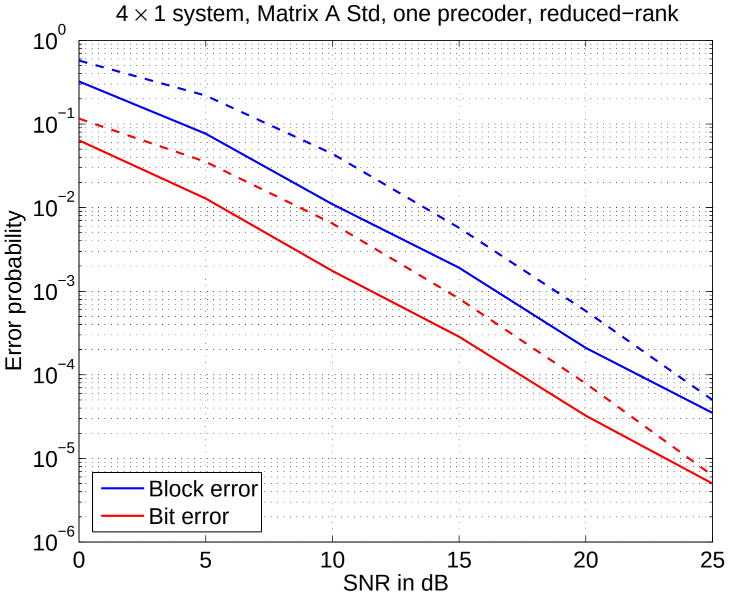


Figure 1. Precoding on channel mean and correlation.

3.2. Precoding on both aged channel estimate and long-term correlation

Figure 2 shows the performance results for precoder employing both aged channel estimates and long-term transmit antenna correlation. The underlying channel in this plot has zero-mean and a transmit correation of

$$R_t = \begin{bmatrix} 1 & 0.7 & 0.49 & 0.343 \\ 0.7 & 1 & 0.7 & 0.49 \\ 0.49 & 0.7 & 1 & 0.7 \\ 0.343 & 0.49 & 0.7 & 1 \end{bmatrix}$$

The estimate correlation factor ρ is varied to show the respective gains at different channel estimate qualities.

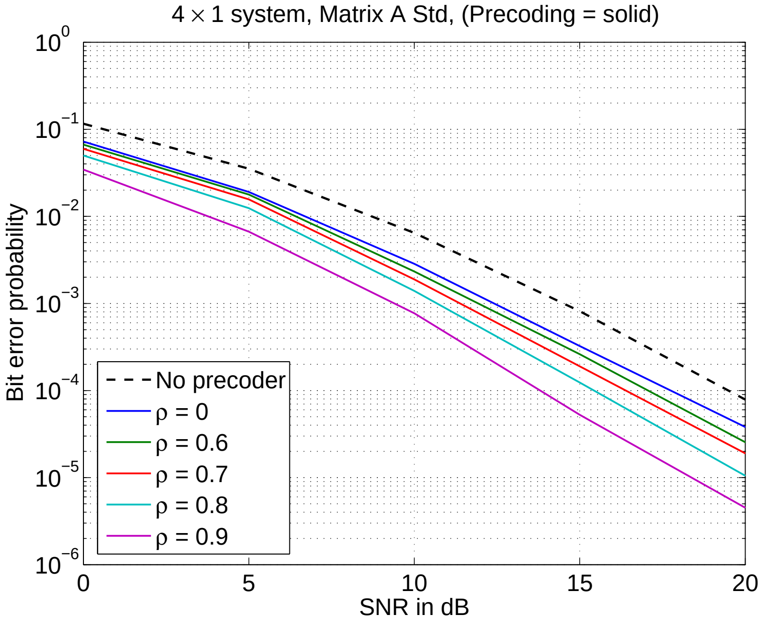


Figure 2. Precoding on both channel estimate and long-term correlation.

Figure 3 shows the comparison of the precoder performance based on the proposed scheme, versus beamforming based on the instantaneous channel estimate only. Again the factor ρ shows the correlation between the channel estimate and the true channel at the time of use, ρ=0.99 reflects a very accurate channel estimate, and ρ=0 means the estimate has completely de-correlated with the true channel.

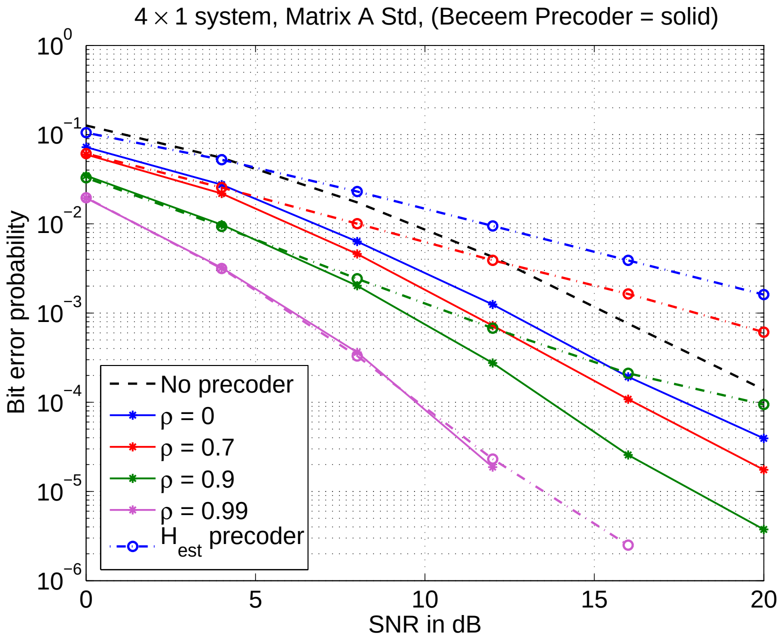


Figure 3. Comparison between proposed scheme and beamforming on channel estimate only.

References

[1] Mai Vu and Arogyaswami Paulraj, “Linear precoding for MIMO channels with non-zero mean and transmit correlation in orthogonal space-time coded systems”, IEEE Vehicular Technology Conf. (VTC-Fall), Sept 2004.

[2] B. Hochwald et al, “Systematic Design of Unitary Space-Time Constellations”, IEEE Transactions on Information Theory, Vol 46, No 6, pp1962-1973, Sept 2000