A Link Model for Equalized Optical Receivers

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Overview

- It is not a *simulation* tool
- It uses well established and well documented theory to compute the signal to noise ratio at the slicer and the bit error rate for DFE-based receivers
- Written in Matlab (it also uses the Signal Processing Toolbox), which facilitates computations needed for signal processing, such as convolutions, Fourier transforms, power spectral density calculations, etc.
- Although parameters values can be changed by the user, in this presentation we provide a table of default values
- The numerical results we present later are based on default parameters

Assumptions

- The optical channel is assumed linear and time invariant
- Noise sources accounted for are RIN, thermal noise in the transimpedance amplifier (TIA), jitter, and quantization noise of the A/D converter
- RIN is assumed white and Gaussian at the transmitter, but it is spectrally shaped by the frequency response of the fiber and the receiver front-end
- Thermal noise is assumed white and Gaussian at the input of the receiver, but it is spectrally shaped by the frequency response of the receiver front-end
- Phase jitter is assumed white and Gaussian, and it is converted into (colored) amplitude noise using the methodology described later
- The sampling phase of the receiver is the phase that maximizes the sampled energy of the received pulse
- A/D quantization noise is assumed white and uniformly distributed
- All signals before the A/D are sampled at a high rate f_S typically much larger than the symbol rate (for a symbol rate of 10.3125GBaud, the sampling rate could be f_S=165GHz)

Transmitted Signal

- **Transmitted pulses are trapezoidal, with specified rise and fall times**
- **Pulses are treated as binary antipodal (they assume values** $\pm ta$**), superim-posed on an average optical power** tpl
- **The pulse amplitude (in milliwats of optical power) is given by**

$$ta = tpl \cdot \frac{1 - erl}{1 + erl}$$

- **where** $tpl = 10^{(tp/10)}$**,** tp **is the average optical power in dBm,** $erl = 10^{(-er/10)}$**, and** er **is the extinction ratio in dB**

Fiber Model

- **The fiber impulse response can be read from a file, or the default model can be used**
- **The default impulse response is a Gaussian [ref. 2], given by**

$$h_0(n) = A \cdot e^{-[n^2 T_s^2 / 2\alpha^2 T^2]}$$

- **where** $T_s = 1/f_s$ **is the sampling period,** T **is the symbol period,**

$$\alpha = \frac{1}{2\pi BT}\sqrt{2\ln 2}$$

- **and** B **is the 6dB electrical (3dB optical) bandwidth of the fiber**
- **The normalization constant is chosen as**

$$A = 1/\left(\sum_{n=-\infty}^{\infty} h_0(n)\right)$$

Fiber Model (cntd)

- **The 3dB optical bandwidth is computed from the fiber length as**

$$B = B_0 / l$$

- **where B_0 is the bandwidth X length product for the fiber and l is the fiber length**
- **As a result of the normalization assumed for the impulse response, this model of the fiber is *dispersive* but *lossless***
- **Fiber and connector losses are accounted for separately, as**

$$A = A_0 + lA_1$$

- **where A_0 represents the connector losses and A_1 is the fiber loss per unit length (in optical dB per Km)**

Receiver Front End

- **The receiver front end is modeled as an n^{th} order Butterworth filter (where n is user-selectable) with a 3dB electrical bandwidth TIABW**

RIN Model

- Relative Intensity Noise (RIN) is assumed white and Gaussian, with a (two-sided) power spectral density

$$rpd_{Hz} = 0.5 \times 10^{(rin/10)} tpl^2$$

- where the power spectral density is expressed in milliwatts per hertz, and it is assumed to correspond to a two-sided spectrum (a spectrum that extends over negative as well as positive frequencies), tpl is the average transmitted power, and rin is the laser RIN parameter in dB/Hz (for example -120dB/Hz). The factor 0.5 is used because the laser RIN parameter is specified as corresponding to a one-sided spectrum, whereas rpd_{Hz} is a two-sided power spectral density

- In power spectral density calculations in the link model, it is often convenient to measure frequency in units of the Fast Fourier Transform (FFT) frequency step, $f_s/ffts$, where $ffts$ is the number of points of the FFT

$$rpd_{FFT} = 0.5 \times 10^{(rin/10)} tpl^2 (f_s/ffts)$$

RIN Model ***(cntd)***

- **RIN is applied at the output of the transmitter (input of the fiber) and it is filtered and spectrally shaped by the fiber and TIA frequency responses**
- **It is also attenuated by the fiber and connector losses**

TIA Noise Model

- The TIA noise performance is described in terms of its *input-referred noise current* I_n
- The *noise bandwidth* of the TIA is defined as

$$NBW_{TIA} = \frac{1}{2\pi}\int_0^{\infty}\left|F_0(\omega)\right|^2 d\omega$$

- where $F_0(\omega)$ is the TIA transfer function
- Then the power spectral density of the TIA input noise is

$$rpd_{Hz} = (I_n/k_P)^2/(2NBW_{TIA})$$

- where k_P is the responsivity of the photodetector, and the factor 2 in the denominator appears because rpd_{Hz} is a two-sided spectral density
- As in the case of RIN, it is often convenient to measure power spectral densities in units of the FFT frequency step $f_s/ffts$

$$rpd_{FFT} = (I_n/k_P)^2/(2NBW_{TIA})(f_s/ffts)$$

- TIA noise is spectrally shaped by the frequency response of the receiver front end

Quantization Noise Model

- It is assumed that the A/D full range is equal to the peak to peak value of the input signal
- This condition can be achieved by an Automatic Gain Control (AGC)
- The quantization step is

$$\Delta = \frac{V_{pp}}{2^{nbits}}$$

- where V_{pp} is the peak to peak value of the input signal and $nbits$ is the number of bits of the A/D
- Assuming that quantization noise is uniformly distributed, its power is

$$\sigma_q^2 = \frac{\Delta^2}{12}$$

- The (two-sided) power spectral density of quantization noise (where frequency is expressed in units of the FFT frequency step) is $\sigma_q^2 / ffts$

Jitter Model

- The samples of the signal taken at the symbol rate at the input of the A/D converter can be expressed as

$$y(nT) + \Delta y_n = \sum_{k=0}^{N-1} h[kT + \tau_{n-k}] a_{n-k}$$

- where τ_n is the instantaneous value of the sampling phase jitter at sampling instant n
- Expanding this expression in Taylor series up to second order terms, we get

$$\Delta y_n = \sum_{k=0}^{N-1} h'(kT) a_{n-k} \tau_{n-k} + \frac{1}{2} \sum_{k=0}^{N-1} h''(kT) a_{n-k} \tau_{n-k}^2$$

- Multiplying this expression by itself, taking expected value, and using the facts that the symbols a_k and the samples of the phase jitter τ_n are uncorrelated, that the third moment of a normal distribution is zero, and that the fourth moment is twice the squared variance, we get

$$E\{\Delta y_n \Delta y_m\} = \sum_{k=0}^{N-1} h'(kT) h'((m-n+k)T) \sigma_J^2 + \frac{1}{4} \sum_{k=0}^{N-1} h''(kT) h''((m-n+k)T) \sigma_J^4 + \frac{1}{2} \delta(m-n) \sum_{k=0}^{N-1} h''^2(kT) \sigma_J^4$$

- where $\sigma_J^2 = E\left\{\tau_n^2\right\}$ is the variance of the phase jitter
- The power spectral density of the jitter-induced amplitude noise at the output of the sampler is given by the discrete Fourier transform of the above autocorrelation

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Intersymbol Interference

- **The Mean Squared Error (MSE) solution for the DFE automatically takes into account any residual intersymbol interference not removed by the equalizer in the expression for the mean squared error, so no explicit ISI calculation is needed**

- **However, for comparison we also compute the signal to noise ratio for a non-equalized receiver. In this case we evaluate the ISI as**

$$ISI = \sum_{k=0}^{N-1} |h(kT)| - |h(pT)|$$

- **where** $h(pT)$ **is the largest sample of the impulse response**

Power Spectral Density of the Noise at the Input of the DFE

- **The RIN and TIA noise power spectral densities are computed by multiplying the power spectral density of the input noise by the square magnitude of the corresponding transfer function**
- **These computations are done at the high sampling rate f_s**
- **The power spectral density of the noise after being sampled at the symbol rate by the A/D converter is computed by "folding" (repeating periodically with period f_B, where f_B is the symbol rate) the spectrum at rate f_s**
- **The power spectral densities of quantization noise and jitter are already computed at rate fB (therefore they are already "folded")**
- **The total noise power spectral density is computed by adding the power spectral densities of RIN, TIA noise, jitter, and quantization noise**

Power Spectral Density of the Signal

- The power spectral density of the signal is computed as the FFT of the autocorrelation function of the (symbol-rate sampled) received signal

Mean Squared Error and SNR for the DFE

- **The mean squared error for the DFE is given by**

$$MSE_{DFE} = GeometricMean\left(\frac{s}{s + h_2}\right)$$

- **where s is the power spectral density of the noise, and h_2 is the power spectral density of the received signal (see [ref.1] for a comprehensive treatment of the DFE and a derivation of the formula for the mean squared error)**

- **The signal to noise ratio at the slicer is**

$$SNR_{DFE} = -10\log(MSE_{DFE})$$

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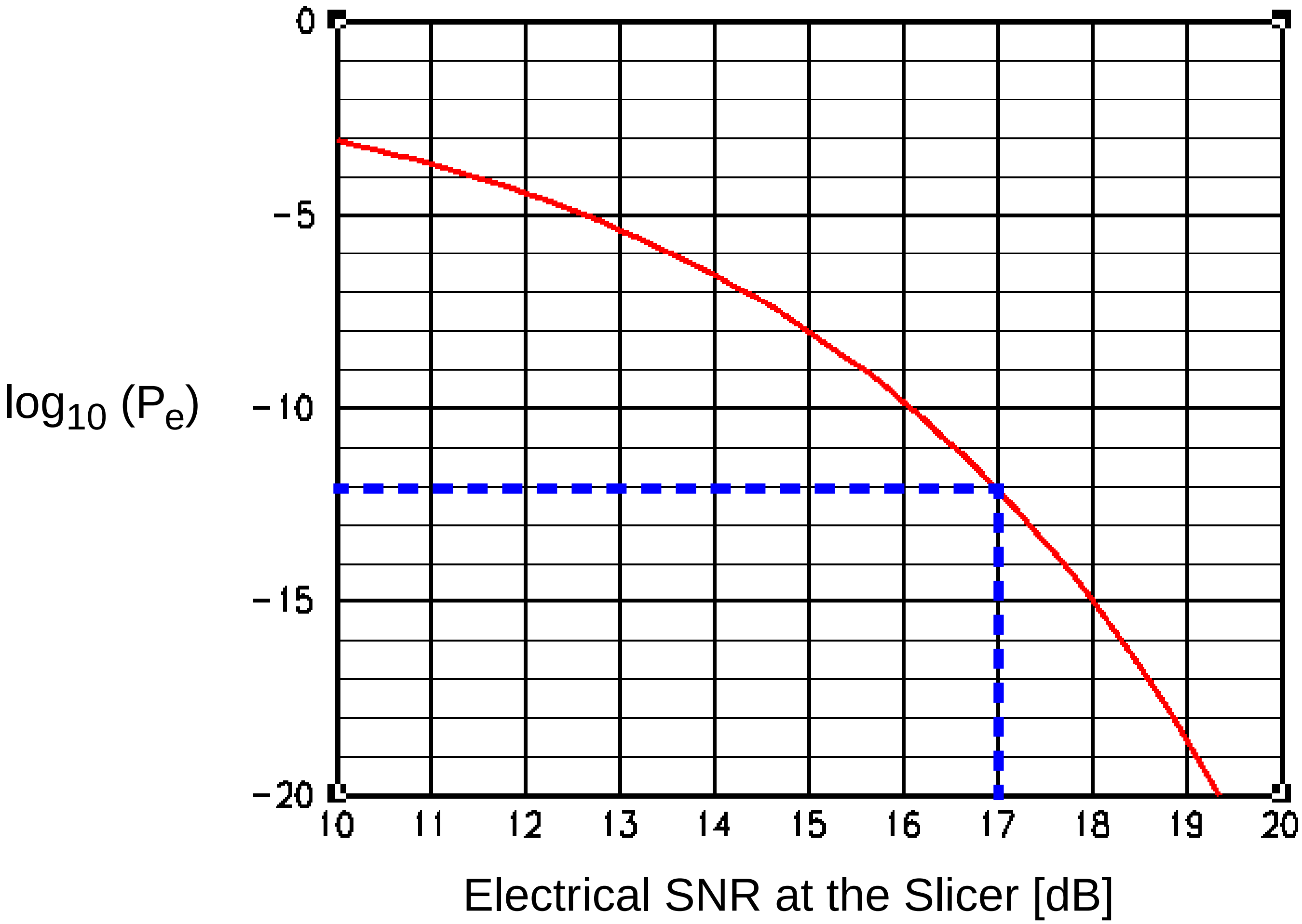
SNR at the Slicer for Non-Equalized Receiver

- **The total noise power is computed by integration of the total noise power spectral density (already computed)**
- **The signal amplitude is the peak sample of the impulse response minus the ISI**

Error Rate Versus SNR for Binary Signaling



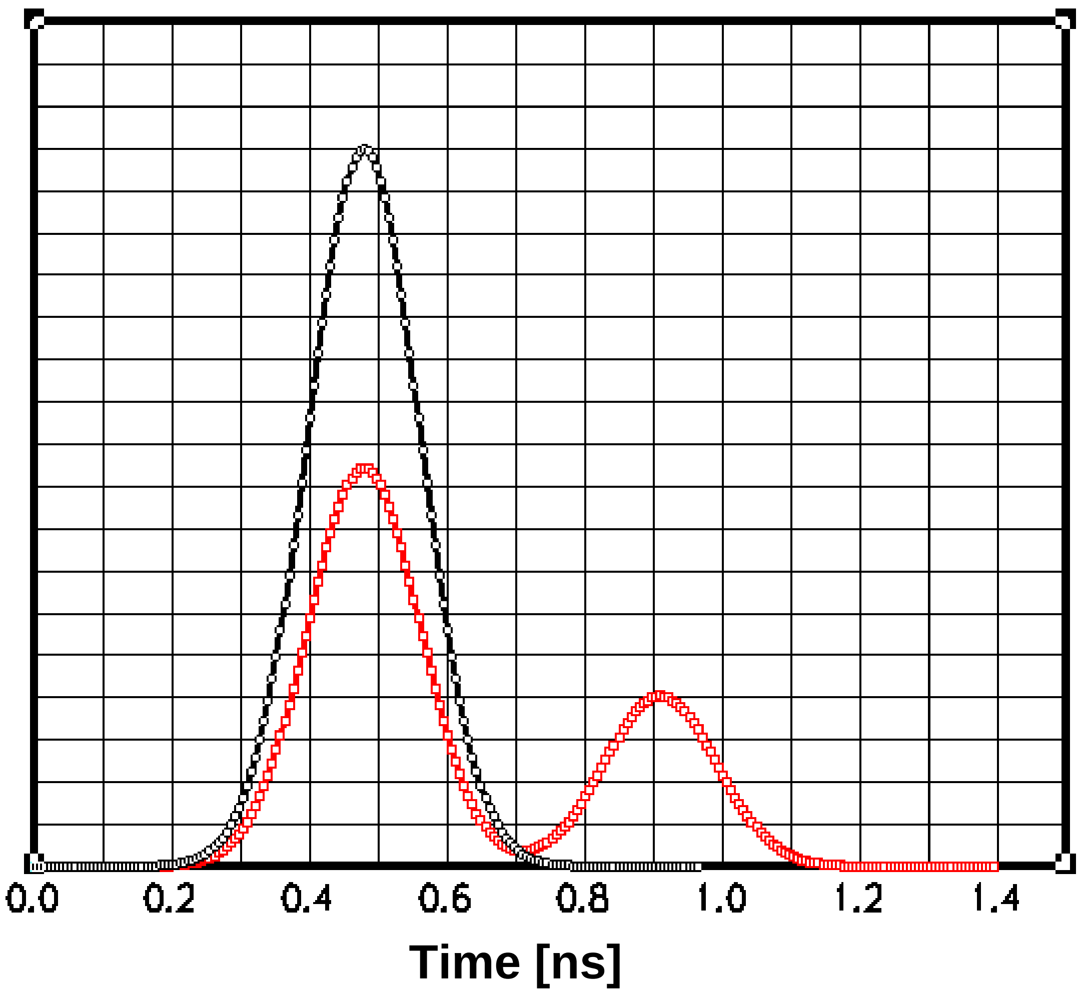
Default Values of Parameters

PARAMETER	VALUE
Symbol Rate	10.3125 Gb/s
Sampling Rate	165 GHz
Laser Wavelength	1310 nm
Fiber Bandwidth	500 MHz x Km
Connector Loss	2 dB
Fiber Loss	2 dB/Km
Transmit Power	0 dBm
Extinction Ratio	5 dB
Transmitter Rise Time	35 ps
RIN	-130 dB/Hz
Photodetector Responsivity	0.7 Amp/Watt
TIA Bandwidth	7.5 GHz
Order of Butterworth Filter Modeling TIA	1
TIA Input-referred noise current	1 µA
RMS Jitter	5 ps
A/D Resolution	5.5 bits
FFT Size	4096

Fiber Impulse Responses

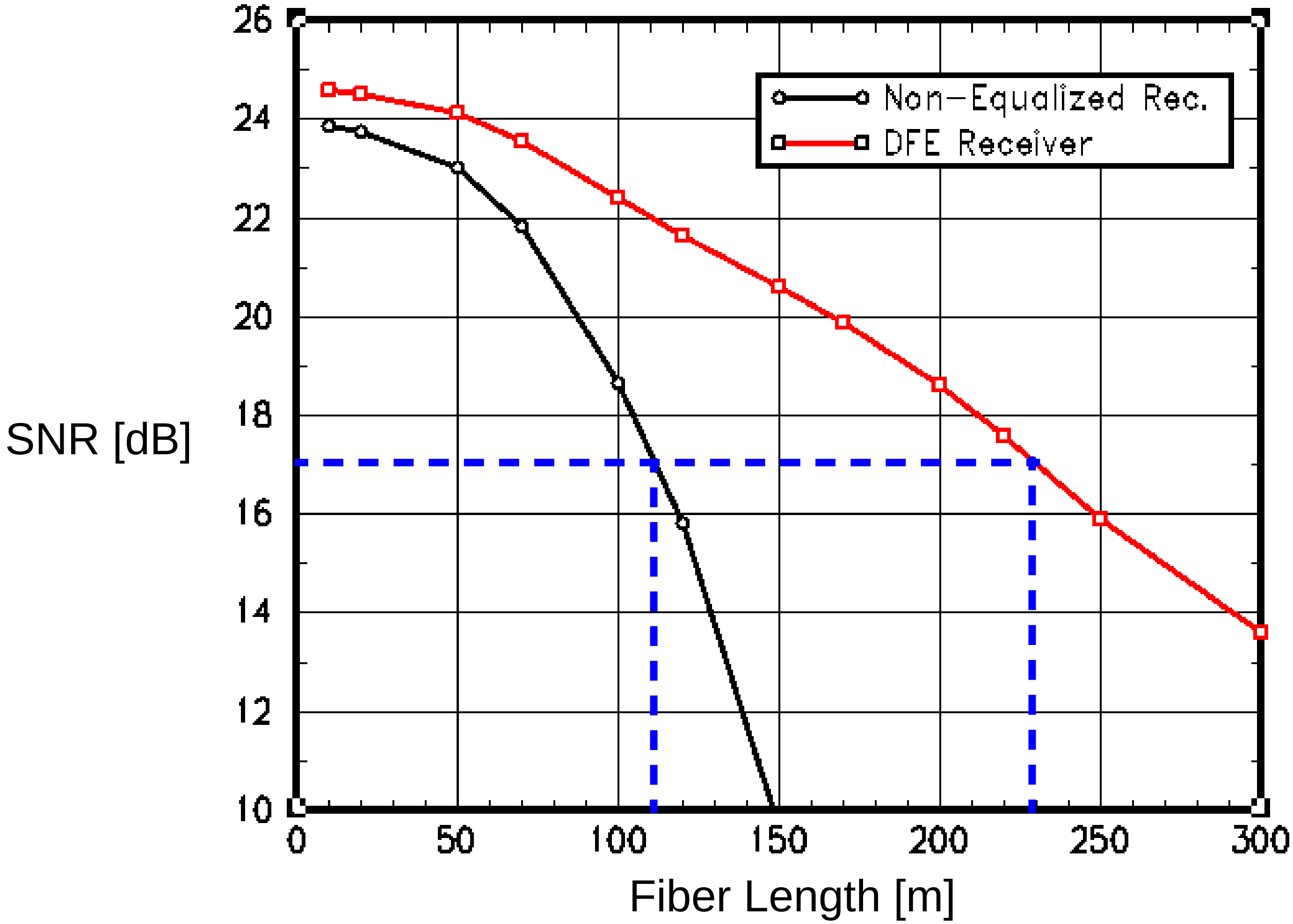
- **In the following viewgraphs we present some numerical results for two different fiber impulse responses:**
  - A Gaussian pulse as described in page 5
  - A split pulse formed by the addition of two Gaussian pulses weighted by factors 0.7 and 0.3 respectively, with a time separation of 2ns/Km

Fiber Impulse Responses Used in the Numerical Examples that Follow

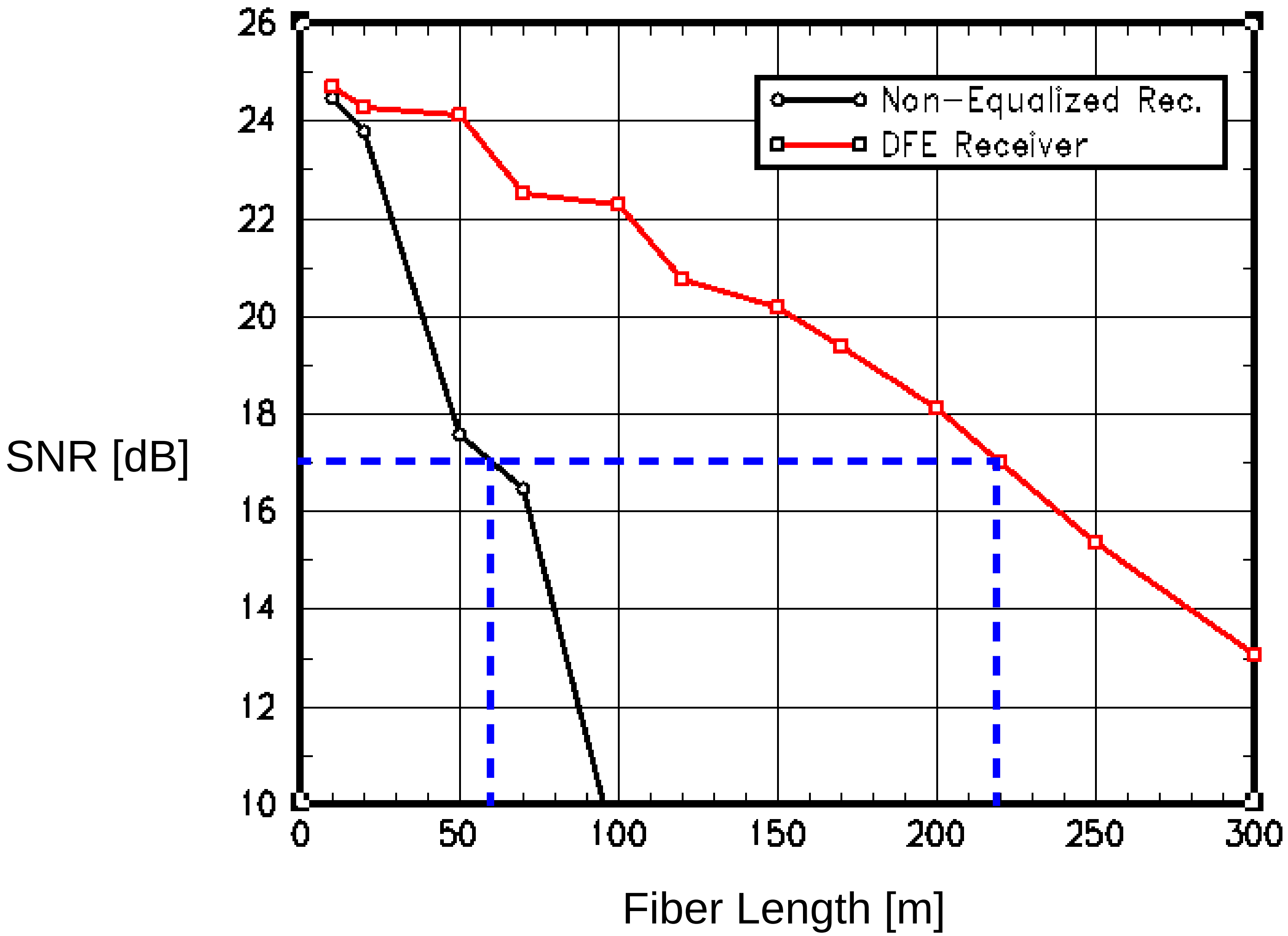


NOTE: These are examples for 230m fiber length

SNR versus Length (1310nm, OFL, 62.5/125μm fiber)



SNR versus Length (1310nm, DMD, 62.5/125μm fiber)

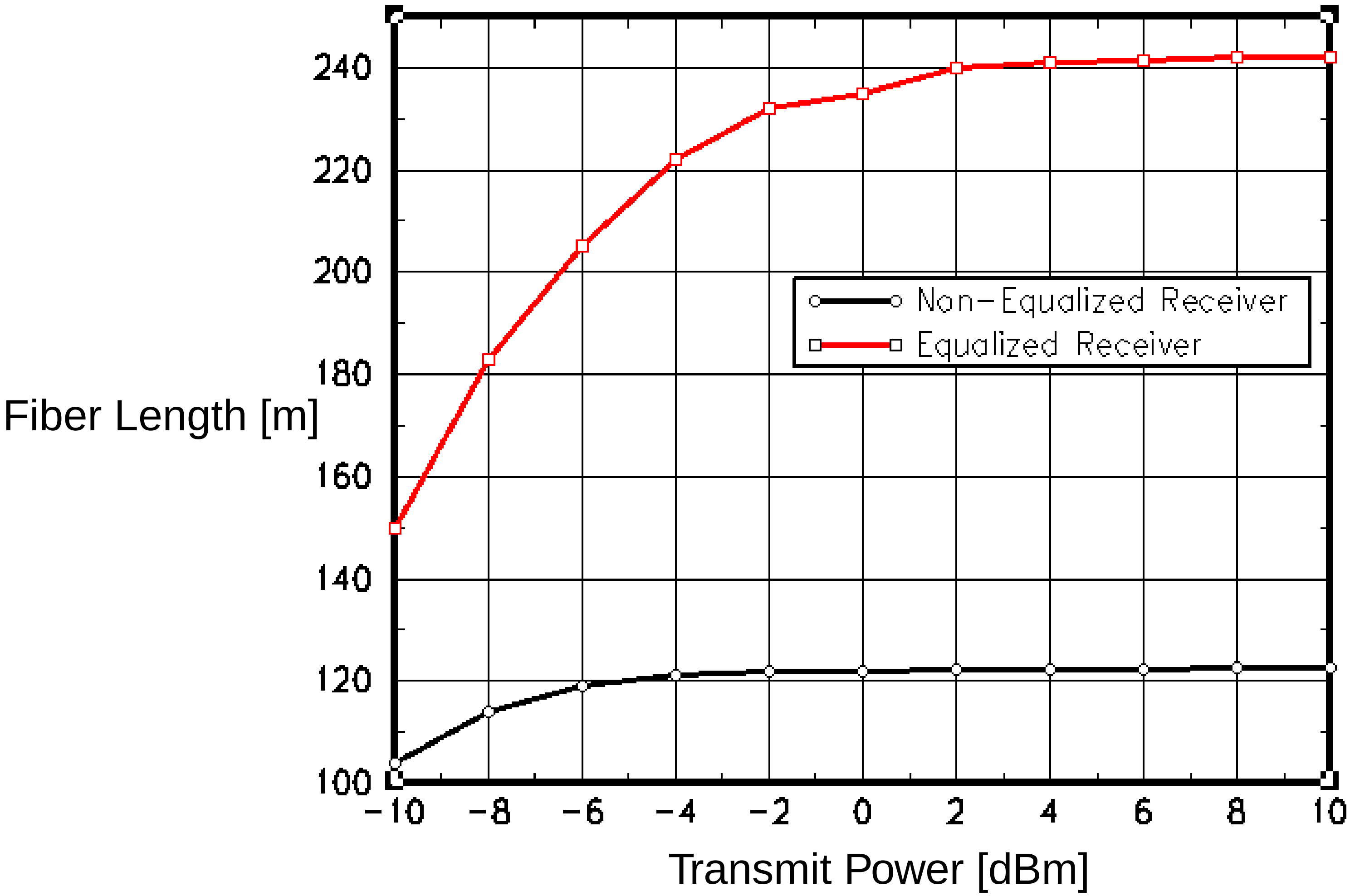


Sensitivities

- **Fiber length versus:**
  - Transmit Power
  - RIN
  - Jitter
  - A/D Resolution

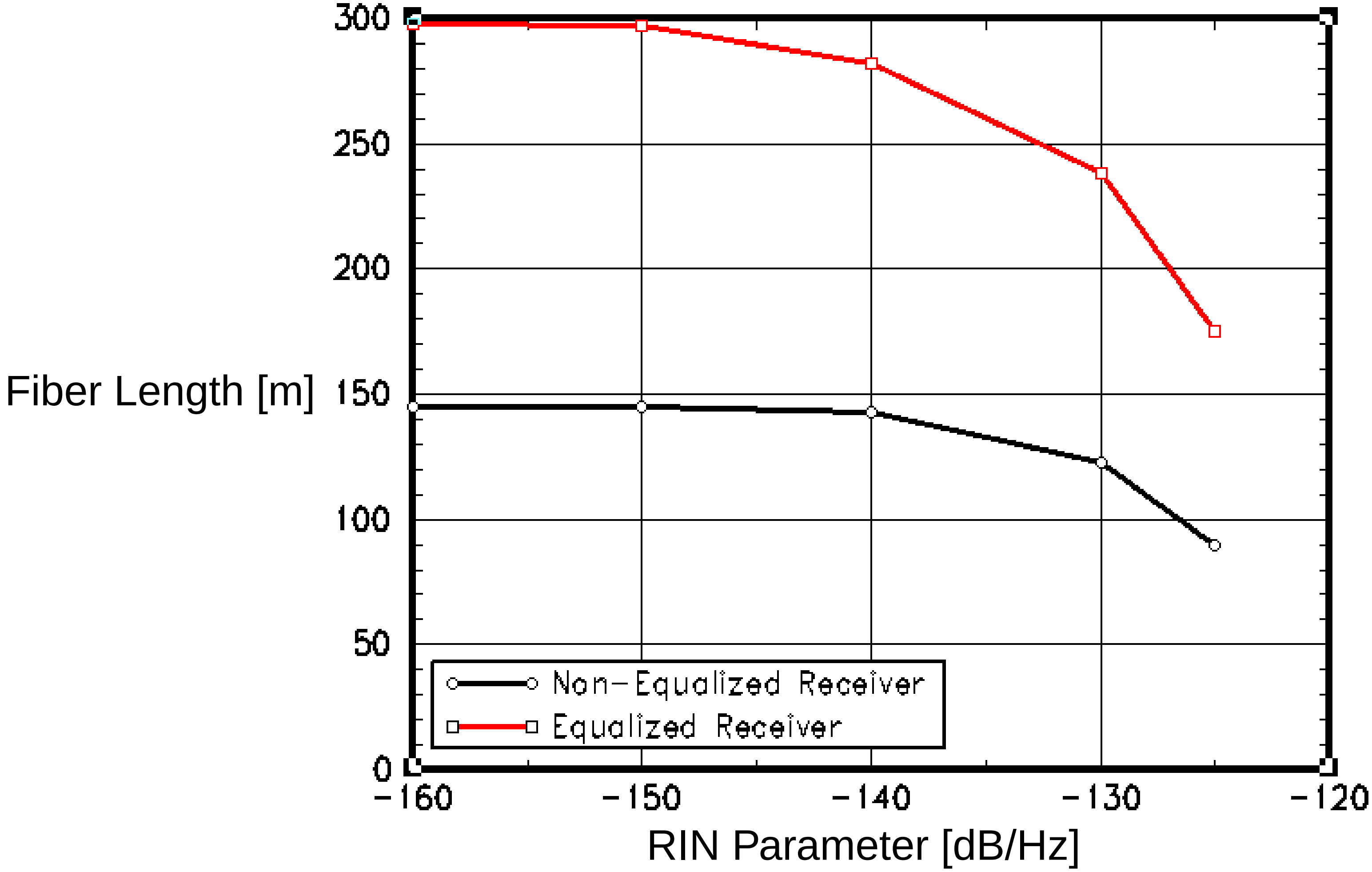
Fiber Reach versus Transmit Power

(1310nm, 62.5/125µm fiber)



Fiber Reach versus RIN

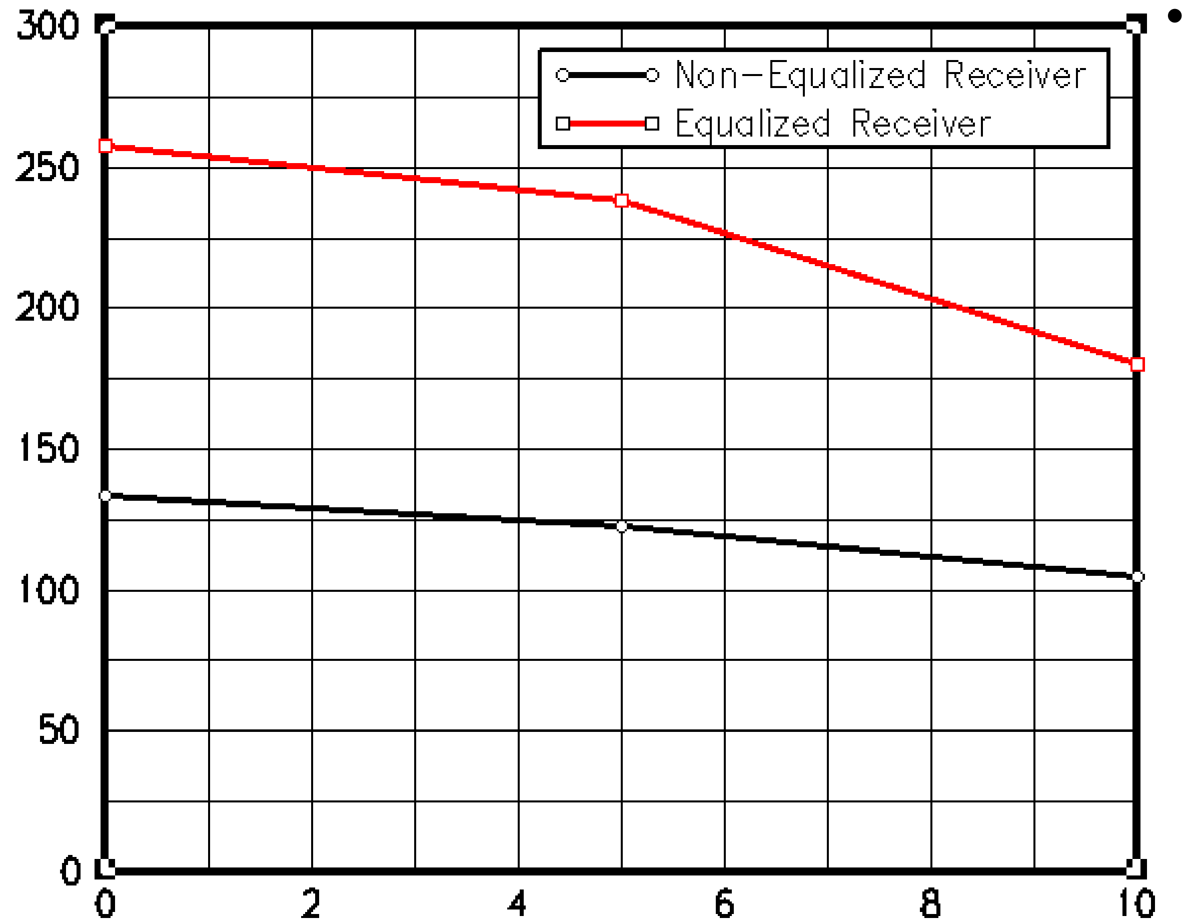
(1310nm, 62.5/125μm fiber)



Fiber Reach versus Jitter

(1310nm, 62.5/125μm fiber)

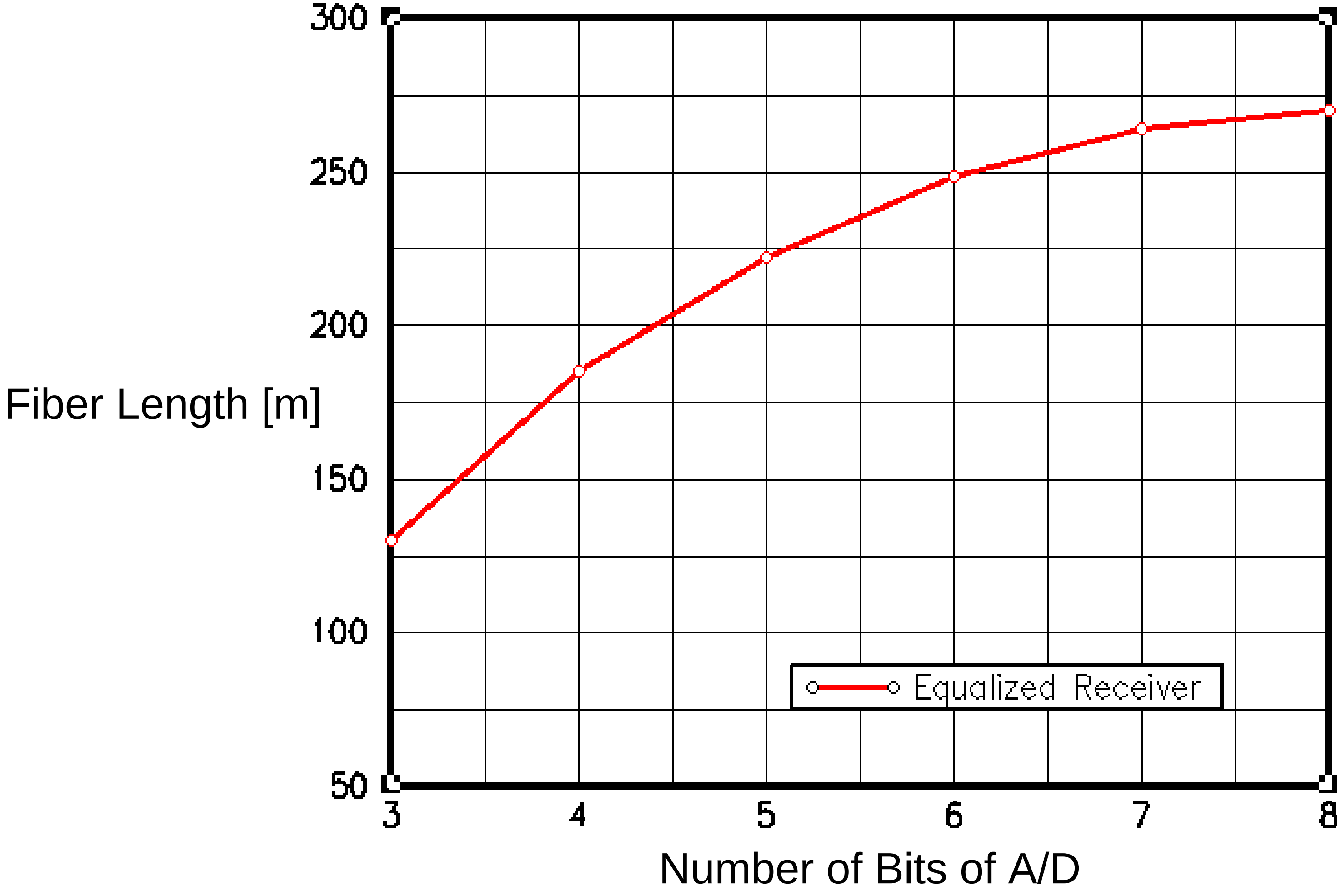
Fiber Length [m]



RMS Jitter [ps]

NOTE: Phase jitter is assumed Gaussian

Fiber Reach versus Number of Bits of A/D Converter
(1310nm, 62.5/125μm fiber)



Conclusions

- **A link model for equalized optical receivers was described**
- **Based on the well established and well documented theory of the DFE**
- **Written in Matlab, uses Signal Processing Toolbox**
- **Allows users to predict cable reach, signal to noise ratio, bit error rate, sensitivities to variations of the parameters, etc.**
- **Also allows users to quickly explore the entire design space before detailed simulations**

References

1. E.A. Lee and D.G.Messerschmitt, *"Digital Communication"*, Second Edition, Kluwer Academic Publishers, 1994

2. B.L.Kasper, "Equalization of Multimode Optical Fiber Systems", *Bell System Technical Journal,* Vol.61, No.7, September 1982, pp.1367-1388