A New Method to Accurately Estimate (A_{DD}, σ_{RJ}) of Dual Dirac Jitter Model from (J3u, J_{RMS}), and to Elucidate Limitation of Current Methods

(A presentation related to draft 2.0 comments #134, 135)

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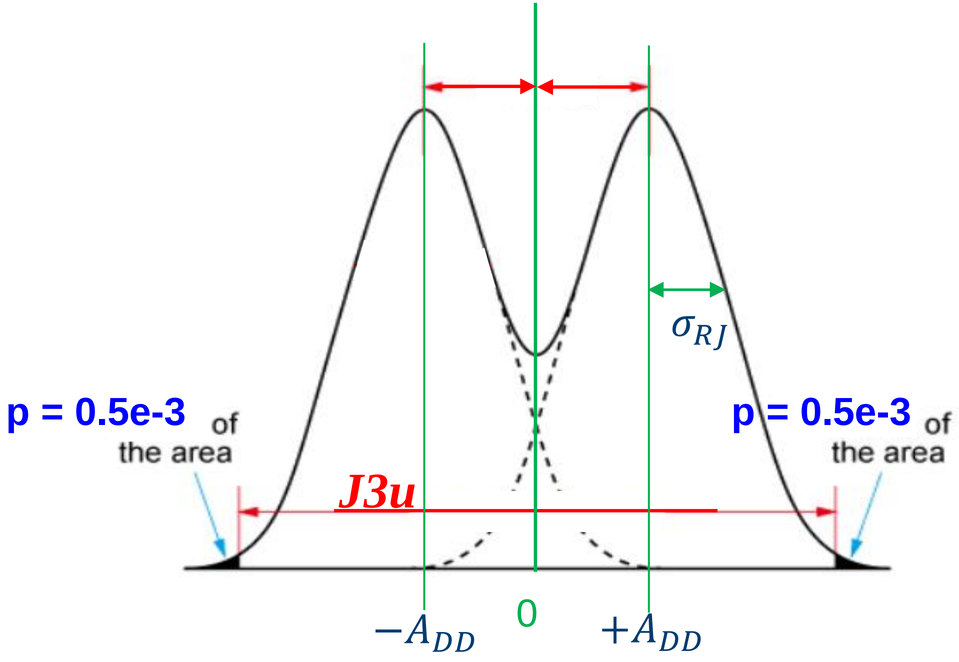
Outline

- Problem Statement
- (A_{DD}, σ_{RJ}) Estimation Methods
  - Accurate Relation among (J3u, J_{RMS}) and (A_{DD}, σ_{RJ}, Q3)
  - Accurate Estimation with Lookup Table (LUT)
  - Current Approximation Methods and Analysis of Two Issues
  - Proposal of New Hybrid Approximation Method
  - Accuracy Evaluation of Three Approximation Methods
- Summary

Note: (A_{DD} , Add), and (σ_{RJ} , RJ) may be interchangeably used in this slides

Problem Statement (1/2)

- The method to estimate (A_{DD}, σ_{RJ}) from ($J3u$,J_{RMS}) specified by (802.3: 163-2&3) can be derived from Dual-Dirac model (DD-Model Eq.1&2), but **one critical information is lost (issue #1)** as explained later.
- Another issue is, solving the set of equations (DD-Model Eq.1&2) for (A_{DD},σ_{RJ}) for a given ($J3u$,J_{RMS}) is not mathematically straightforward because of the 3rd unknown Q3.
  - **Three unknowns/variables with two equations, an under-determined non-linear system (issue #2) !!!**



$$\begin{cases} A_{DD} = \dfrac{\dfrac{J3u}{2} + Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\dfrac{J3u}{2}\right)^2}}{Q3^2+1} & \text{(802.3: 163-2)} \\ \sigma_{RJ} = \dfrac{\dfrac{J3u}{2} - A_{DD}}{Q3} & \text{(802.3: 163-3)} \end{cases}$$

$$\begin{cases} \dfrac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} & \text{(DD-Model Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 & \text{(DD-Model Eq.2)} \end{cases}$$

Problem Statement (2/2)

- To overcome the issue #2, 802.3 D2.0 assumes Q3 = 3.2905 ≈ norminv(1-0.5*10^{-3})
  - The accuracy degradation with this was pointed out by Yasuo, and he proposed revised equations to improve the estimation accuracy.
- Yasuo's proposal assumes Q3 = 3.0902 ≈ norminv(1-1*10^{-3}) as default, and conditionally changes to $Q3 = \sqrt{\left(\frac{J3u}{2J_{RMS}}\right)^2 - 1}$
- Yasuo's proposal is an improvement over the D2.0 method. It, however, still suffers from accuracy degradation when $_{ADD}$ is very small compared with σ_{RJ} as it does not address the issue #1, which also affects the Q3 approximation .
- Thus, we show a new method to accurately estimate (A_{DD},$\sigma_{RJ,}$Q3) from (J3u,J_{RMS}) and Dual-Dirac model look up table (LUT)
  - without solving quadratic equation addressing the issue #1
  - without any assumption on Q3 addressing the issue #1
- Then, we propose a new hybrid approximation method to improve the accuracy further over Yasuo's proposal
  - utilizing the insights obtained by comparing accurate solution, D2.0 method and Yasuo's proposal

Accurate Relation among (J3u, J_{RMS}) and (A_{DD}, σ_{RJ}, Q3) (1/2)

- Directly solve the non-linear Dual-Dirac equation for x =J3u/2 using given A_{DD} and σ_{RJ}

$$\frac{normcdf(x,-A_{DD},\sigma_{RJ})+normcdf(x,+A_{DD},\sigma_{RJ})}{2}=1-0.5\times 10^{-3}$$

- Calculate true Q3 using J3u (known from the above), and given A_{DD} and σ_{RJ}

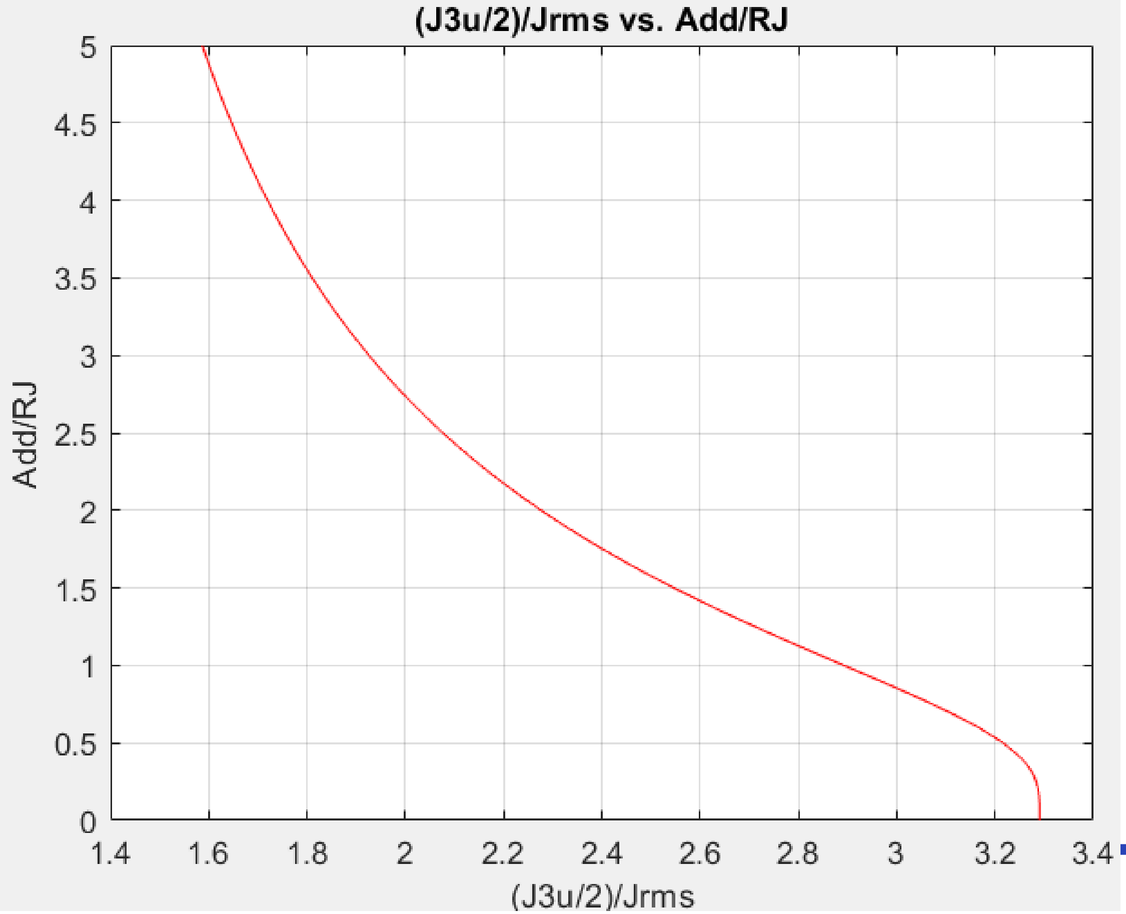
$$\frac{J3u}{2}=A_{DD}+Q3\cdot\sigma_{RJ} \qquad \text{(DD-Model Eq.1)} \qquad \Longrightarrow \qquad Q3=\frac{\frac{J3u}{2}-A_{DD}}{\sigma_{RJ}}$$

- Calculate J_{RMS} using given A_{DD} and σ_{RJ}

$$J_{RMS}^2=A_{DD}^2+\sigma_{RJ}^2 \qquad \text{(DD-Model Eq.2)} \qquad \Longrightarrow \qquad J_{RMS}=\sqrt{A_{DD}^2+\sigma_{RJ}^2}$$

Accurate Relation among (J3u, J_{RMS}) and (A_{DD}, σ_{RJ}, Q3) (2/2)

- Now we know the accurate relation between measurement value (J3u/2)/J_{RMS} and Dual-Dirac model parameter A_{DD}/σ_{RJ}
- This relationship can be utilized for accurate (A_{DD}, σ_{RJ}) estimation from (J3u, J_{RMS})



$$\frac{\frac{J3u}{2}}{J_{RMS}} \equiv \alpha \equiv f\left(\frac{A_{DD}}{\sigma_{RJ}}\right)$$

$$\therefore \frac{A_{DD}}{\sigma_{RJ}} = f^{-1}\left(\frac{\frac{J3u}{2}}{J_{RMS}}\right) \equiv g(\alpha) \equiv g_{\alpha}$$

- A_{DD}/σ_{RJ} can be obtained from the measured (J3u,J_{RMS})

Accurate Estimation of (A_{DD},$\sigma_{RJ,}$Q3) with LUT (1/2)

For a given / measured data set: ($J3u$,J_{RMS})

- Create Look up table (LUT): $(J3u/2)/J_{RMS}$ vs. A_{DD}/σ_{RJ}
- Find g_α for a given / measured ($J3u$,J_{RMS}) using LUT

$$\frac{\frac{J3u}{2}}{J_{RMS}} \equiv \alpha \equiv f\left(\frac{A_{DD}}{\sigma_{RJ}}\right) \qquad \therefore \frac{A_{DD}}{\sigma_{RJ}} = f^{-1}\left(\frac{\frac{J3u}{2}}{J_{RMS}}\right) \equiv g(\alpha) \equiv g_\alpha$$

(DD-Model Eq.1&2) can be written as follows

$$\begin{cases} \frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} & \text{(DD-Model Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 & \text{(DD-Model Eq.2)} \end{cases}$$

⇒

$$\begin{cases} \frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} = (g_\alpha + Q3)\sigma_{RJ} & \text{(New Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 = \left(g_\alpha^2 + 1\right)\sigma_{RJ}^2 & \text{(New Eq.2)} \end{cases}$$

Then

$$\frac{\left(\frac{J3u}{2}\right)^2}{J_{RMS}^2} = \alpha^2 = \frac{(g_\alpha + Q3)^2}{g_\alpha^2 + 1}$$

$$\therefore Q3 = -g_\alpha + \alpha\sqrt{g_\alpha^2 + 1} \qquad \text{(New Eq.3)}$$

- Estimated Q3 accurately with
  $\alpha = (J3u/2/J_{RMS})$ and g_α obtained with LUT

- Estimated σ_{RJ} and A_{DD} as follows

$$\begin{cases} \sigma_{RJ} = \frac{\frac{J3u}{2}}{Q3 + g_\alpha} & \text{(New Eq.4)} \\ A_{DD} = g_\alpha \sigma_{RJ} & \text{(New Eq.5)} \end{cases}$$

Accurate Estimation of (A_{DD},σ_{RJ},Q3) with LUT (2/2)

$$(J3u, J_{RMS}) \rightarrow \frac{\frac{J3u}{2}}{J_{RMS}} (\equiv \alpha) \xrightarrow{\text{with LUT}} \frac{A_{DD}}{\sigma_{RJ}} (\equiv g_\alpha)$$

$$Q3 = -g_\alpha + \alpha\sqrt{g_\alpha^2 + 1}$$

$$\sigma_{RJ} = \frac{\frac{J3u}{2}}{Q3 + g_\alpha}$$

$$A_{DD} = g_\alpha \sigma_{RJ}$$

Example Look Up Table (LUT): $(J3u/2)/J_{RMS}$ vs. A_{DD}/σ_{RJ}

$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}
3.29052673	0.00	3.28612098	0.22	3.03807708	0.80	1.92590056	3.00
3.29052671	0.01	3.28532567	0.23	2.96602742	0.90	1.90041853	3.10
3.29052639	0.02	3.28443584	0.24	2.89226854	1.00	1.87621900	3.20
3.29052500	0.03	3.28344629	0.25	2.81866747	1.10	1.85321463	3.30
3.29052126	0.04	3.28235203	0.26	2.74654361	1.20	1.83132499	3.40
3.29051341	0.05	3.28114832	0.27	2.67677303	1.30	1.81047605	3.50
3.29049919	0.06	3.27983066	0.28	2.60989485	1.40	1.79059962	3.60
3.29047590	0.07	3.27839483	0.29	2.54620285	1.50	1.77163289	3.70
3.29044038	0.08	3.27683688	0.30	2.48581818	1.60	1.75351795	3.80
3.29038907	0.09	3.27515320	0.31	2.42874420	1.70	1.73620140	3.90
3.29031802	0.10	3.27334047	0.32	2.37490675	1.80	1.71963392	4.00
3.29022293	0.11	3.27139570	0.33	2.32418290	1.90	1.70376998	4.10
3.29009919	0.12	3.26931625	0.34	2.27642109	2.00	1.68856745	4.20
3.28994190	0.13	3.26709981	0.35	2.23145516	2.10	1.67398735	4.30
3.28974592	0.14	3.26474438	0.36	2.18911370	2.20	1.65999355	4.40
3.28950592	0.15	3.26224833	0.37	2.14922637	2.30	1.64655257	4.50
3.28921639	0.16	3.25961034	0.38	2.11162781	2.40	1.63363331	4.60
3.28887171	0.17	3.25682944	0.39	2.07616016	2.50	1.62120690	4.70
3.28846618	0.18	3.25390495	0.40	2.04267435	2.60	1.60924644	4.80
3.28799406	0.19	3.21682624	0.50	2.01103080	2.70	1.59772695	4.90
3.28744960	0.20	3.16660322	0.60	1.98109951	2.80	1.58662509	5.00
3.28682712	0.21	3.10594193	0.70	1.95275992	2.90		

(A_{DD},σ_{RJ},Q3) Estimation Algorithm of 802.3ck D2.0 Method

$(J3u, J_{RMS})$

$Q3 = norminv(1 - 0.5 \cdot 10^{-3}) \cong 3.2905$

$$D_3 \equiv (Q3^2 + 1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2$$

if $D_3 \geq 0$

$$\begin{cases} A_{DD} = \left(\frac{J3u}{2} + Q3\sqrt{(Q3^2+1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}\right) / (Q3^2 + 1) & \text{(802.3: 163-2)} \\ \sigma_{RJ} = \left(\frac{J3u}{2} - A_{DD}\right) / Q3 & \text{(802.3: 163-3)} \end{cases}$$

else $(i.e. D_3 < 0)$

a different transmitter should be use in the test setup

(A_{DD}, σ_{RJ}, Q3) Estimation Algorithm of Yasuo's Proposal

Change #1 from 802.3ck D2.0

$(J3u, J_{RMS})$

$Q3 = norminv(1 - 10^{-3}) \cong 3.0902$

$$D_3 \equiv (Q3^2 + 1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2$$

if $D_3 \geq 0$

$$\begin{cases} A_{DD} = \left(\frac{J3u}{2} + Q3\sqrt{(Q3^2+1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}\right) / (Q3^2 + 1) & \text{(802.3: 163-2)} \\ \sigma_{RJ} = \left(\frac{J3u}{2} - A_{DD}\right) / Q3 & \text{(802.3: 163-3)} \end{cases}$$

else $(i.e. D_3 < 0)$

Change #2 from 802.3ck D2.0

$$\begin{cases} Qx = \sqrt{\left(\frac{J3u/2}{J_{RMS}}\right)^2 - 1} \\ A_{DD} = \left(\frac{J3u}{2}\right) / (Q_x^2 + 1) \\ \sigma_{RJ} = \sqrt{J_{RMS}^2 - A_{DD}^2} \end{cases}$$

Issue #1: Solving Quadratic Equation (1/3)

Let's solve (DD-Model Eq.1&2) for A_{DD} assuming that Q3 is known.

$$\begin{cases} \frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} & \text{(DD-Model Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 & \text{(DD-Model Eq.2)} \end{cases}$$

$$sqrt(D) \equiv Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}$$

Then, the straightforward A_{DD} solution is slightly different from Eq (802.3: 163-2).

$$A_{DD} = \frac{\frac{J3u}{2} \pm Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1}$$

vs. $$A_{DD} = \frac{\frac{J3u}{2} + Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1} \quad \text{(802.3: 163-2)}$$

σ_{RJ} solution is the same formula as Eq (802.3: 163-3), but its value is affected the by the estimated A_{DD}.

$$\sigma_{RJ} = \frac{\frac{J3u}{2} - A_{DD}}{Q3} \quad \text{(802.3: 163-3)}$$

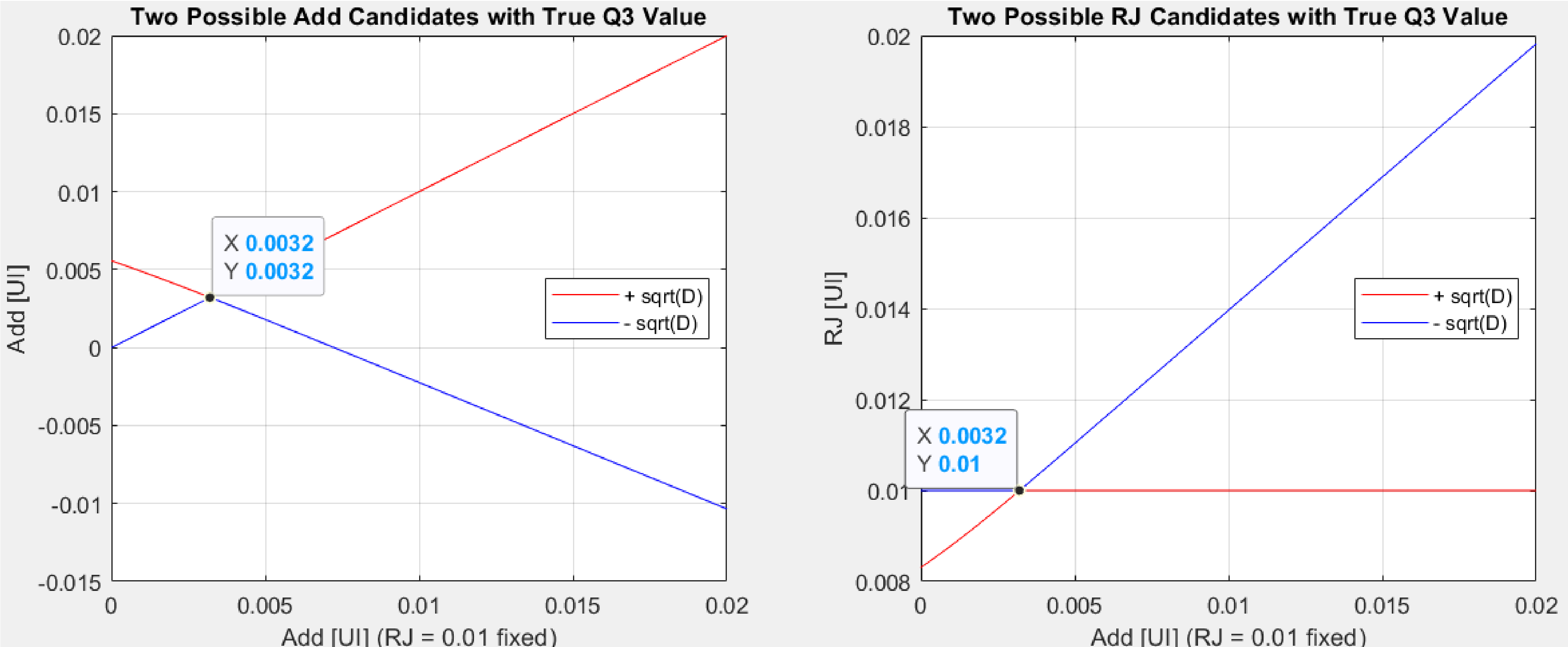
Issue #1: Solving Quadratic Equation (2/3)

Which of two possible ADD candidates should be used depends on A_{DD}/σ_{RJ} , which means $(J3u/2)/J_{RMS}$ vs. true Q3 value.

$$A_{DD} = \frac{\frac{J3u}{2} \pm Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1} \qquad sqrt(D) \equiv Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}$$

Note. The threshold for choosing either "+sqrt(D)" or "-sqrt(D)" is Add~0.0032 (w/RJ=0.01)

Note. Estimated σ_{RJ} is affected by estimated A_{DD}



Numerical example

- σ_{RJ} = 0.01UI and A_{DD} = 0 to 0.02 UI
- Their ratio is essential, or jitter may be considered as normalized with σ_{RJ}

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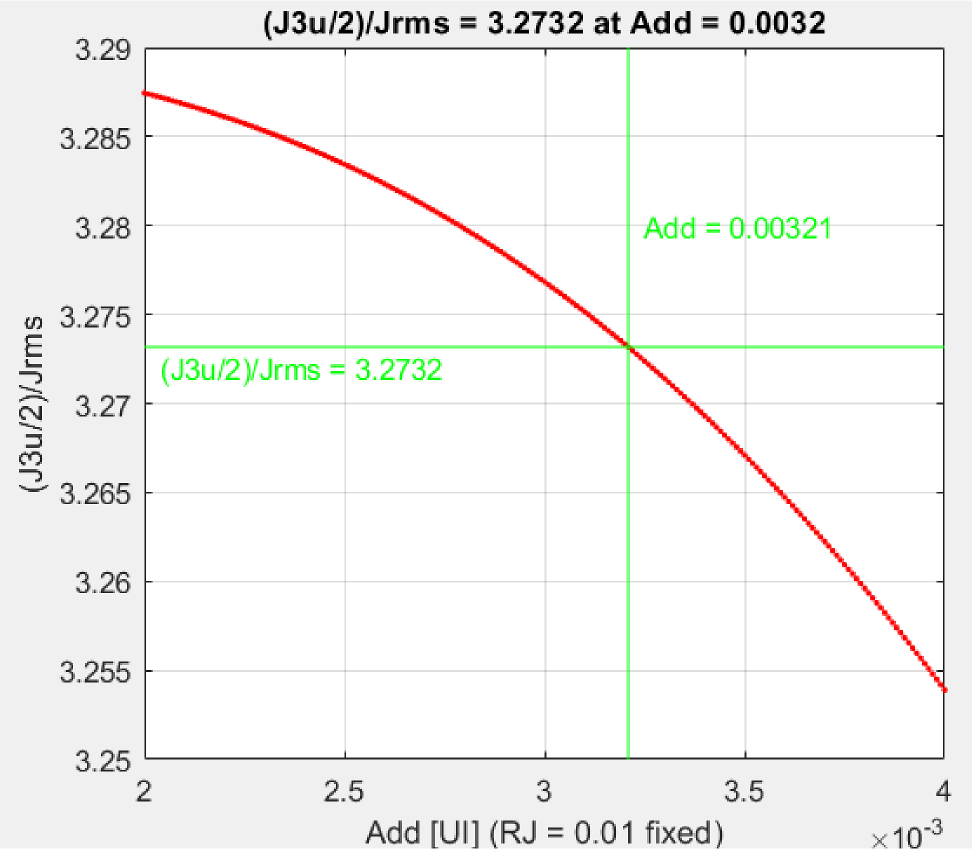
Issue #1: Solving Quadratic Equation (3/3)

- Threshold for choosing either "+sqrt(D)" or "-sqrt(D) is Add~0.0032, i.e. (J3u/2)/JRMS ~ 3.2732

When $\frac{J3u/2}{J_{RMS}} = Q3$

$$A_{DD} = \frac{\frac{J3u}{2} \pm Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1} = \frac{\frac{J3u/2}{2} \pm \frac{J3u/2}{2}}{Q3^2+1}$$

Then, A_{DD}=0 if "+sqrt(D)" is chosen.



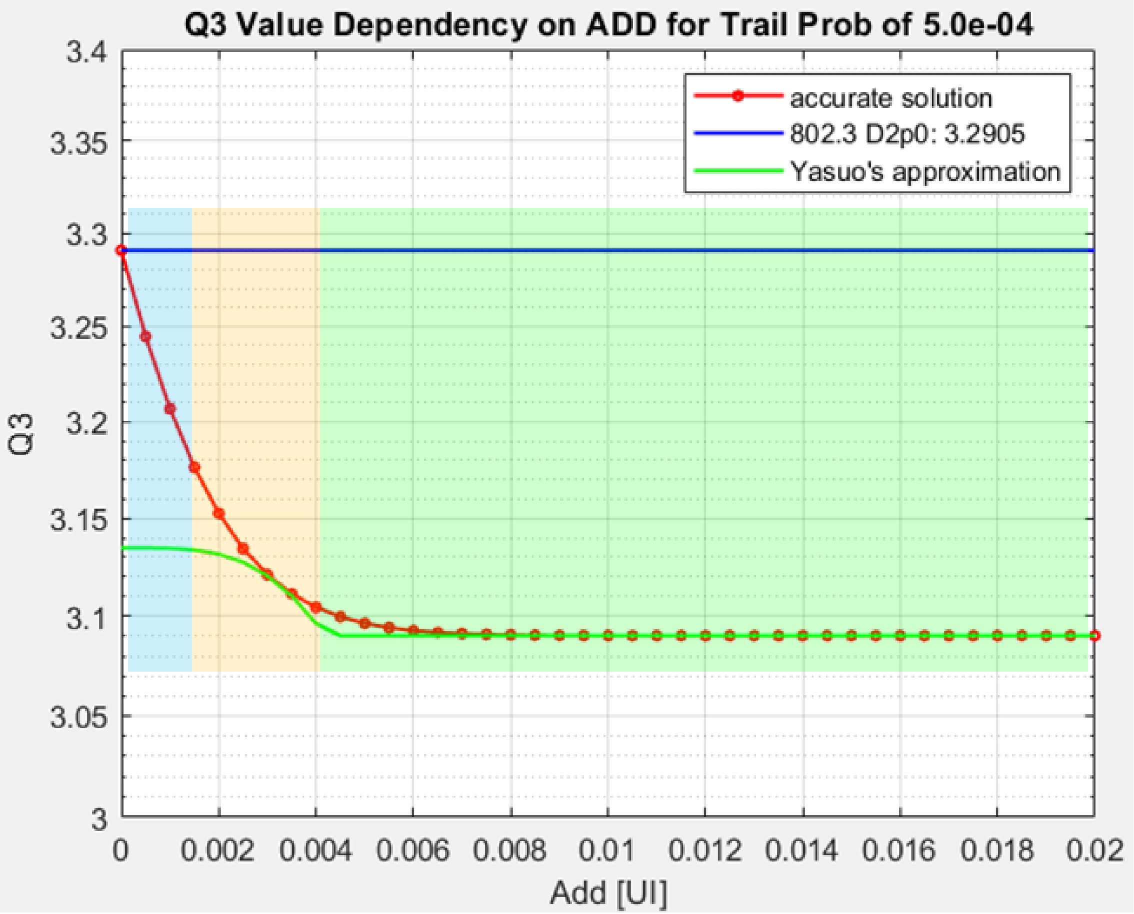
Numerical example

- σ_{RJ} = 0.01UI and A_{DD} = 0 to 0.02 UI
- Their ratio is essential, or jitter may be considered as normalized with σ_{RJ}

- Issue #1: Always using "+sqrt(D)" is the reason why 802.3ck D2.0 method and Yasuo's proposal never result in Add~0 regardless of ($J3u, J_{RMS}$) values

Issue #2: Q3 Estimation Accuracy

- Our issue #2 is true Q3 estimation accuracy if we are to stick to Dual-Dirac model
  - Q~3.2905 when Add is much smaller than σ_{RJ}
  - Q is smaller than 3.2905 when Add is relatively small compared with σ_{RJ}
  - Q~3.0902 when Add is relatively large compared with σ_{RJ}



Numerical example

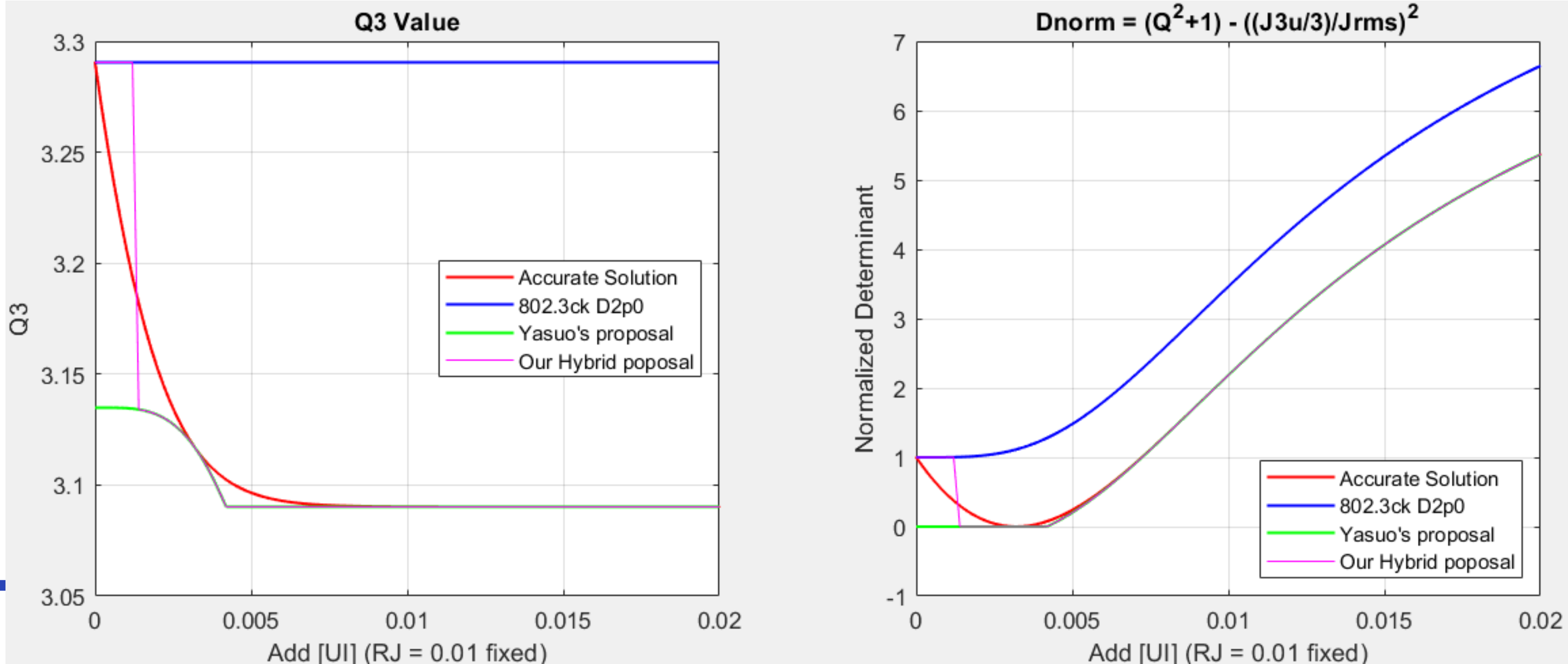
- σ_{RJ} = 0.01UI and A_{DD} = 0 to 0.02 UI
- Their ratio is essential, or jitter may be considered as normalized with σ_{RJ}

Improving Yasuo's Proposal by Addressing Issue #1 (1/2)

Let's define Dnorm as follows.

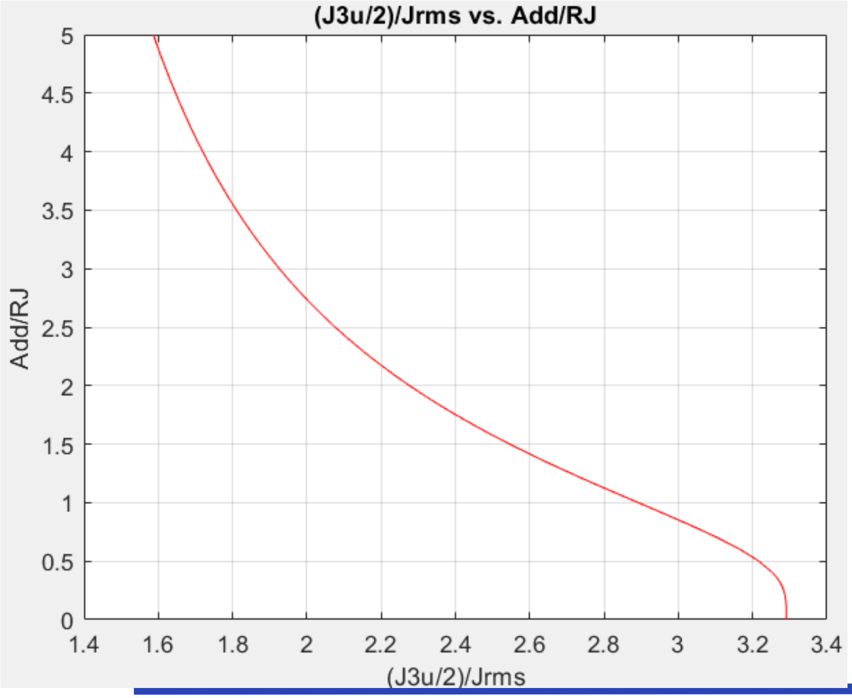
$$A_{DD} = \frac{\frac{J3u}{2} \pm Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1} = \frac{\frac{J3u}{2} \pm Q3 \cdot J_{RMS}\sqrt{(Q3^2+1) - \left(\frac{J3u/2}{J_{RMS}}\right)^2}}{Q3^2+1} \equiv \frac{\frac{J3u}{2} \pm Q3 \cdot J_{RMS}\sqrt{D_{norm}}}{Q3^2+1}$$

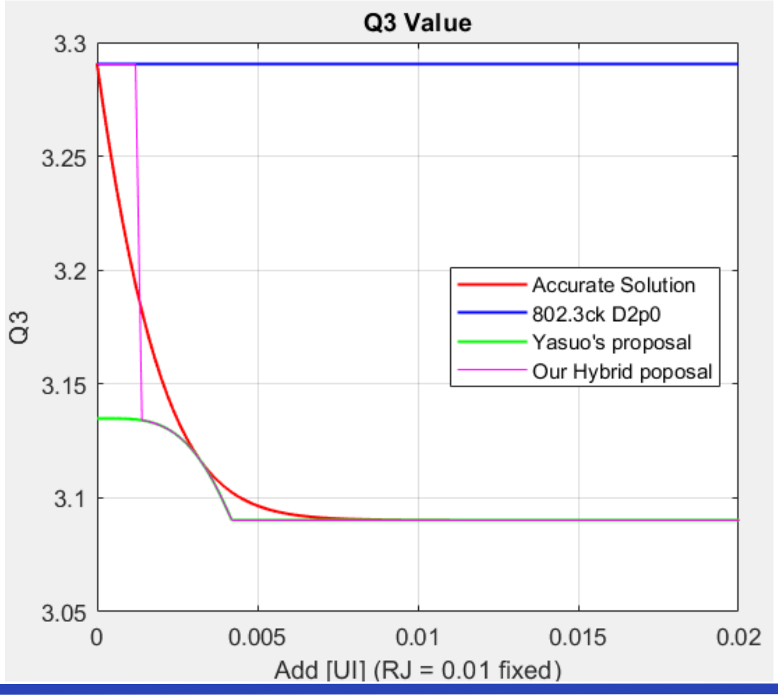
- As shown on the right graph below, Dnorm becomes 0 at around Add=0.0032, but Dnorm becomes larger than 0 when Add becomes smaller than this threshold.
- Since Yasuo's proposal sets D=0 when Add becomes smaller than around this threshold, which prevents 1) estimated Add from becoming smaller, and 2) estimated Q3 from becoming larger as shown on the left graph.



Improving Yasuo's Proposal by Addressing Issue #1 (2/2)

- When Add (i.e. A_{DD}/σ_{RJ}, and therefore (J3u/2)/$_{JRMS}$) is close to 0, the jitter pdf is much closer to single Gaussian than bimodal Gaussian.
- By forcing estimated Add to 0 when (J3u/2)/J_{RMS} is smaller than a certain threshold, Yasuo's proposal can be improved. We propose this threshold value of 3.290 by having examined resulting estimation errors.
- Forcing Add to 0 under this condition results in 1) Q3 = $\mathrm{norminv}(1 - 0.5 \cdot 10^{-2}) \cong 3.2905$, and 2) Dnorm = 1 as shown in the graphs in the previous slide.





Notes.

- Slight difference in (J3u/2)/J_{RMS} results in large difference in Add/RJ as observed in the true Dual Dirac model characteristic (shown in the left graph).
- Once consequence is a large difference in the estimated Q3 value (shown in the right graph)

(A_{DD}, σ_{RJ}, Q3) Estimation Algorithm of Our Hybrid Method

$(J3u, J_{RMS})$

$Q3 = norminv(1-10^{-3}) \cong 3.0902$

Change #1 from 802.3ck D2.0

$J3uh2Jrms = 3.290$

Change #2 from 802.3ck D2.0 adding another conditional processing to Yasuo's proposal

$$D_3 \equiv (Q3^2+1)\times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2$$

if $D_3 \geq 0$

$$\begin{cases} A_{DD} = \left(\frac{J3u}{2} + Q3\sqrt{(Q3^2+1)\times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}\right)/(Q3^2+1) \\ \sigma_{RJ} = \left(\frac{J3u}{2} - A_{DD}\right)/Q3 \end{cases}$$

(802.3: 163-2)

(802.3: 163-3)

else if $\frac{J3u/2}{J_{RMS}} \leq J3uh2Jrms$

Change #2-A
Use Yasuo's proposal

$$\begin{cases} Qx = \sqrt{\left(\frac{J3u/2}{J_{RMS}}\right)^2 - 1} \\ A_{DD} = \left(\frac{J3u}{2}\right)/(Q_x^2+1) \\ \sigma_{RJ} = \sqrt{J_{RMS}^2 - A_{DD}^2} \end{cases}$$

else $\left(i.e.\ D_3 < 0\ and\ \frac{J3u/2}{J_{RMS}} > J3uh2Jrms\right)$

$$\begin{cases} Qy = norminv(1-0.5\cdot 10^{-3}) \cong 3.2905 \\ A_{DD} = 0 \\ \sigma_{RJ} = \frac{J3u/2}{Qy} \end{cases}$$

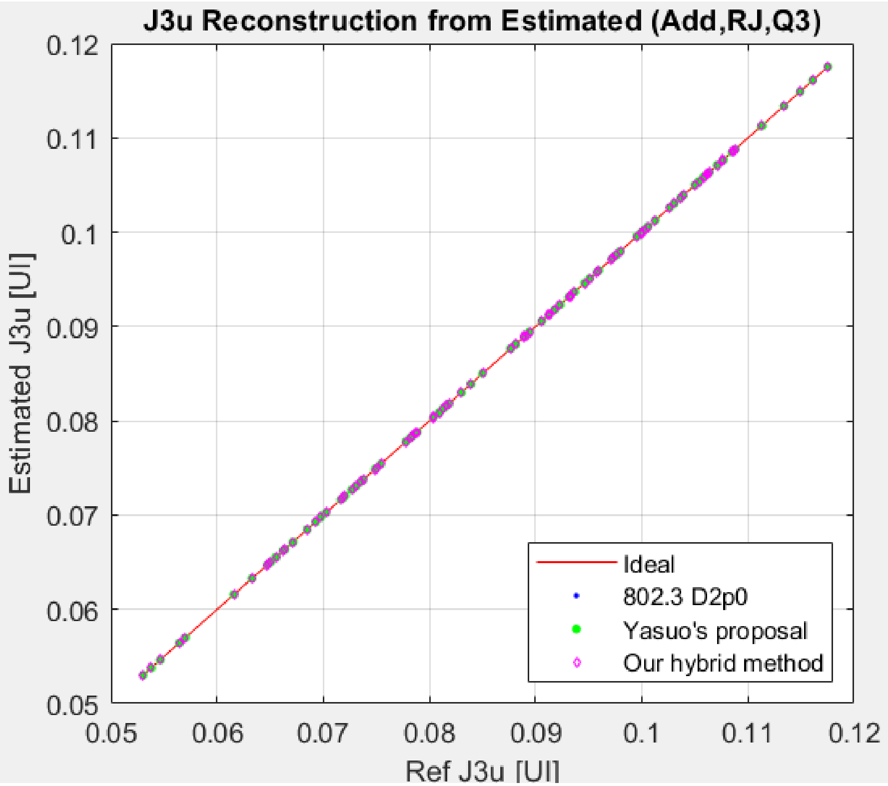
Change #2-B
Added by our hybrid proposal

J3u and J4u Accuracy Evaluation

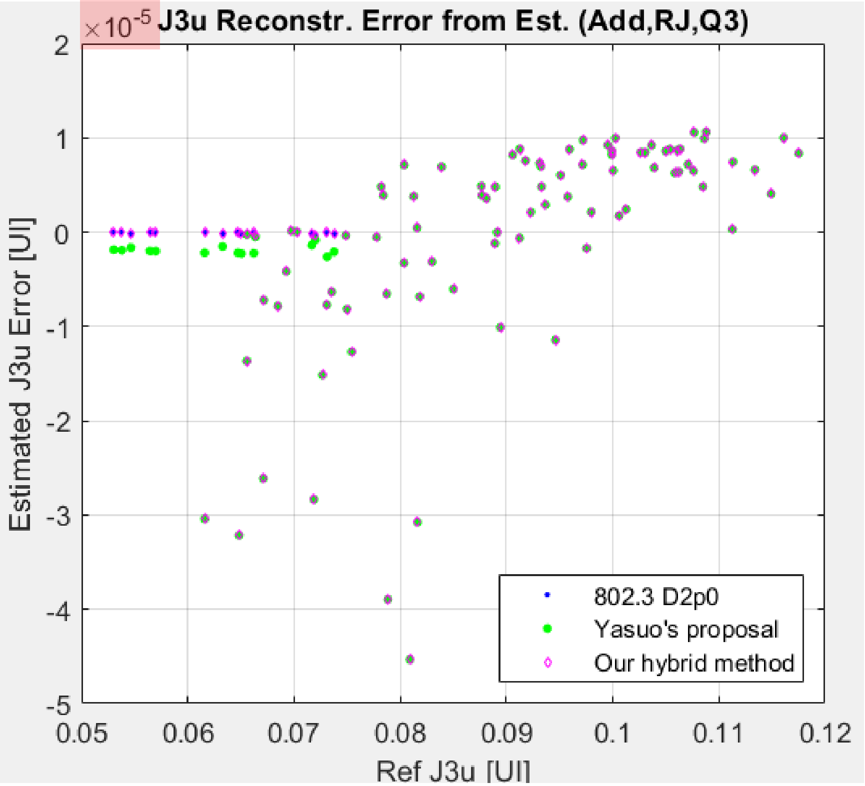
- 100 test cases (J3u,J_{RMS}) were generated from A_{DD} ~ unif(0,0.024)UI and σ_{RJ} ~ unif(0.008, 0.012)UI
- J3u Reconstruction Accuracy
  - Reconstruct J3u from the estimated (A_{DD} , σ_{RJ}, Q3)
- J3u Re-estimation Accuracy
  - Find/estimate true J3u by solving non-linear Dual Dirac model with the estimated (A_{DD} , σ_{RJ})
- J4u Estimation Accuracy
  - Find/estimate true J3u by solving non-linear Dual Dirac model with the estimated (A_{DD} , σ_{RJ})
  - The reason for this test is because DER is set to 10^{-4} (FEC symbol error ration < 10^{-3}) for 802.3ck Interference Tolerance Test

J3u Reconstruction Accuracy Evaluation

- J3u (and J_{RMS}) is very accurately reconstructed by all the three methods using the estimated (A_{DD} , σ_{RJ}, Q3) because the errors in some parameters are compensated by the errors in opposite direction of the other parameters.
- This is not a good method to evaluate the accuracy of the estimation methods.

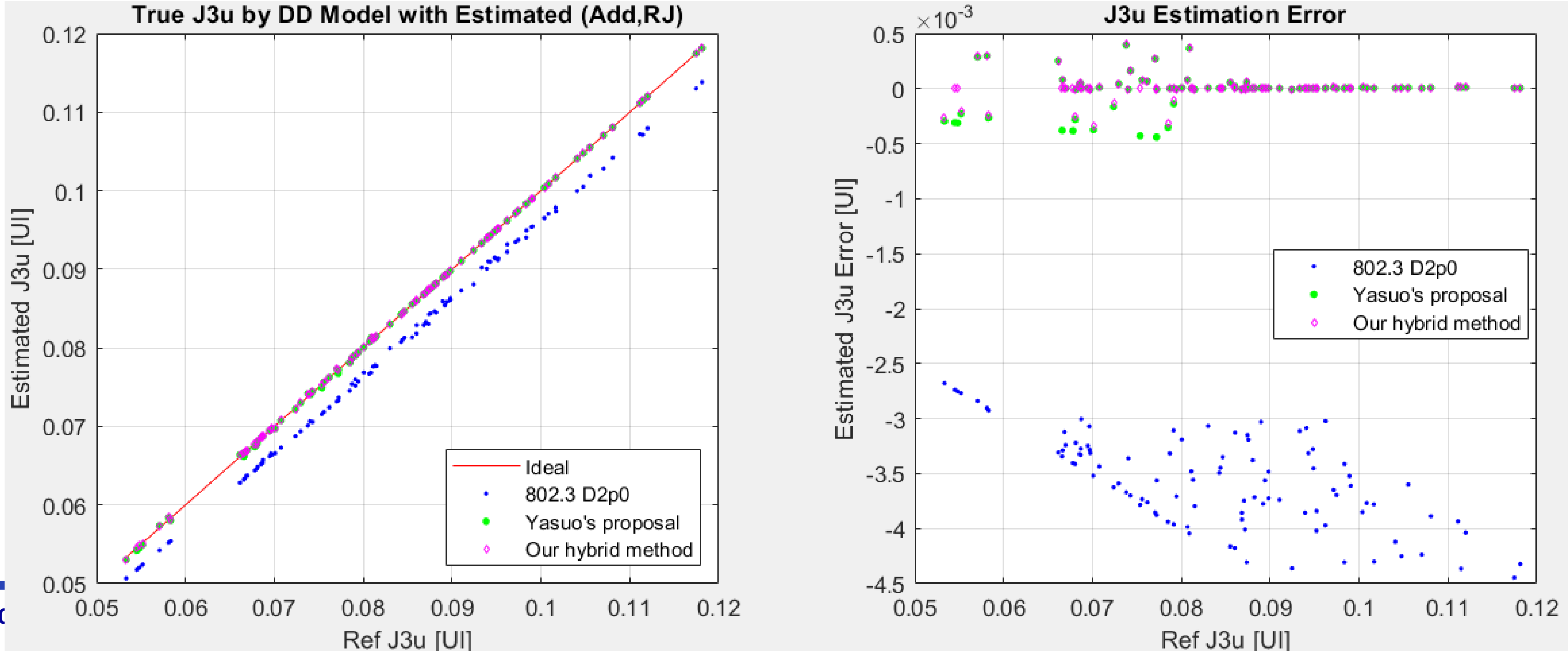


Negligibly small error



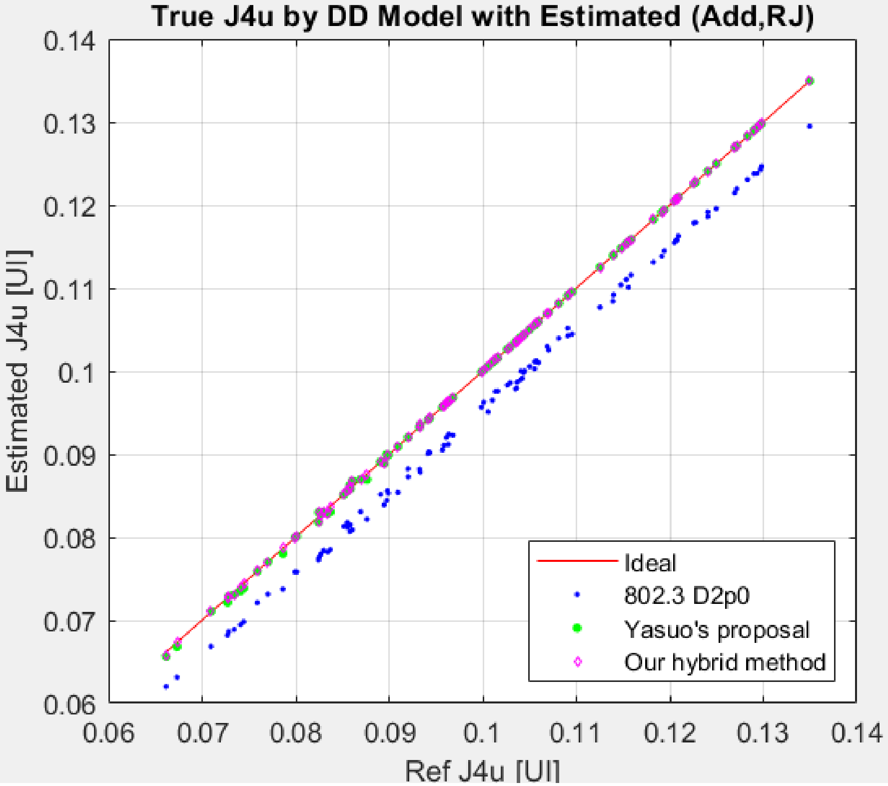
J3u Re-Estimation Accuracy Evaluation

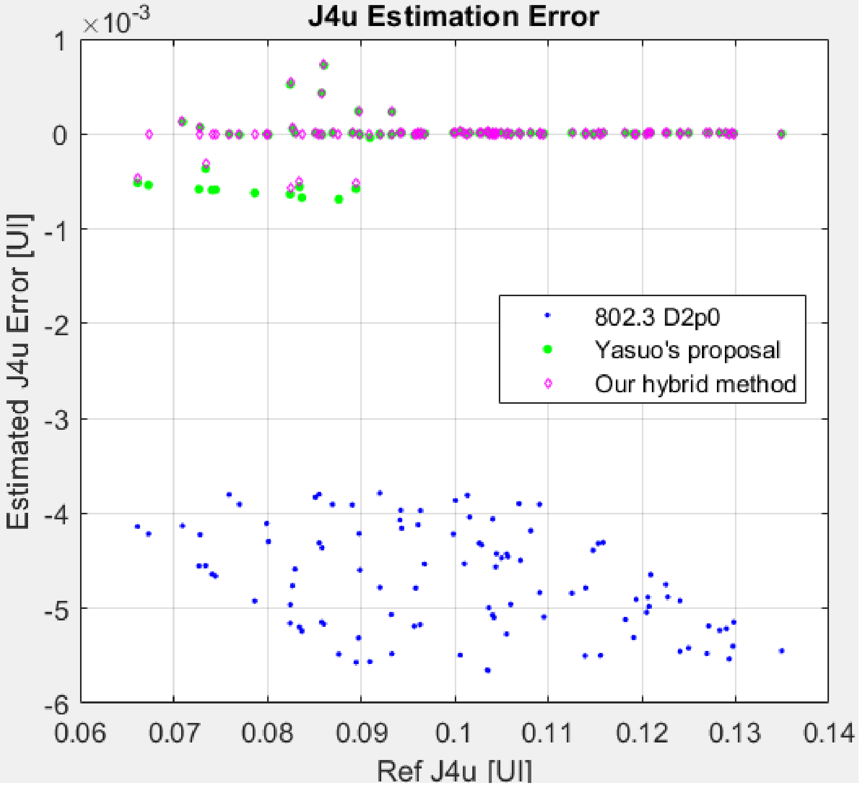
- Since estimated Q3 is not used here, estimated (A_{DD} , σ_{RJ}) accuracy determines the J3u re-estimation accuracy. If estimated (A_{DD} , σ_{RJ}) are used to generate jitter as done in COM method, this is the accuracy we like to evaluate.
- 802.3ck D2.0 method: Because of the larger estimated Q3 than actual, therefore smaller estimated σ_{RJ} , J3u is almost always underestimated.
- Yasuo's proposal significantly improves J3u estimation accuracy except for some small number of the test cases where intrinsic A_{DD}/σ_{RJ} is very small. Our hybrid method further reduces the error for those small number of the test cases.



J4u Estimation Accuracy Evaluation

- The error trends and their reasons are the same for J3u estimation.
- 802.3ck D2.0 method almost always underestimates J4u.
- Yasuo's proposal accurately estimates J4u in many cases, and our hybrid method further increases the accurate estimation range.





Summary

- Problem / Challenge
  - Estimating (A_{DD},$\sigma_{RJ,}$Q3) for a given (J3u,J_{RMS}) assuming Dual Dirac jitter model is not mathematically straightforward
    - **Three unknowns with two equations problem, an underdetermined non-linear system !!!**
    - 802.3ck D2p0 method of linear approximation is too simple and has room to improve
- Resolution Option#1: use LUT
  - We have shown a new method to accurately estimate (A_{DD},$\sigma_{RJ,}$Q3) from (J3u,J_{RMS}) and Dual-Dirac model look up table (LUT) without any assumption on Q3
    - This results elucidate the reasons for the limitation of the 802.3ck D2.0 method and Yasuo's proposal
- Resolution Option#2: hybrid approximation
  - We have proposed a new "hybrid approximation" method to further improve the estimation accuracy of Yasuo's proposal with a small modification of adding one more conditional processing
    - In case LUT-based method is considered too much change from 802.3ck D2.0 method, or too complicated/elaborated in practice

Proposal Option #1: Algorithm Using LUT

$$(J3u, J_{RMS}) \longrightarrow \frac{\frac{J3u}{2}}{J_{RMS}} (\equiv \alpha) \xrightarrow{\text{with LUT}} \frac{A_{DD}}{\sigma_{RJ}} (\equiv g_\alpha)$$

$$Q3 = -g_\alpha + \alpha\sqrt{g_\alpha^2 + 1}$$

$$\sigma_{RJ} = \frac{\frac{J3u}{2}}{Q3 + g_\alpha}$$

$$A_{DD} = g_\alpha \sigma_{RJ}$$

Proposal Option #2 Algorithm: Hybrid Approximation

$(J3u, J_{RMS})$

$Q3 = norminv(1 - 10^{-3}) \cong 3.0902$

$J3uh2Jrms = 3.290$

Change #1 from 802.3ck D2.0

Change #2 from 802.3ck D2.0 adding another conditional processing to Yasuo's proposal

$$D_3 \equiv (Q3^2 + 1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2$$

if $D_3 \geq 0$

$$\begin{cases} A_{DD} = \left(\frac{J3u}{2} + Q3\sqrt{(Q3^2+1) \times J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}\right) / (Q3^2+1) \\ \sigma_{RJ} = \left(\frac{J3u}{2} - A_{DD}\right) / Q3 \end{cases}$$

(802.3: 163-2)

(802.3: 163-3)

else if $\frac{J3u/2}{J_{RMS}} \leq J3uh2Jrms$

Change #2-A
Use Yasuo's proposal

$$\begin{cases} Qx = \sqrt{\left(\frac{J3u/2}{J_{RMS}}\right)^2 - 1} \\ A_{DD} = \left(\frac{J3u}{2}\right) / (Q_x^2 + 1) \\ \sigma_{RJ} = \sqrt{J_{RMS}^2 - A_{DD}^2} \end{cases}$$

else $\left(i.e.\ D_3 < 0\ and\ \frac{J3u/2}{J_{RMS}} > J3uh2Jrms\right)$

$$\begin{cases} Qy = norminv(1 - 0.5 \cdot 10^{-3}) \cong 3.2905 \\ A_{DD} = 0 \\ \sigma_{RJ} = \frac{J3u/2}{Qy} \end{cases}$$

Change #2-B
Added by our hybrid proposal

Backup: Estimation Accuracy Evaluation: A_{DD}, σ_{RJ} and Q3

- 100 test cases ($J3u$,J_{RMS}) were generated from A_{DD} ~ unif(0,0.024)UI and σ_{RJ} ~ unif(0.008, 0.012)UI
- For the three approximation methods (802.3ck D2.0, Yasuo's proposal, our hybrid proposal), estimated Q3 values and the ($A_{DD,}$ σ_{RJ}) estimation errors are shown below.
  - Q3 estimation results simply show what are expected from the discussion in the previous slides
  - Yasuo's proposal significantly improves ($A_{DD,}$ σ_{RJ}) estimation accuracy except for the test cases with very small inherent A_{DD}/σ_{RJ} .
  - Our hybrid proposal reduces the ($A_{DD,}$ σ_{RJ}) estimation error the test cases with very small inherent A_{DD}/σ_{RJ} .