## Optical Link Model for PAM-4 Multimode Channels with equalizers

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## Introduction and Objectives

- Provide a link model tool to estimate reasonable worst-case channel reaches for multimode fiber links using VCSEL based transceivers and PAM-4 modulation
- 50GBASE-SR, 100GBASE-SR2, 200GBASE-SR4, 400GBASE-SR8, 400GBASE-SR4.2, BiDi and others operating in the spectral window $840-953 \mathrm{~nm}$
- Previous link models compute penalties based on worst-case eye opening
- Too pessimistic for equalized PAM-4 links
- The statistical eye amplitude method (castro_3cm_01_0518) provides a fast and accurate estimation.
- This presentation describes the eye amplitude estimation method of all the penalties (ISI and noise penalties) in the link model.
- An implementation of the statistical eye amplitude link model in Matlab will be shown during the presentation.

Current approach "Worst-Case eye opening"


Proposed "statistical eye amplitude" method: the information from the eye amplitude PDF is included in the evaluation to provide a reasonable worst-case link

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## Equalized Link model



## Implementation

- Evaluated several options shown in Table.

| Platform | Speed | Required License | Complexity | Size |
| :---: | :---: | :---: | :---: | :---: |
| Excel ${ }^{\text {® }}$ | Very Slow | MS Office | Very High | Very small |
| Excel ${ }^{\circledR}$ with Visual Basic Application (VBA) | Slow | MS Office | Medium | Very small |
| Excel ${ }^{\circledR}$ with Dynamic time libraries | Fast | MS Office | High | Small |
| MATLAB ${ }^{\text {® }}$ compiled | Fast | None | Medium | Large 500 Mbytes min |

- Excel with simple VBA could be good for worst case eye closure but not for eye amplitude method.
- Latest version, presented here implemented in MATLAB (compiled)


## Implementation

- Spreadsheet approach
- Top menu for basic functions (save load,...)
- Section for input parameters
- Section for results
- Eye diagram (ISI-Jitter penalties)
- Histograms at 2 m and for the maximum reach



## Implementation

- Computation of link penalties for PAM-2 and PAM-4
- Transmitter equalizer only 3 taps (symmetric) with fixed options
- Receiver equalizer MMSE 3 or 5 Taps
- Histograms computed at the center of the eye for 2 m and maximum reach
- Show the histograms due to ISI-Jitter component (red trace) and also the noise components
- Noise enhancement due equalizer included
- Eye compression knob only allow the range of values shown in dial
- Eye diagram (ISI-Jitter penalties only)
- Receiver equalizer MMSE 3 or 5 Taps
- Noise enhancement due equalizer included


## Implementation

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## Real time demo shown during the presentation

## File Verification Tools Version

| a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 a9 | a10 | a11 | a12 | a13 | a14 | a |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tx pwr OMA= |  | dBm | RIN_Coef= | 0.7000 |  | Connections= | 1.5000 dB | at |  |  | C_IX= |  | ns. ${ }^{\wedge}$ |
| Min. Ext Ratio= | 2.9960 |  | DJ+ \& TP4eye= | 4.2500 | s inc... | Pwr.Bud.-Conn.L... | 6.4000 dB | Disp. min. $\mathrm{Uo}=$ | 1316 |  | T_rx(10-90) $=$ | 17.2 |  |
| "Worst"ave.TxPwr= | 1.6800 |  | DCD_DJ= |  | ps TP3 | $\mathrm{C} 1=$ | 480 ns.MHz | Disp. So= | 0.1027 | ps/n... | TP4 Eye= | 7.75 |  |
| Ext. ratio penalty= | 4.7903 |  | Effect. DJ= | 0.1228 | UI) e... | Reflection Noise f... | 0 no units | Disp. D1= | -108.41 | S/(n... | Opening |  | = $\dagger$ |
| Tx mask= |  |  | MPN k(OMA) = | 0.1000 |  | Effective Rate= | 2.8900... MBd | Fraction Jitter T... | 0.7500 |  | RMS Baseline wa... | 0.012 |  |
| X1= | 0.3000 | UI | Tx eye height= | \#REF! |  | Tb_eff= | 34.6021 ps | (not in use) | 10 |  | SECQ_rx= | 0.900 |  |
| X2= | 0.4000 | UI | Refl Tx= | -12 |  | Effective Rec Eye= | 0.2000 UI | BWm= | 4400 | Hz*... | P_BLW(no ISI)= | 0.00 |  |
| $\mathrm{Y} 1=$ | 0.2500 |  | ModalNoisePen= | 0.4488 |  | Tx mask top= | 0.2000 UI | Eff. BWm= | 4400 | $M H z^{*} \ldots$ | P_BLW= | 0.005 | dB - |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  | - |


| L (km) | Patt (dB) | ChiL | D. L (p... | BWcd (... | effBWm... | Te (ps) | Tc (ps) | Pisi-C. | Peye-... | P_DJ... | P_DC.. | P_Re... | Beta | SDmpn | Pmpn... | Prin (... | P_M... | P_BLW | Pc | P_total | Ma |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0020 | 0.0072 | 1.5072 | 0.2168 | 1724 | 1357 | 24.2900 | 29.8300 | 0.9463 | 0 | 0.4148 | 0 | 0 | 0.0098 | 0 | 0 | 0.1400 | 0.0300 | 0.7800 | 0.0900 | 2.4155 | 3 |
| 0.0114 | 0.0414 | 1.5414 | 1.2390 | 301 | 237 | 24.3700 | 29.8900 | 0.9515 | 0 | 0.4179 | 0 | 4.821... | 0.0562 | 0 | 0 | 0.1400 | 0.0300 | 0.7900 | 0.0900 | 2.4599 | 3 |
| 0.0171 | 0.0621 | 1.5621 | 1.8585 | 201 | 158 | 24.4700 | 29.9800 | 0.9583 | 0 | 0.4221 | 0 | 0 | 0.0843 | 0 | 0 | 0.1400 | 0.0300 | 0.8000 | 0.0900 | 2.4986 | 3 |
| 0.0229 | 0.0828 | 1.5828 | 2.4780 | 150 | 118 | 24.6200 | 30.1000 | 0.9677 | 0 | 0.4277 | 0 | 0 | 0.1124 | 0 | 0 | 0.1400 | 0.0300 | 0.8100 | 0.0900 | 2.5457 | 3 |
| 0.0286 | 0.1035 | 1.6035 | 3.0975 | 120 | 95 | 24.8000 | 30.2500 | 0.9799 | 0 | 0.4350 | 0 | 4.821.. | 0.1405 | 0 | 0 | 0.1500 | 0.0200 | 0.8200 | 0.0900 | 2.6022 | 3 |
| 0.0343 | 0.1242 | 1.6242 | 3.7170 | 100 | 79 | 25.0300 | 30.4300 | 0.9946 | 0 | 0.4439 | 0 | 4.821.. | 0.1687 | 0 | 0 | 0.1500 | 0.0200 | 0.8400 | 0.0900 | 2.6679 | 3 |
| 0.0400 | 0.1449 | 1.6449 | 4.3365 | 86 | 67 | 25.2900 | 30.6500 | 1.0124 | 0 | 0.4544 | 0 | 0 | 0.1968 | 0 | 0 | 0.1500 | 0.0200 | 0.8600 | 0.1000 | 2.7442 | 3 |




TDECQ 17.9






## Summary \& Conclusions

- We described Panduit's link model for PAM-4 signals over multimode fiber
- Statistical eye amplitude method
- A general description of the functionalities and a demonstration shown during the presentation
- Plot of histograms, eye diagrams and BER
- Tested response time for any change in input parameters

- Model details of the model included in annexes:
- Fast \& accurate method to compute the eye diagram histograms
- Signal dependent noise treatment
- Computation Method for Margin and Penalties
- The model was implemented in Matlab GUI (compiled)
- GUI implemented
- Compiles, (no license required by user)

- Modular


## - Penalties (currently being reviewed such as MN) could be updated

- Could be available in IEEE website if solicited.


## References

## PANDUIT

- Previous work for 1Gbps and 10Gbps using NRZ link models
- Del Hanson, David Cunningham, Piers Dawe and David Dolfi (for 10G)
- Prior works for equalized channels :
- D. Cunningham proposed a 3-tap equalizer for PI-6 (12-044v1, 12-123v0)
- However, required several sheets (one per link length) and valid only for NRZ
- PAM-4 power budget penalties require more sophisticated equations than NRZ
- Equalization taps need to be efficiently computed for each length in one sheet
- In Fibre Channel, PAM-4 has been modeled using additional software packages
- For Python languages 16-013 v0, 16-012v0 (Richard Johnson)
- For Matlab 15-263v0
- An Excel VBA was proposed in T11-2016-065v0
- Fully implemented PI-6P (32GFC NRZ)
- Investigation of $60 \mathrm{~Gb} / \mathrm{s} 4-\mathrm{PAM}$ Using an 850 nm VCSEL and Multimode Fiber
- Journal of Lightwave Tech. Vol 34 (16) , April 2016
- Optical Link Model for PAM-4
- castro_3cm_01_0518

Backup

## PanduIt

## Annex I Direct Method to compute the Eye PDF from the input (ideal) PDFs and from the channel response

## Description

- The method shown in this annex provides an very accurate and fast computation of the eye diagram amplitude histogram.
- It only needs the input distribution, i.e. for PAM-4 4 delta functions with $25 \%$ probability each, and the impulse response of the channel.
- It can include the deterministic jitter of the channel
- The proposed method provides a general solution that does not need the simulation of the channel (sending billion of symbols). Therefore, is significantly faster and used in some parts of the presented link models.
Examples, and more details of the operation in shown in next slides.


## Eye Diagram Statistics

- For a time-invariant channel the PDF of the eye at a specific sampling time can be obtained using the statistical properties of the transmitted data (PDF) and the impulse response of the channel.
- The resultant PDF using this method is equivalent of sending a pseudo random sequence of "infinite length"
- A very simple example to illustrate this method is shown in the following 3 steps

1. A PAM-4 signal in a channel with infinite bandwidth
mpulse
Response


## Eye Diagram Statistics

1. A PAM-4 signal in a channel with infinite bandwidth (previous slide)
2. A PAM-4 signal with a channel with response $1+0.1 z$


The input PFD is modified by the adjacent pulse that add $0,0.1,0.2$ or 0.3 each $25 \%$ of the time.

## Eye Diagram Statistics

- A very simple example to illustrate the method is shown

1. A PAM-4 signal in a channel with infinite bandwidth
2. A PAM-4 signal with a channel with response $1+0.1 z$
3. A PAM-4 signal with a channel with response $1+0.1\left(z+z^{-1}\right)$


- The input PFD is modified by the adjacent pulse to the left that adds $0,0.1,0.2$ or 0.3 each $25 \%$ of the time and by the adjacent pulse to the right that adds $0,0.1,0.2$ or 0.3 each $25 \%$ of the time.
- What is the final PDF?
- The modification to the PDF can be obtained by convolving the effects from the left and right side. In some way, equivalent to launching two four-sided dies (tetrahedral die) each face marked with 0,0.1,0.2,0.3.


## Eye Diagram Statistics

- The received optical signal is given by,

$$
y\left(k T_{p}\right)=Y_{k}=\sum_{i} X_{i} h_{e}\left([k-i] T_{p}+J T_{p}\right)=h_{0} X_{k}+\sum_{i>k} X_{i} h_{i-k}+\sum_{i<k} X_{i} h_{-(k-i)}
$$

where $J$ is the effective jitter

- The Probability Density function (PDF) of Y can be obtained using

$$
P D F_{\text {OUTPUT }}\left(Y_{k}\right)=P D F\left(h_{0} X_{k}+\sum_{i>k} X_{i} h_{i-k}+\sum_{i<k} X_{i} h_{k-i}\right)
$$

- We can define the PDF of $X$ as: $\Omega_{X}=P D F_{I N P U T}(X)$
- Assuming, equiprobable symbols, the PDF for PAM-2 and PAM-4 are shown below,




## Eye Diagram Statistics: General method

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- The PDF of y can be obtained using:
$P D F_{\text {OUTPUT }}(Y)=h_{0} \Omega_{X} \otimes h_{1} \Omega_{X} \otimes h_{-1} \Omega_{X} \otimes h_{2} \Omega_{X} \otimes h_{-2} \Omega_{X} \otimes h_{3} \Omega_{X} \otimes h_{-3} \Omega \otimes \ldots \otimes h_{7} \Omega_{X} \otimes h_{-7} \Omega_{X}$


Where $\otimes$ represents the convolution operator

$$
\Omega_{X}=P D F_{\text {INPUT }}(X)
$$

and $h_{0}, h_{1}, h_{-1}, h_{2}, h_{-2}, \ldots$ represents the impulse response of the channel sampled using the symbol period.

- The exact PDF of the eye diagram at a specific sampling time can be obtained without the need of sending a pseudorandom sequence, we just need to convolve scaled versions of the input signal PDF.
- The scaling factor is the value of the impulse response at $T, \pm T, \pm 2 T$
- This method works also when dual-Dirac jitter is present.


## Examples Eye Diagram Statistics: No Jitter

- Using this method the PDFs for the received signal at the decision time can be computed as shown in the examples below.





## Example Eye Diagram Statistics: dual-Dirac Jitter

- Using this method the PDFs for the received signal at the decision time can be computed as shown in the examples below.


Annex II
Treatment of signal dependent noises example for RIN

## Example of signal dependent noises $\mathrm{RIN}_{\text {ОмА }}$

- IEEE 802.3ae (clause 52.9.6) defines RIN $_{\text {OMA }}$ as the ratio of the electrically noise power normalized to a 1 Hz bandwidth and the electrical power of a square wave modulation
- The RIN ${ }_{\text {OMA }}$ (outer) can be measured using a scope and a square wave pattern (i.e., 8 zeros and 8 threes as indicated in 802.3 bs Table 139-9). The equations to relate the measured RIN parameters to our PAM-4 equalized optical link will be shown in this section.
- The normalized variance due to RIN can be obtained using

$$
\begin{equation*}
\sigma_{R I N-O M A}^{2}=\frac{n_{0}^{2}+n_{3}^{2}}{2(\Delta / 2)^{2} \rho_{I S I-J}^{2}}=\frac{\mathrm{K}_{R I N} 10^{-R I N / 10} 10^{6}}{\sqrt{\left.0.477 / B W_{R X}^{2}\right)}} \tag{D.1}
\end{equation*}
$$

- Where $3 \Delta$ is the optical modulation amplitude without any dispersion penalties, $\mathrm{K}_{\mathrm{RIN}}=0.7$, $\mathrm{BW}_{\mathrm{Rx}}$ is the bandwidth of the receiver, and $\rho_{T_{e}}=A_{i}\left(J T_{p}\right)$ is the reduction in the optical signal attributed only to fiber and laser ( $\mathrm{T}_{\mathrm{e}}$ ).


## RIN ${ }_{\text {OMA }}$ without eye compression

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The RIN variances (no-normalized) for each signal level is derived

$$
\begin{align*}
& n_{0}{ }^{2}=\rho_{I S I-J}{ }^{2} \frac{2(\Delta / 2)^{2}}{1+E R^{2}} \sigma_{R I N-O M A}{ }^{2}=\left(\frac{\Delta}{2} \sigma_{0}\right)^{2}  \tag{A.1}\\
& n_{1}^{2}=\rho_{I S I-J}^{2} 2(\Delta / 2)^{2} \frac{(E R+2)^{2}}{9\left(1+E R^{2}\right)} \sigma_{\text {RIN-OMA }}{ }^{2}=\left(\frac{\Delta}{2} \sigma_{1}\right)^{2}  \tag{A.2}\\
& n_{2}{ }^{2}=\rho_{I S I-J}{ }^{2} 2(\Delta / 2)^{2} \frac{(2 E R+1)^{2}}{9\left(1+E R^{2}\right)} \sigma_{\text {RIN-OMA }}{ }^{2}=\left(\frac{\Delta}{2} \sigma_{2}\right)^{2}  \tag{A.3}\\
& n_{3}^{2}=\rho_{I S I-J}^{2} \frac{2(\Delta / 2)^{2}}{1+E R^{2}} E R^{2} \sigma_{R I N-O M A}{ }^{2}=\left(\frac{\Delta}{2} \sigma_{3}\right)^{2}  \tag{A.4}\\
& \Delta=O M A_{T_{X} \text { _outer_linear }} 10^{- \text {Losses/10 }}  \tag{A.5}\\
& \text { Losses }=\text { Connector_Losses }+ \text { Att Length }  \tag{A.6}\\
& \text { Losses }=\text { Connector _Losses }+ \text { Att Length }
\end{align*}
$$

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## Annex III <br> Computation Method for Margin and Penalties

## Computation of Margin \& Penalties PAM-2

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- PAM-2 is used to explain the model ( $\mathrm{M}=2$ to facilitate the description)
- The Q factor for a $y$ value in the histogram as a function of the $\mathrm{OMA}_{\text {tx_linear }}$ and transmission penalties $\mathrm{P}_{\mathrm{z}}$ is given by,

$$
\begin{equation*}
Q_{a}\left(y, P_{Z}\right)=\Delta \frac{y-V_{\text {threshold }}}{\sqrt{\left(P_{Z} n_{\mathrm{R}}\right)^{2}+\left(n_{S}\right)^{2}}}=\Delta \frac{y-V_{\text {threshold }}}{\sqrt{\left(P_{Z} n_{\mathrm{R}}\right)^{2}+\left(\frac{\Delta}{2} \sigma_{S}\right)^{2}}} \tag{B.}
\end{equation*}
$$

- Where $y$ is the normalized received signal, $\mathrm{V}_{\text {threshold }}$ is the decision threshold for the eye, $\Delta=O M A_{T x_{-} \text {outer_linear } 10^{\text {-Losses } 10} \text { is }}$ the OMA before the O/E, $n_{s}$ is the signal (level) dependent noise (RIN, MPN, MN, baseline wander) for 0's or 1's, $Q_{t}$ is the Q factor for a desired BER, $P_{z}$ represents how much the AWGN can be increased from current values and still achieve the $Q_{t}$ (more on $Q_{t}$ in next slide)
- The AWGN, $n_{R}$, is given by,


PDF $(y, 0)$

$$
\begin{equation*}
n_{R}=\frac{1}{M-1} \frac{\text { OMA }_{R x_{-} \text {outer_linear }}}{2 Q_{T}}=\frac{10^{\text {Receiver_Sens-SECQ }_{T x}}}{2 Q_{T}} \tag{B. 2}
\end{equation*}
$$

## Computation of Margin \& Penalties

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- The allowable power that can be used to mitigate for transmission penalties is the remaining power budget after connector losses and attenuation as shown,

$$
\begin{equation*}
P_{B-L}=\frac{O M A_{\text {outer_linear }}}{O M A_{R x_{-} \text {outer_linear }}} L_{\text {con }} 10^{- \text {att } L / 10}=\frac{\Delta}{O M A_{R x_{-} \text {outer_linear }}} \tag{B.}
\end{equation*}
$$

- Using B. 2 to B. 3 in B.1, we obtain the $Q$ factor for each trace in the sampled eye

$$
\begin{equation*}
Q_{a}\left(y, P_{Z}\right)=\frac{y-V_{\text {threshold }}}{\sqrt{\left(P_{z} \frac{O M A_{R x_{-} \text {outer_linear }}}{2 Q_{T} \Delta}\right)^{2}+\left(0.5 \sigma_{S}\right)^{2}}}=\frac{y-V_{\text {threshold }}}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{1}{2 Q_{T}}\right)^{2}+\left(0.5 \sigma_{S}\right)^{2}}} \tag{B. 4}
\end{equation*}
$$

- The BER is obtained by integrating the normalized PDFs for each level, 0 and 1 as shown,

$$
\begin{equation*}
\operatorname{BER}\left(P_{z}\right)=0.5\left[\int \operatorname{PDF}(y, 0) \operatorname{erfc}\left(Q_{a_{-} 01}\left(y, P_{z}\right) / \sqrt{2}\right) d y+\int \operatorname{PDF}(y, 1) \operatorname{erfc}\left(Q_{a_{-} 10}\left(y, P_{z}\right) / \sqrt{2}\right) d y\right] \tag{B. 5}
\end{equation*}
$$

- where

$$
\int P D F(y, 0) d y=\int P D F(y, 1) d y=0.5
$$

## Computation of Margin \& Penalties

- At a given length, the actual BER of the system is obtaining by using all the power budget (minus losses). This is equivalent of setting $P_{z}=1(0 \mathrm{~dB})$

$$
\begin{equation*}
B E R_{\text {Link }}=B E R\left(P_{z}=1\right) \tag{B. 6}
\end{equation*}
$$

- The Margin are obtained by solving B. 5 (numerically) to obtain a BER that match the BER targeted.

$$
\begin{equation*}
B E R\left(P_{z}=M \arg \mathrm{in}\right)=B E R_{\text {target }}=0.5 \operatorname{erfc}\left(Q_{T} / \sqrt{2}\right) \tag{B. 7}
\end{equation*}
$$

- The total penalties are obtained using,

$$
\begin{equation*}
\text { Penalties }=P_{B-L} / M \text { arg in } \quad \rightarrow \quad P_{\text {Budget_dB }}-\text { Losses }-M \text { arg in_dB } \tag{B. 8}
\end{equation*}
$$

- What is the meaning of Margin and Penalties in this model?
- If there are no penalties (ISI, JITTER RIN, MPN... ), the margin should be equal to the power budget minus losses. This is the ideal system with TDECQ=1 (0dB).
- If the margin =0 all the budget (minus) losses is used to compensate for ISI, Jitter, RIN, MPN, and MN penalties


## Computation of individual Penalties

- The methods to compute ISI, Jitter, RIN, MPN, MN , BL are given below
- For the ISI penalty, the jitter and all signal dependent noises are set to zero and $P_{z}$ is numerically found. $\mathrm{P}_{\mathrm{ISI}}=\mathrm{P}_{\mathrm{B}-\mathrm{L}}-\mathrm{P}_{\mathrm{z}}$ (in dB);
- For Jitter penalty, all signal dependent noises are set to zero and $P_{z}$ is numerically found. $P_{|S|-\text {-itter }}=P_{B-L}-P_{z}$ (in dB); $P_{j i t t e r}=-P_{|S|}+P_{|I|-\text {-jiter }}$
- For RIN penalty all signal dependent noises with exception of RIN are set to zero and $P_{z}$ is numerically found. $P_{\text {RIN }}=P_{B-L}-P_{z}-P_{\text {III-Iiter }}$ (in dB)
- For MPN penalty all signal dependent noises with exception of MPN are set to zero, $P_{z}$ is numerically found. $P_{M P N}=P_{B-L}-P_{z}-P_{\text {ISI-Iiter }}$ (in dB)
- For MN penalty all signal dependent noises with exception of $M N$ are set to zero, $P_{z}$ is numerically found. $P_{M N}=P_{B-L}-P_{Z}-P_{\text {III-Jitter }}$ (in dB)
- For BL penalty all signal dependent noises with exception of $B L$ are set to zero, $P_{z}$ is numerically found. $P_{B L}=P_{B-L}-P_{Z}-P_{I S I-\text {-itter }}$ (in $d B$ )
- Cross term penalties is the difference between total penalty (previous slide) minus the sum of all the individual penalties (in dB).


## Computation of Margin \& Penalties

- Summary of the model and differences compared to the worst-case eye?
- The $Q$ factors are computed for each eye trace (at the center of the eye $+/$-Jitter).
- The BER for each trace is computed
- The total BER is the average of trace BER weighted using the PDFs of level 0 and 1
- The margins are computed numerically. The total penalty is equal to the power budget minus losses minus margin (in dB).
- The individual penalties are computed as shown in the previous slide
- Fast checking with previous link models
- The margin and penalties solution for the proposed method (statistical eye are mainly numerically). However, if we select the worst case eye trace, the numerical solutions and worst-case eye method are identical.
- For example if we use B. 4 and make $Q_{a}=Q_{t}$

$$
\begin{aligned}
& Q_{a}\left(y_{\text {worst_trace }}, P_{Z}\right)=Q_{T}=\frac{0.5}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{1}{2 Q_{T}}\right)^{2}+\left(0.5 \sigma_{S}\right)^{2}}} \\
& \text { Penalties_worst_case }- \text { eye }=\frac{P_{z}}{P_{B-L}}=\frac{1}{\sqrt{1-\left(Q_{T} \sigma_{S}\right)^{2}}}
\end{aligned}
$$

## Link Model Margin \& Penalties for PAM-4

The model is now applied for PAM-4

$$
Q_{01}\left(y, P_{z}\right)=\frac{y-\text { threshold }(0,1)}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{O M A_{\text {outer }}}{6 Q_{T}}\right)^{2}+\left(0.5 \sigma_{s}(0)\right)^{2}}}
$$

$$
Q_{10}\left(y, P_{z}\right)=\frac{y-\text { threshold }(0,1)}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{O M A_{\text {outer }}}{6 Q_{T}}+\left(0.5 \sigma_{s}(1)\right)^{2}\right.}}
$$

$$
Q_{23}\left(y, P_{z}\right)=\frac{y-\text { threshold }(2,3)}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{O M A_{\text {outer }}}{6 Q_{T}}\right)^{2}+\left(0.5 \sigma_{s}(2)\right)^{2}}}
$$

$$
Q_{32}\left(y, P_{z}\right)=\frac{y-\text { threshold }(2,3)}{\sqrt{\left(\frac{P_{z}}{P_{B-L}} \frac{O M A_{\text {outer }}}{6}\right)^{2}+\left(0.5 \sigma_{s}(3)\right)^{2}}}
$$

- The thresholds are computed numerically from the histograms. They are placed at the locations where the PDFs of each level intercept.
- The traditional equations for threshold does not apply here due to the arbitrary shape of the PDFs



## Link Model Margin \& Penalties

The $S E R$ is computed as follows,

$$
\begin{aligned}
& \operatorname{SER}\left(P_{z}\right)=0.5\left[\int \operatorname{PDF}(y, L 0) \operatorname{erfc}\left(Q_{01}\left(y, P_{z}\right) / \sqrt{2}\right) d y+\right. \\
& \quad \int P D F(y, L 1)\left\{\operatorname{erfc}\left(Q_{10}\left(y, P_{z}\right) / \sqrt{2}\right)+\operatorname{erfc}\left(Q_{12}\left(y, P_{z}\right) / \sqrt{2}\right)\right\} d y+ \\
& \quad \int P D F(y, L 2)\left\{\operatorname{erfc}\left(Q_{21}\left(y, P_{z}\right) / \sqrt{2}\right)+\operatorname{erfc}\left(Q_{23}\left(y, P_{z}\right) / \sqrt{2}\right)\right\} d y+ \\
& \left.\quad \int P D F(y, L 3) \operatorname{erfc}\left(Q_{32}\left(y, P_{z}\right) / \sqrt{2}\right) d y\right]
\end{aligned}
$$

Where the PDFs of the signal for each level satisfies

$$
\int P D F(y, L 0) d y=\int P D F(y, L 1) d y=\int P D F(y, L 2) d y=\int P D F(y, L 3) d y=0.25
$$

The BER is computed assuming Gray coding

$$
\operatorname{BER}\left(P_{z}\right)=0.5 \operatorname{SER}\left(P_{z}\right)
$$

## Link Model Margin \& Penalties

- The BER of the system is given by,

$$
B E R_{\text {Link }}=B E R\left(P_{z}=1\right)
$$

- The Margins are computed numerically by finding the $P_{z}$ value that makes the link BER equal to the target BER. Provide the margin

$$
B E R\left(P_{z}=M \arg \text { in }\right)=B E R_{\text {target }}=0.5 \operatorname{erfc}\left(Q_{T} / \sqrt{2}\right)
$$



- Therefore all the eyes are solved at once. There is only one margin value for the system instead of three margins for each eye!
- The penalty for all signal dependent noises is given by,

$$
\text { Penalties }=P_{\text {Budget }}-M \arg \text { in }
$$

- The individual ISI, Jitter, RIN... penalties are computed using the same methodology as previously shown for the PAM-2 case

Annex IV Relationship of Link Model with TDECQ

## TDECQ from link model margin derivation

The value of TDECQ can be computed as follows,
$T D E C Q=\frac{O M A_{\text {outer }}}{\sqrt{n_{G}{ }^{2}+n_{R}{ }^{2}} 6 Q_{T}}=\frac{O M A_{\text {outer }}}{n_{T} 6 Q_{T}}$
Where $n_{G}$ is the maximum noise amount that can be added (before equalizer) while maintaining the BER target, . Therefore,
$n_{T}=\sqrt{\left(\frac{O M A_{\text {outer }} M \arg i_{\text {TDECQ }}}{6 P_{B-L} Q_{T}}\right)^{2}-n_{R}{ }^{2}+n_{R}{ }^{2}}=\left(\frac{O M A_{\text {outer }} M \arg \operatorname{in}_{\text {TDECQ }}}{6 P_{B-L} Q_{T}}\right)$
$T D E C Q=\frac{O M A_{\text {outer }}}{\frac{M M A_{\text {outer }}}{6} \frac{M \arg \text { in } n_{\text {TDECQ }}}{P_{B-L} Q_{T}} 6 Q_{T}}=\frac{P_{B-L}}{M \arg \text { in }_{\text {TDECQ }}} \approx$ Penalties $_{{\text {At } 2 m, w i t h_{-} B W_{-} T D E C Q}}$

Notes:

- TDECQ in this computation only consider the noises at 2 m using the specified BW.
- Therefore is does not guaranty that longer reaches with more noise will work.
- Moreover, is a receiver with better sensitivity is used $n_{r}{ }^{\prime}=X n_{r}$, where $X<1$, the TDECQ is also should reduced by $X$

