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Title	**Method for Capturing Noises in Digital Radio Receiver**	
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Re:	Evaluation criteria.	
Abstract	This document proposes a method for capturing the noises in a digital radio receiver, which is necessary for the evaluation of link level performance of the receiver via computer simulation.	
Purpose	Discuss and adopt.	
Notice		
Release		
Patent Policy		

Method for Capturing Noises in Digital Radio Receiver

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1 Introduction

The digital signal received by a receiver through a fading channel can be modeled by

$$r(t) = s(t) * h(t) + n_0(t) \tag{1}$$

where $s(t)$ is the analog signal conveying the modulated symbol sequence $\{c_i\}_{i=0}^{L}$ such that

$$s(t) = \sum_i c_i p(t - t_i) \tag{2}$$

and $h(t)$ is the channel impulse response and $n(t)$ is the additive noise. A digital radio receiver has to perfrom hardware and software operation to recover the symbol sequence. Those operations introduce additional noise to the signal, so that $n(t)$ and $h(t)$ alone are not sufficient to determine the signal-to-noise ratio per symbol, which is important for the evaluation of the link level performance. The methodology of separate simulation model for link performance and system performance requires the interface be a simple relation of packet error rate versus signal-to-noise ratio. Necessarily, the received modulation symbols should be able to expressed as

$$r_k = c_k + n_k \tag{3}$$

so that the signal-to-nose ration can be determined as

$$\frac{S}{N} = \frac{\bar{c_k^2}}{\bar{n_k^2}} \tag{4}$$

where integer k refers to discrete time for integer $t = t_k$ and $\bar{x}$ refers to the appropriate short term average of the variable x. The random variate $n(t_k)$ depends on the input noise $n(t)$ and fading $h(t)$ as well as the radio system. Generally speaking, it is not possible to give an explicit expression for $n(\bar{t_k})$ without reference to the specific design of the system and the receiver/transmitter. It is, however, possible to find a common methodology for the determination of this expression and quantity. This common methodology shall be based on an agreed-up model for the noises introduced by the different parts of the system. Noise of different parts of the system has different causes. The purpose of this contribution is to propose a method to be used to model the noises and to determine the final noise value in (1).

2 Physical Background

The term $n_0(t)$ in (1) includes noise generated by the receiver as well as undesired radio signals, i.e. interference. Interference can, again, be decomposed into exterior and interior. The exterior interference are signals transmitted by an transmitter others than the expected, while the interior interference is caused by the imperfection of the receiver, e.g. non-linearity of the amplifier. By the performance evaluation, the exterior interference is normally assumed known. Hence, its modeling is straightforward. Only the interior interference belongs to the object of noise modeling in the sequel.

The noise generated within the receiver have two major causes: shot noise and thermal noise. The shot noise refers to the random currents due to the active devices in the receiver, e.g. transistor and diode. By a linear model of the receiver, the power spectrum of the shot noise is

$$G_s(f) = |H(f)|^2 G_{s,i}(f) \tag{5}$$

where $H(f)$ is the transfer function of the linear system including the channel and filters, $G_s(f)$ is the power spectrum of the received shot noise and $G_{s,i}(f)$ is the power spectrum of the random currents. For narrow band system, i.e. $f << B$ with noise-equivalent bandwith B, $G_{s,i}(f) \sim eI_{dc}$, i.e. the noise power is proportional to the DC component and can be regarded white.

The thermal noise is believed to be caused by the thermal interaction between electrons and vibrating ions. The spontaneous fluctuation voltage is found, experimentally by J.B. Johnson in Bell Labs 1928, to have mean-squared value

$$\bar{v^2} = 4kTRB \tag{6}$$

where $k = 1.38 \times 10^{-23}$ joule/°K is the Boltzmann constant, T the absolute temperature in Kelvin and R teh resitance in ohms and B the bandwith. The derivaion of this relation, which involves statistic thermal dynamic arguments and the quantum mechanic notions for oscillators, shows that the relation applies to any linear passive device. Thus, by linear system model, we have the power spectrum for thermal noise

$$G_v(f) = \frac{\bar{v^2}}{2B} = 2kTR \tag{7}$$

and the thermal noise is considered white.

In a digital system, the received analog signal is converted into digits, on which numerical algorithms are applied to recover the sent digits. The A/D converter introduce quantization noise and the algorithm introduce numerical errors which can be regarded as a kind of digital noise. The digital noise is so far more or less ignored and handled in a case by case basis. The goal here is to include the numerical noises in the noise quantification, so that it can be handled equally with classic physical noises.

3 The Model

Let's review the notion system noise figure used as a metric to characterize the noise introduced by the receiver. It is defined as the ratio of the input SNR versus output SNR of a given system, e.g. a radio receiver,

$$F := \frac{SNR_{in}}{SNR_{out}} \tag{8}$$

where SNR_{in} is the SNR measured at the input of the system and SNR_{out} is the SNR measured at the output of the system. For a mobile receiver within a simulation model on symbol level the input can be the antenna feeder and the output can be the base band symbols. When the system has a power gain G, the output noise power is

$$GN_0 + N_i \tag{9}$$

where N_i is the noise power generated by the system and N_0 the noise power coming with the signals. Hence, representing the signal power by S and the noise power by N

$$SNR_{out} = \frac{GS}{GN_0 + N_i} = \frac{1}{1 + N_i/GN_0} SNR_{in} \tag{10}$$

with $SNR_{in} = S/N_0$, and the noise figure is equivalent to

$$F = 1 + \frac{N_i}{GN_0} \tag{11}$$

Equivalently, the noise figure can be expressed as the ratio of the output total noise power versus the amplified input noise power.

For the study of the concatenated systems, it is necessary to impose additional contraints to the definition of the noise figure of a system. The noise figure is defined with reference to a common input noise power: N_0. Consider at first a system that consists of two concatenated network stages, with interior noise power N_1 and N_2, and gain G_1 and G_2, respectively, i.e. $i = 1, 2$. Then

$$\begin{aligned} N_{o,1} &= G_1 N_0 + N_1 \\ N_{o,2} &= G_2(G_1 N_0 + N_1) + N_2 \end{aligned}$$

designate the ouput noise power of the first stage and the second sage, respectively. By this definition, the noise figure of each stage can be measured independently

$$F_1 = 1 + \frac{N_1}{G_1 N_0} \quad (12)$$

$$F_2 = 1 + \frac{N_2}{G_2 N_0} \quad (13)$$

with reference to the same fixed input noise power N_0. Thus, given the noise figures, the interior noise power and the total output noise power of the cancatenated system can be determined by

$$\begin{aligned} N_1 &= (F_1 - 1)G_1 N_0 \\ N_2 &= (F_2 - 1)G_2 N_0 \end{aligned}$$

respectively. The relation is illustrated as following

$$\begin{array}{ccc|ccc|ccc|ccc|cc} S \rightarrow & \oplus & & \rightarrow & G_1 \rightarrow & \oplus & \rightarrow & N_{o,1} & \rightarrow & G_2 \rightarrow & \oplus & & \rightarrow & G_1 G_2 S + N_{out} \\ & \uparrow & & & & \uparrow & & + & & & \uparrow & & & \\ & N_0 & & & & N_1 & & G_1 S & & & N_2 & & & \end{array} \quad (14)$$

where $N_{out} = N_{o,2}$. The noise figure of the system of 2 concatenated stages is

$$\begin{aligned} F : &= SNR_{in}/SNR_{out} = \frac{N_{out}}{G_1 G_2 N_0} \\ &= \frac{G_2 G_1 N_0 + G_2 N_1 + N_2}{G_1 G_2 N_0} \\ &= \frac{G_1 G_2 N_0 + G_1 G_2 (F_1 - 1) N_0 + G_2 (F_2 - 1) N_0}{G_1 G_2 N_0} \\ &= F_1 + \frac{F_2 - 1}{G_1} \end{aligned}$$

Now we can prove the following

Theorem 1 *A system consisting of n stages, each with G_i and N_i for $i = 1, 2, ...n$, has the noise figure*

$$F = F_1 + \sum_{i=2}^{n} \frac{F_i - 1}{\Pi_{k=1}^{i-1} G_k} \quad (15)$$

where F_i is the noise figure for stage i and defined by

$$F_i = 1 + \frac{N_i}{G_i N_0}, \quad (16)$$

for $i = 1, 2, ..., n$.

Proof: The statement is true for $n = 2$, as is already shown. Assume it is true for $n > 2$. Then consider a system consisting of $n + 1$ stages, which can be viewed as a two stage system: one comprises n stages with gain $G = \Pi_{i=1}^{n} G_i$ and noise figure F as given by eq.(15), and the other comprises only one stage with gain G_{n+1} and noise figure F_{n+1}. Then, the composite noise figure becomes

$$F + \frac{F_{n+1} - 1}{G_n} = F_1 + \sum_{i=2}^{n+1} \frac{F_i - 1}{\Pi_{k=1}^{i-1} G_k} \quad (17)$$

The assertion is true, since the natural integer n is arbitrarily chosen. Q.E.D.

4 Noise Figure of a Digital Receiver

A special case for the result achieved above is when $G_i = 1$ for $i > 1$ and $G_1 = G$. We have then

$$F = F_1 + \frac{\sum_{i=2}^{n} (F_i - 1)}{G} = F_1 + \frac{\sum_{i=2}^{n} N_i}{G N_0} = 1 + \frac{\sum_{i=1}^{n} N_i}{G N_0} \quad (18)$$

considering relation (12). This is consistent with the assumption that all interior noises are additive, and the composite system becomes a system of a single stage when all interior noises are lumped together. In this case it is easy to verify

$$G = \frac{\sum_{i=1}^{n} N_i}{N_0(F-1)} \tag{19}$$

which turns out to be consistent with the definition as shown by

$$SNR_{out} = \frac{GS}{GN_0 + \sum_{i=1}^{n} N_i} = \frac{S}{N_0}\frac{1}{F} = \frac{1}{F} SNR_{in} \tag{20}$$

In addition, in order to find the noise figure of each stage for the worst case scenario, one can consider to use the upper bound given by

$$F_i = 1 + \frac{N_i}{GN_0} \leq 1 + \frac{\max(N_i)}{GN_0} =: F_{i,max} \tag{21}$$

yielding a F_{max}.

Now that a mobile radio receiver entails different noise sources, each with a different physical background, e.g. antenna circuits, radio frequency ciruits, base band circuits, A/D converter, digital signal processor, etc. For the purpose of link level simulation, we consider receiver noise as coming from two classes of sources: the electronic circuits and the digital signal processor. The first includes the analog noise, e.g. the thermal noise within and fed into the receiver, while the latter covers the digital noise introduced by the signal processing begining from the A/D converter. With this principle, the receiver can be modeled as system of two stages: the stage 1 comprises the electronic circuits and the stage 2 comprises the DSP. The first stage is given by the receiver hardware, hence its noise figure is a given parameter. For the simulation, the second stage is of more concern; it corresponds to the base band digital processing. With this notion, a model of receiver can be illustrated as follows

$$\begin{array}{cc|ccccccc|c} S \rightarrow & \oplus & \rightarrow\ G \rightarrow & \oplus & \rightarrow & \oplus & \rightarrow & \oplus & \ldots \rightarrow & P \\ & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\ & N_0 & & Quantization & & Inter-Symbol & & AdjacentChannel, ..., etc. & & \end{array} \tag{22}$$

where S is the transmitted symbol power and P the received total power per symbol including noise. Let N_i denote the additive noise generated by the DSP, where the index relates the noise term with the physical source when necessary. For instance, N_i represents noise source contributed by quantization, inter-symbol interference and adjacent channel interference, etc., for $i = 1, 2, 3, ...$, respectively. These noise powers are computed in a mannter that is consistent with the receiver structure and the numerical algorithms applied within the receiver. The noise figure of this system is given by (18), assumed there are n interior noise sources.

5 Recommendation

For the evaluation of the link level performance, the noise power added to the received signal have to be decomposed and computed individually according to the nature of the noise source and the receiver structure and algorithms, before the overal noise value can be determined. A modularized approach would highly reduce the work load and facilitate the analysis: For each physical noise source, a noise figure will be computed pertinent to the source nature. All noise components are then combined based on the model presented above in (15) and (18). Therefore, using a list of noise components from individual noise souces, the proponent of a proposal can specify the values and the derivation, when applicable, of the item in the list. As this facilitates the comprehension of the design and comparability of different proposals, we propose to adopt this method into the evaluation criteria document.