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Abstract	In this document, we describe a structured approach to irregular LDPC code construction based on "seed" matrices that are expanded using permutation matrices for purposes of error correction control. These codes have small storage requirements with good block error rate performances over a wide range block sizes. Also described in this document is a structured approach to puncturing irregular LDPC codes facilitate rate-compatibility without having to modify the connective net in the encoder or decoder while still offering a wide range of code rates for link optimization.
Purpose	[Adoption of proposed text as optional feature.]
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Irregular Structured LDPC Codes

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Explanation of design

Some random irregular LDPC constructions based on edge ensemble designs have error correcting capabilities measured in Bit Error Rate (BER) that are within 0.05dB of the rate distorted Shannon limit for the AWGN channel [3].

Unfortunately, these random LPDC code constructions require long codeword constructions (on the order of 10^6 to 10^7 bits) in order to achieve these error rates, and despite good BER performance, these random code constructions often have poor Block Error Rate (BLER) performances. Hence, these random constructions do not lend themselves well to packet-based communication systems. Yet another disadvantage of random constructions based on edge distribution ensembles is for each codeword length another random construction is needed. Thus, communication systems employing variable block sizes would require multiple code definitions that could consume a significant about of non-volatile memory storage for a large combination of codeword lengths and code rates.

An alternative to random LDPC construction is structured LDPC constructions that rely on a general algorithmic approach to constructing LDPC matrices and require much less non-volatile memory than random constructions. Thus, the problem is to design irregular structured LDPC codes that have good overall error performance (both BER and BLER) for a wide range of code rates and block sizes with attractive storage requirements. The result of such LDPC codes is a better performing communication system with lower cost terminals. These factors make such a FEC attractive for application over a wide range of products including but not limited to IEEE802.16 and IEEE802.11n compliant products.

Thus, this exact description below for LDPC code construction succeeds at solving the above said problem while providing excellent performance for all block sizes without the non-volatile memory requirements of randomly constructed codes.

The LDPC codeset design below fits within the LDPC framework as proposed by TGn Sync and provides an optimisation of consumed power per bit while meeting the specified datarate with a negligible variation in SNR for constant PER performance for different packet lengths. It is designed such that it allows efficient near-linear encoding and various decoding algorithms including layered belief propagation, which is a preferred implementation due to its convergence rate.

The codeset below is designed for $L_{\text{cword\_inc}} = 48$ (or any multiple thereof by omission) and $z = N_{\text{spread}}$ per the TGn Sync definition.

Proposed Text Change

8.4.9.2.5.3 LDPC parity check matrix construction

The parity check matrices ***H*** are derived from binary seed matrices $\boldsymbol{H}_b$ of size $m_b \times n_b$ by expanding each ‘1’ in $\boldsymbol{H}_b$ with $\boldsymbol{P}^{(i\times(j+1-i))\bmod p}$ for all rows i, $i \in \{1,\dots,m_b\}$, and columns j, $i \in \{1,\dots,n_b\}$, for which $i \le j+1$ holds, where $\boldsymbol{P}^1$ is the singular cyclicy left shifted identity matrix of size *z* and by expanding every other element with the all-zeros matrix of the same size as $\boldsymbol{P}^1$. The modulo factor *p* is the smallest positive primitive for which the relations $p \ge n_b + 2$ and $p \ge z$ hold.The codeword length is therefore $z \times n_b$.

A parity check matrix ***H*** is hence entirely defined by the selection of $\boldsymbol{H}_b$, the code rate *R* and the value z.

$$\boldsymbol{H}_b^1 = \begin{bmatrix} \text{0x80000102 0385} \\ \text{0xC0000040 2039} \\ \text{0x60000040 203C} \\ \text{0x30000048 B010} \\ \text{0x18000018 4032} \\ \text{0x0C000023 2820} \\ \text{0x06000000 232C} \\ \text{0x03000007 2008} \\ \text{0x01800048 4240} \\ \text{0x00C00084 0026} \\ \text{0x00600040 022C} \\ \text{0x00300140 4048} \\ \text{0x001800A0 2480} \\ \text{0x000C0044 0301} \\ \text{0x00060004 8214} \\ \text{0x00030042 9004} \\ \text{0x00018012 0A08} \\ \text{0x0000C002 021C} \\ \text{0x00006002 A410} \\ \text{0x00003042 2102} \\ \text{0x00001880 2814} \\ \text{0x00000C02 04B0} \\ \text{0x00000643 1020} \\ \text{0x00000330 0248} \end{bmatrix}, \quad \boldsymbol{H}_b^2 = \begin{bmatrix} \text{0x80000101 0951} \\ \text{0xC0000000 8398} \\ \text{0x60000030 1026} \\ \text{0x300000C0 2094} \\ \text{0x18000014 440A} \\ \text{0x0C000003 00CA} \\ \text{0x06000050 08A1} \\ \text{0x03000000 2268} \\ \text{0x01800000 9070} \\ \text{0x00C00064 0005} \\ \text{0x00600100 401A} \\ \text{0x00300000 0456} \\ \text{0x00180004 00A6} \\ \text{0x000C0000 1033} \\ \text{0x00060082 0184} \\ \text{0x00030001 0489} \\ \text{0x00018088 0062} \\ \text{0x0000C002 A081} \\ \text{0x00006008 400D} \\ \text{0x00003000 0455} \\ \text{0x00001800 0343} \\ \text{0x00000C00 02A9} \\ \text{0x00000600 0836} \\ \text{0x00000328 004C} \end{bmatrix}, \quad \boldsymbol{H}_b^3 = \begin{bmatrix} \text{0x80000140 B042} \\ \text{0xC0000020 806A} \\ \text{0x60000090 4114} \\ \text{0x30000000 2E41} \\ \text{0x18000028 1142} \\ \text{0x0C000008 121A} \\ \text{0x0600001A 1220} \\ \text{0x03000062 0202} \\ \text{0x01800040 0702} \\ \text{0x00C00008 5804} \\ \text{0x00600001 5041} \\ \text{0x00300002 8890} \\ \text{0x00180004 10C8} \\ \text{0x000C00C0 4480} \\ \text{0x00060069 4000} \\ \text{0x00030048 0142} \\ \text{0x00018048 0023} \\ \text{0x0000C068 5000} \\ \text{0x00006028 4240} \\ \text{0x00003034 6000} \\ \text{0x00001804 1224} \\ \text{0x00000C60 0206} \\ \text{0x00000728 4040} \\ \text{0x000003E1 0200} \end{bmatrix},$$

$$\boldsymbol{H}_b^4 = \begin{bmatrix} \text{0x80010050 127B} \\ \text{0xC0002050 301F} \\ \text{0x60000022 067D} \\ \text{0x30004802 08DD} \\ \text{0x18001308 006F} \\ \text{0x0C001848 031E} \\ \text{0x06001006 C153} \\ \text{0x03004084 406F} \\ \text{0x01800300 88BB} \\ \text{0x00C0A401 0433} \\ \text{0x00612430 0065} \\ \text{0x00300004 64EC} \\ \text{0x00180100 296E} \\ \text{0x000C86A1 1014} \\ \text{0x00060809 0077} \\ \text{0x0003C080 807A} \end{bmatrix} \quad \boldsymbol{H}_b^5 = \begin{bmatrix} \text{0x8001814A 085C} \\ \text{0xC0005010 10D7} \\ \text{0x60000E06 005D} \\ \text{0x30008400 503F} \\ \text{0x18000050 00FF} \\ \text{0x0C00220D 8069} \\ \text{0x06000188 2725} \\ \text{0x03010284 045E} \\ \text{0x01809C00 4263} \\ \text{0x00C02112 882C} \\ \text{0x00601080 A03E} \\ \text{0x00300800 087F} \\ \text{0x00184020 107E} \\ \text{0x000C0021 207B} \\ \text{0x00060061 0783} \\ \text{0x00036000 4173} \end{bmatrix} \quad \boldsymbol{H}_b^6 = \begin{bmatrix} \text{0x80010A00 54E5} \\ \text{0xC0005040 C25C} \\ \text{0x6000200C 403F} \\ \text{0x30008040 027F} \\ \text{0x18001020 04FB} \\ \text{0x0C000110 217E} \\ \text{0x06002AC1 002D} \\ \text{0x03000003 28DB} \\ \text{0x01802488 017A} \\ \text{0x00C1040C 0076} \\ \text{0x00600030 8559} \\ \text{0x00308182 3052} \\ \text{0x00184020 085F} \\ \text{0x000C0406 8227} \\ \text{0x00069300 002F} \\ \text{0x00034811 1825} \end{bmatrix}$$

$$\boldsymbol{H}_b^7=\begin{bmatrix}\text{0x8013065040EF}\\ \text{0xC00F2141C0A4}\\ \text{0x600806D64168}\\ \text{0x30152AC0EC00}\\ \text{0x18031AF05028}\\ \text{0x0C030A1B48B0}\\ \text{0x060051D2D310}\\ \text{0x0303C2BD1020}\\ \text{0x018B61B04422}\\ \text{0x00C792C82502}\\ \text{0x006306D40A25}\\ \text{0x003282D06271}\end{bmatrix}\quad \boldsymbol{H}_b^8=\begin{bmatrix}\text{0x80129DB0E081}\\ \text{0xC001215434A3}\\ \text{0x600051146E31}\\ \text{0x300CA60620CA}\\ \text{0x1812835E00A4}\\ \text{0x0C01919528A1}\\ \text{0x0601805D31E0}\\ \text{0x030CC1161129}\\ \text{0x018E881460A5}\\ \text{0x00C0C729A281}\\ \text{0x0060A900A0F7}\\ \text{0x003001B40F39}\end{bmatrix}\quad \boldsymbol{H}_b^9=\begin{bmatrix}\text{0x80130ABC5901}\\ \text{0xC0087A444381}\\ \text{0x600616076902}\\ \text{0x30088806EC1C}\\ \text{0x18060A821F24}\\ \text{0x0C068A44491A}\\ \text{0x06065A3C4880}\\ \text{0x03070E00D198}\\ \text{0x018F091C0524}\\ \text{0x00C62100EB61}\\ \text{0x0074A3250942}\\ \text{0x00364EC54840}\end{bmatrix}\quad \boldsymbol{H}_b^{10}=\begin{bmatrix}\text{0x80101A2662E9}\\ \text{0xC004314002F7}\\ \text{0x6000A4502CF2}\\ \text{0x30038141823B}\\ \text{0x1819340208CD}\\ \text{0x0C0E402811C7}\\ \text{0x060001A8447F}\\ \text{0x0303008E107D}\\ \text{0x018C4A01219E}\\ \text{0x00C0081184FF}\\ \text{0x0060069411BE}\\ \text{0x0030C000C8FF}\end{bmatrix}$$

In Table 1, the seed matrices and their associated values *z* for the construction of all codewords lengths for all rates are shown.

Table 1 LDPC seed matrices selection per code rate and expansion factors

Code rate $(n_b-m_b)/n_b$	**Seed matrices**	**Expansion factors** *z*
1/2	$\boldsymbol{H}_b^1$	16, 20, 24, 26, 30, 32, 34, 36, 38, 40, 46, 48
	$\boldsymbol{H}_b^2$	12, 14, 22, 28, 42, 44
	$\boldsymbol{H}_b^3$	18
2/3	$\boldsymbol{H}_b^4$	22, 28, 30, 32, 36, 38, 40, 42, 44, 46, 48
	$\boldsymbol{H}_b^5$	12, 16, 20, 26, 34
	$\boldsymbol{H}_b^6$	14, 18, 24
3/4	$\boldsymbol{H}_b^7$	24, 26, 30, 34, 36, 38, 40, 42, 46, 48
	$\boldsymbol{H}_b^8$	20, 32, 44
	$\boldsymbol{H}_b^9$	12, 18, 28
	$\boldsymbol{H}_b^{10}$	14, 16, 22

Simulation results

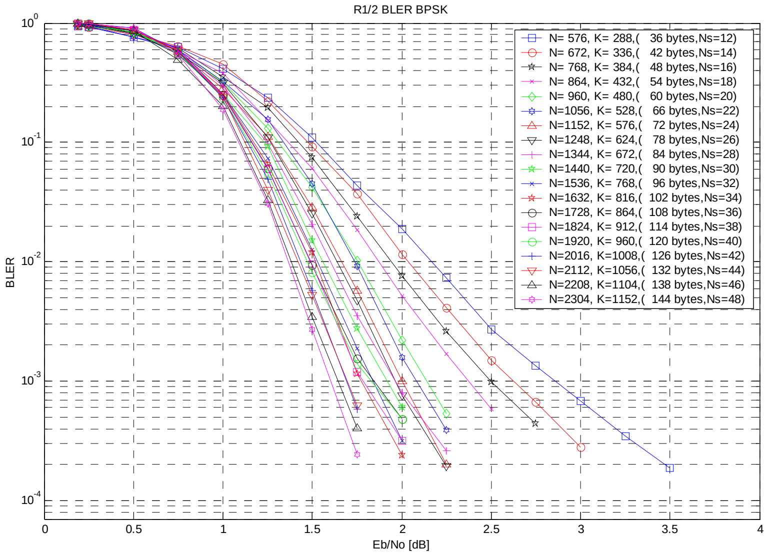

Figure 0-1 Rate 1/2 BPSK results in AWGN using 50 iterations of conventional BP (SPA)

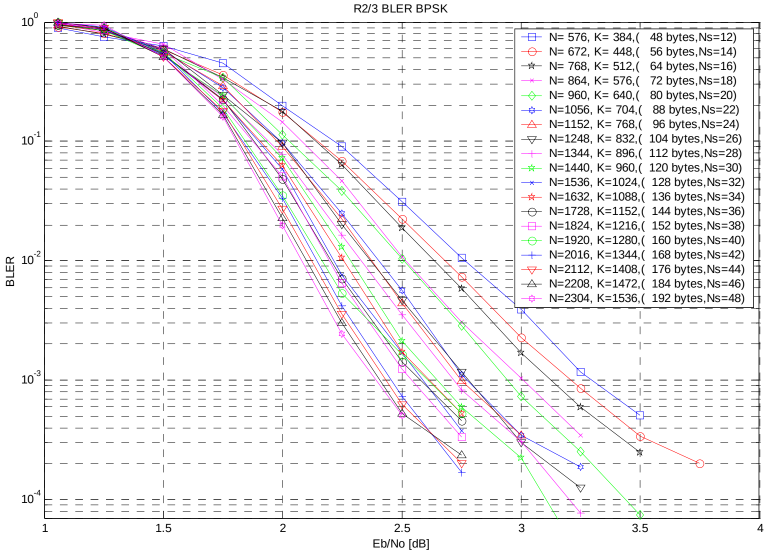

Figure 0-2 Rate 2/3 BPSK results in AWGN using 50 iterations of conventional BP (SPA)

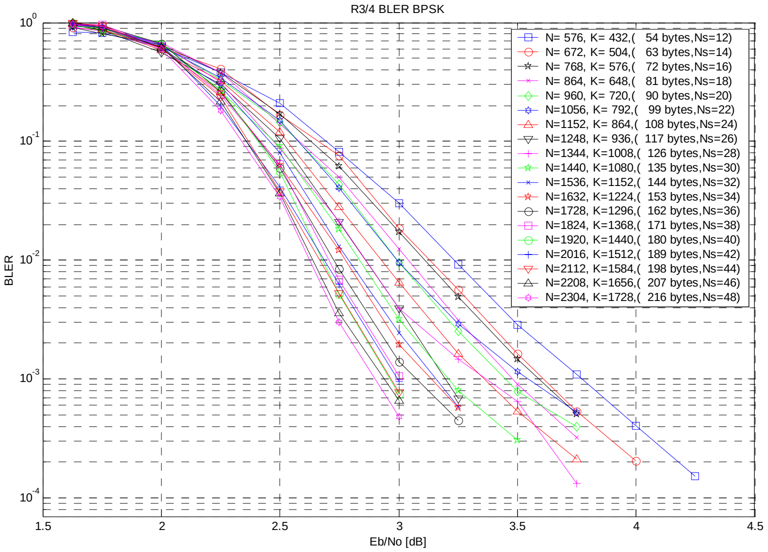

Figure 0-3 Rate 3/4 BPSK results in AWGN using 50 iterations of conventional BP (SPA)

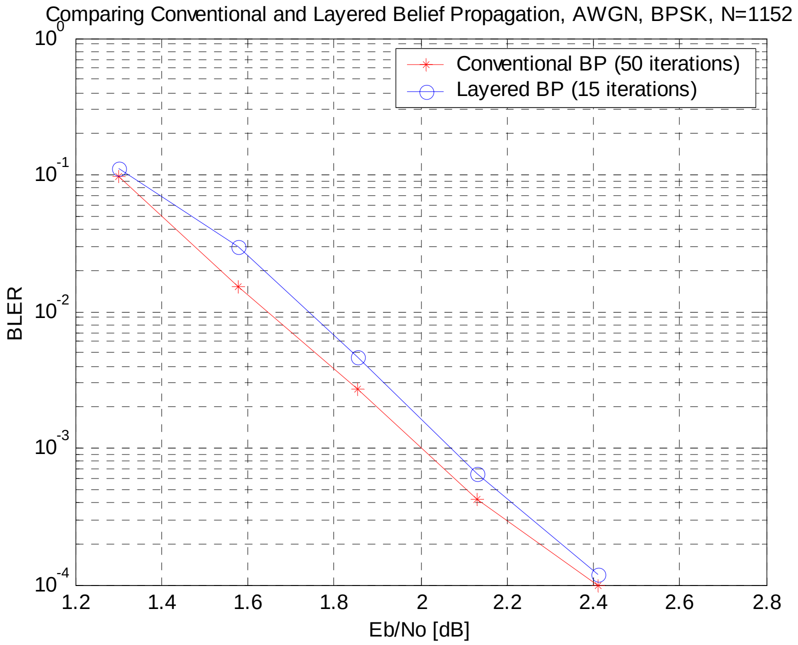

Figure 0-4 BLER comparision of 50 conventional BP (SPA) iterations and 15 layered BP for Rate 1/2 BPSK in AWGN of codeword length of N=1152

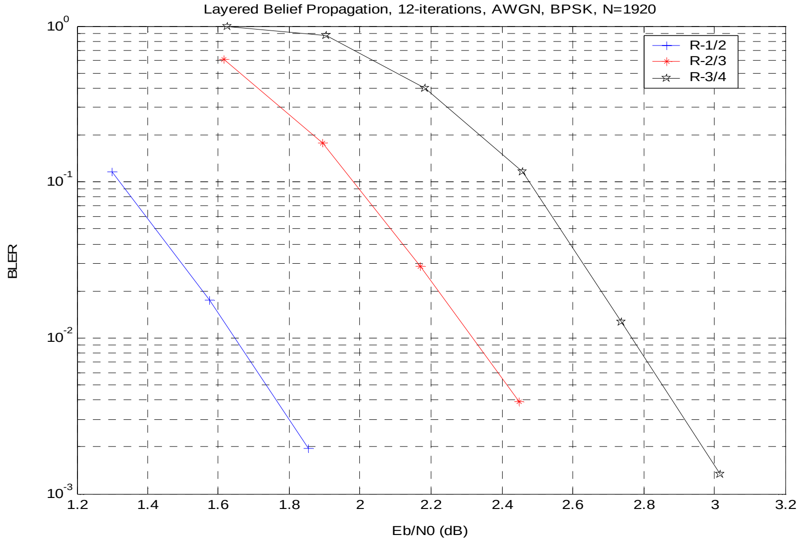

Figure 0-5 BPSK-AWGN results of 12 iterations of layered BP for coderates 1/2, 2/3 and 3/4 of codeword lengths N=1920.

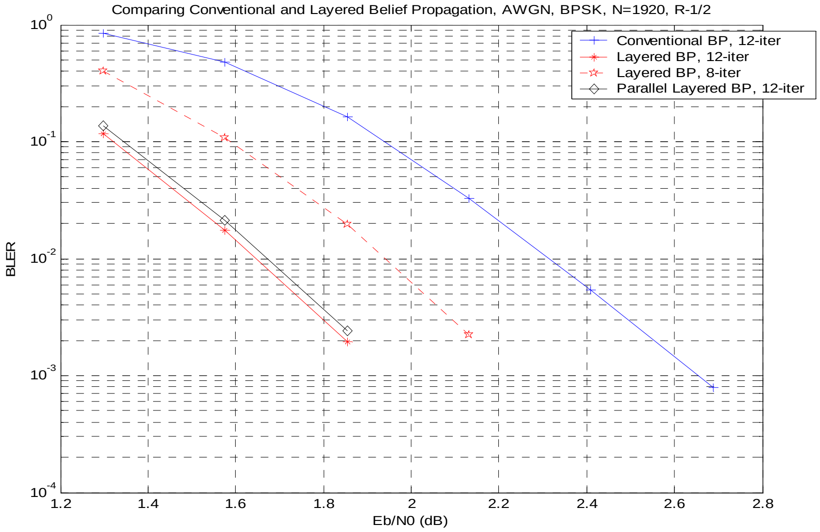

Figure 0-6 BPSK AWGN comparisions of various BP decoding approaches.

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