Partial Response with Bounded Running Disparity

100 Mb/s Long-Reach Single Pair Ethernet Task Force

IEEE 802.3dg

Philip Curran
Brian Murray

Error Propagation

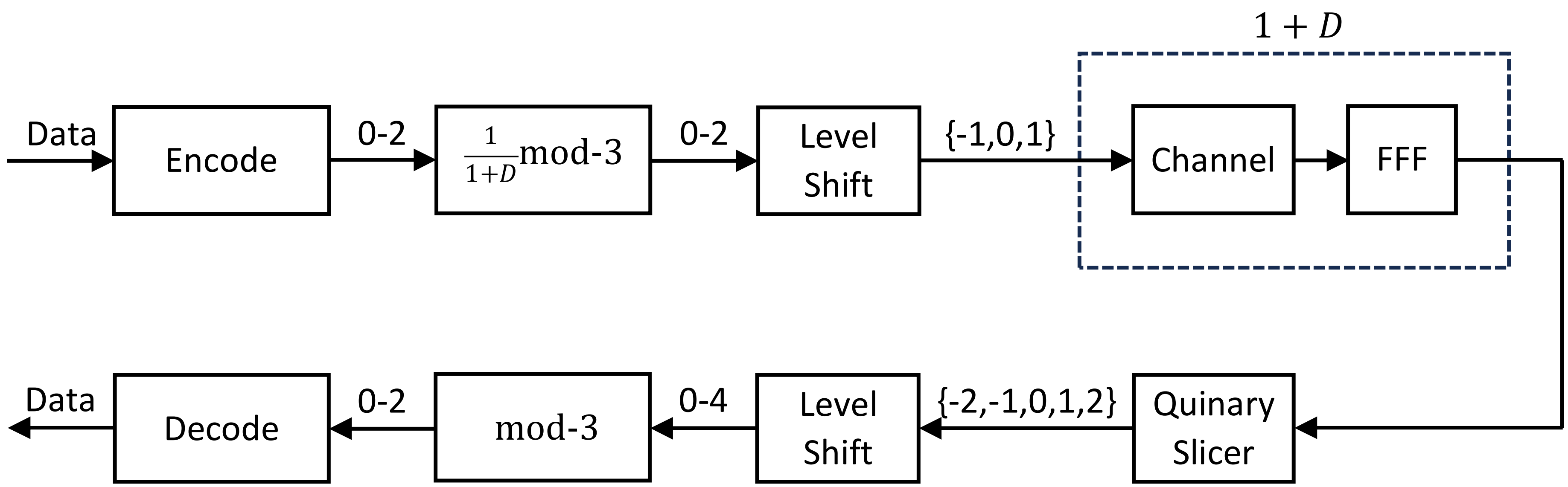
- At long lengths, the 1st coefficient of the DFE feedback filter (FBF) becomes large
  - https://www.ieee802.org/3/dg/public/May_2024/murray_3dg_01_05132024.pdf
- Without mitigation this will cause error propagation
  - For PAM-3 with 1st DFE FBF coefficient equal to 1
    - if the previous decision is incorrect then then the probability of getting another error is 2/3
    - the probability of k consecutive errors is $(2/3)^k$
  - If FEC is used, multiple symbols in a code-word may be corrupted by a single error event
  - Severe error propagation may corrupt consecutive frames
- Could limit the 1st DFE FBF coefficient
  - This increases noise enhancement

Partial Response with Precoding

- Can mitigate error propagation by using a partial response (PR) target for the equalizer
  - https://grouper.ieee.org/groups/802/3/100GCU/public/mar11/bliss_01_0311.pdf
  - https://www.ieee802.org/3/bj/public/sep11/parthasarathy_01_0911.pdf
  - https://www.ieee802.org/3/cd/public/May16/hegde_3cd_01a_0516.pdf
  - https://grouper.ieee.org/groups/802/3/ch/public/adhoc/souvignier_3ch_01_0818.pdf
- PR equalization normally requires a precoder
  - Without this, certain error sequences may propagate indefinitely

Conventional PR Equalization for PAM-3

- We are interested in 1+D PR
  - This matches the characteristics of our system at long lengths
  - The output of the DFE feed-forward filter (FFF) is quinary observed with additive noise
  - The detection process operates directly on the PR samples from the equalizer
  - Errors do not propagate because the detection process is memoryless

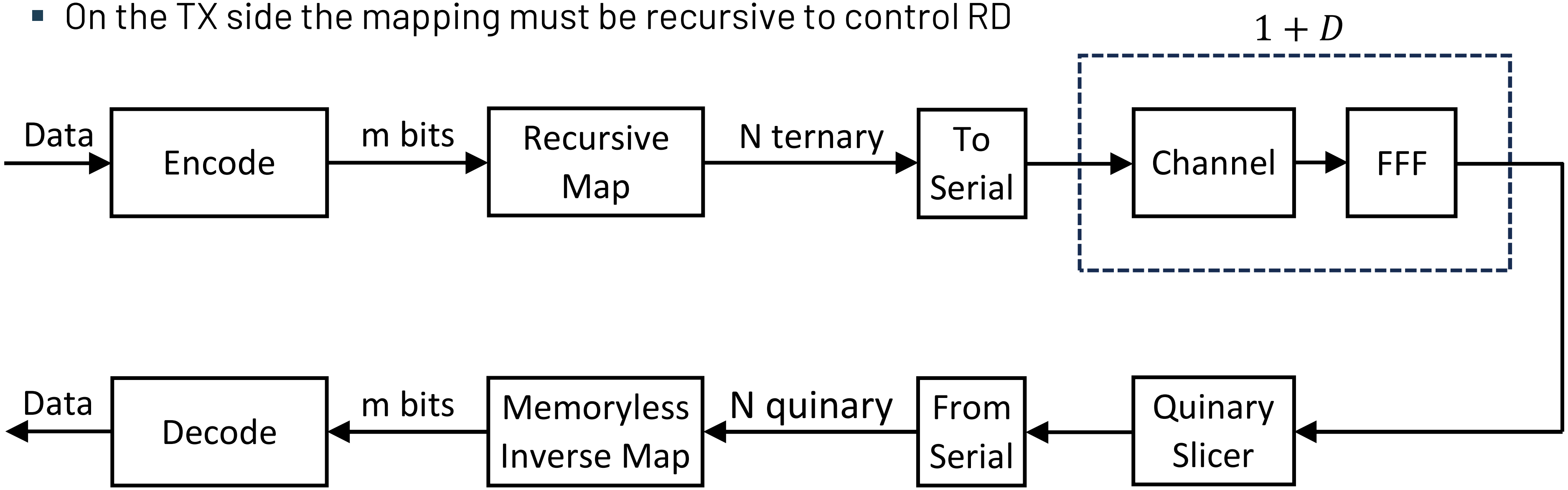


DC Balance

- One of the objectives of the task force relates to intrinsic safety
  - Do not preclude working within an Intrinsically Safe device and system …
- The 802.3cg (10BASE-T1L) task force had the same objective
  - Achieved by including a mechanism to control running disparity (RD) at the transmitter
- Running disparity control reduces droop
  - Beneficial in PoDL applications by allowing smaller power inductors to be used
- Combining precoding with RD control is problematic
  - Each can undo the benefits of the other

Alternative PR Equalization for PAM-3

- We propose to use a line code that maps m bits to N ternary values
  - On the TX side the mapping must be recursive to control RD



- We will show that it is possible to identify maps for which the inverse maps can operate on N quinary values without knowledge of what went before
  - Errors do not propagate because the detection process remains memoryless

Map Construction

- A suitable list of ternary N-tuples with non-negative disparity may be constructed as follows:
  1. Create a list containing all 3^N N-tuples
  2. Remove all N-tuples with negative disparity
  3. Remove all N-tuples whose last ternary element is 0
  4. If N is even, remove the 2 N-tuples of the following form:

     `(-1, +1, -1, +1, … , -1, +1)`

     `(+1, -1, +1, -1, … , +1, -1)`
  5. If N is odd, remove the N-tuple of the following form:

     `(+1, -1, +1, -1, … , +1, -1, +1)`
  6. Check that there are at least 2^m entries remaining
- We will refer to the N-tuples in this list as the NND (non-negative disparity) N-tuples
- Each of the 2^m possible values from the encoder is associated with 1 of the NND N-tuples

Control of Running Disparity

- We have constructed our list of NND N-tuples
  - Each N-tuple in this list with positive disparity has a complementary N-tuple with negative disparity that can be generated by negating it
  - Negating an N-tuple means negating each element
  - If RD is positive, and the m-bit value from the encoder is associated with an N-tuple with positive disparity, then the N-tuple should be negated before transmission
  - If RD is zero, and the m-bit value from the encoder is associated with an N-tuple with positive disparity, then a random Boolean value should determine whether to negate the N-tuple before transmission
  - RD is recomputed after transmission of each N-tuple

Summary of Transmit Process

- The encoder processes data from the MII and sends m-bit values to the mapping process
- Each m-bit value is converted to an NND N-tuple
- If the NND N-tuple has positive disparity it may be negated to control RD before it is presented to the serializer
- The serializer sends the N-tuple to the transmitter one element at a time
  - The leftmost element in each N-tuple is transmitted first
  - The rightmost element in each N-tuple is transmitted last and is never 0

List of Possible PR Sequences

- First let us create a list of what we will call balanced N-tuples
  - We start by copying the list of NND N-tuples
  - For each N-tuple with positive disparity, we append the negated value of that N-tuple to the list
  - The final list contains all N-tuples that may be presented to the serializer after implementing RD control
- Now we create a list of possible PR sequences corresponding to the balanced N-tuples
  - The entries in this list are also N-tuples, but the elements of these N-tuples are quinary
  - There are 2 entries in this list for each balanced N-tuple
    - One for each of the cases where the preceding transmitted ternary value is assumed to be -1 or +1
    - By construction, the preceding transmitted ternary value can never be 0

Characteristics of PR Sequences

- There are no duplicates in the list of possible PR sequences
  - Each allowed PR sequence corresponds to only one balanced ternary N-tuple
  - Each balanced ternary N-tuple corresponds to only one NND ternary N-tuple
  - Each NND ternary N-tuple corresponds to only one m-bit value from the encoder
- The inverse mapping from PR quinary N-tuples received from the deserializer to m-bit values to the decoder is memoryless
  - Errors do not propagate
- The minimum distance between any 2 entries in the list of possible PR sequences is $\sqrt{2}$
  - We will come back to the implications of this shortly

Application to 8b6T at 75 MBaud

- In this case the list of NND 6-tuples has the following properties
  - Total of 286 NND 6-tuples
  - 88 6-tuples have disparity 0
  - 81 6-tuples have disparity 1
  - 60 6-tuples have disparity 2
  - 35 6-tuples have disparity 3
  - 16 6-tuples have disparity 4
- In DATA we propose to associate each of the 256 8-bit values from the encoder with a 6-tuple having disparity not exceeding 3
  - The exact mapping will be covered separately
- In IDLE we propose to substitute 16 6-tuples with disparity 4 for 16 of the 6-tuples with disparity 3 that we use in DATA
  - The rationale for this and the exact mapping will be covered separately

Baseline SNR Assumptions

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- We would like to establish a baseline SNR requirement for 8b6T with PR equalization
  - For this we assume that we slice the equalizer outputs one by one using a quinary slicer
    - We will look at possible improvements later
  - To compare our PR equalizer to a conventional DFE we need a common reference for expressing noise power
    - With the proposed IDLE encoding we see a transmitted ternary symbol power of 0.7122
    - We will use this level as the reference for expressing the noise power in dB
  - We assume that the system noise can be represented by an AWGN signal, w , adding at the output of the equalizer
  - In Ethernet, the bit error ratio is normally inferred from the frame error ratio. We use the following equation for the probability of a bit error with 8b6T line coding

$$P_b^e = \frac{P_f^e}{N_b} \cong \frac{N_s}{N_b} P_s^e = 0.75 \times P_s^e$$

Here P_b^e and P_s^e represent the probabilities of a bit error and a symbol error respectively

Baseline SNR Requirement

- We would like to know the SNR requirement for 10^{-10} BER when using PR equalization
  - With the proposed DATA encoding we see the following probabilities for the 5 quinary levels at the output of the equalizer

$$\begin{aligned} P(-2) &= 0.1091 \\ P(-1) &= 0.2149 \\ P(0) &= 0.3520 \\ P(+1) &= 0.2149 \\ P(+2) &= 0.1091 \end{aligned}$$

  - The probability of an error at the quinary slicer is as follows

$$P_s^e = \big(1 + P(0) + 2 \times P(+1)\big) \times P(w > 0.5) = 1.7818 \times 0.5 \times \mathrm{erfc}\left(\frac{1}{2\sqrt{2}\sigma}\right)$$

  - We require the noise power to be below about -20.7dB with reference to the IDLE ternary symbol power

SNR Requirement without PR

- Let us also calculate the SNR requirement for 10^{-10} BER when using a conventional DFE
  - We use the same set of ternary 6-tuples but this time the output of the equalizer is a ternary value observed with additive noise
  - We assume that there is no error propagation. This assumption will not generally be true at long lengths.
  - With the proposed DATA encoding we see the following probabilities for the 3 ternary levels at the output of the equalizer

$$\begin{aligned} P(-1) &= 0.3509 \\ P(0) &= 0.2982 \\ P(+1) &= 0.3509 \end{aligned}$$

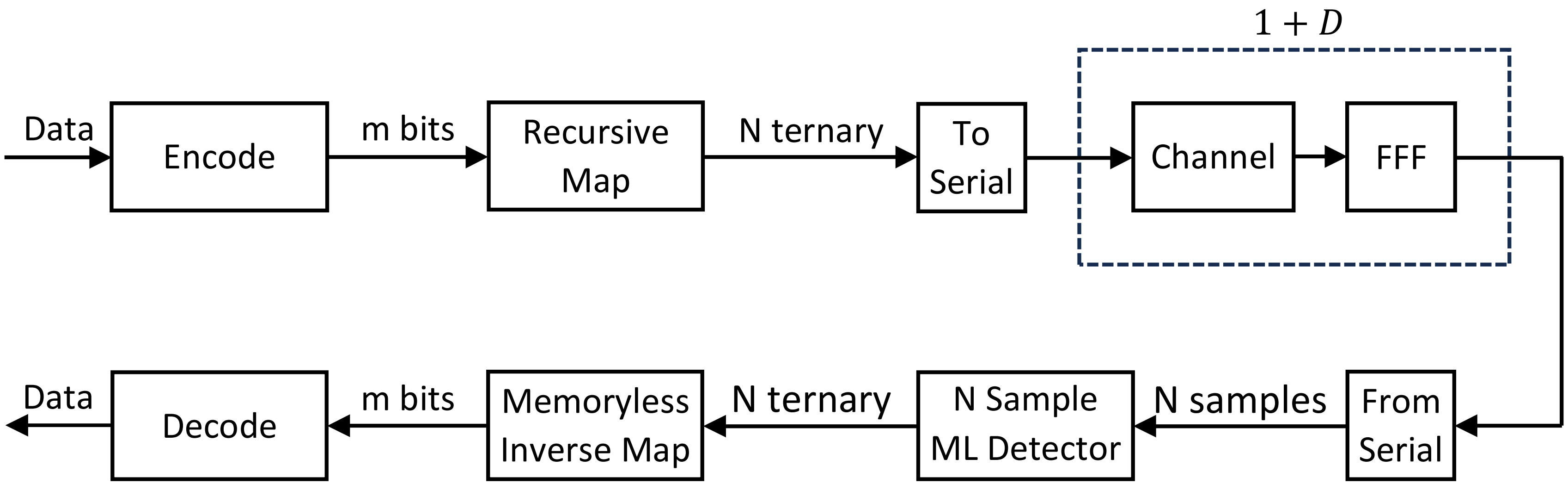
  - The probability of an error at the ternary slicer is as follows

$$P_s^e = \left(1 + P(0)\right) \times P(w > 0.5) = 1.2982 \times 0.5 \times \mathrm{erfc}\left(\frac{1}{2\sqrt{2}\sigma}\right)$$

  - We require the noise power to be below about -20.6dB with reference to the IDLE ternary symbol power
  - There is no real difference between the acceptable noise levels at the slicer in the cases with and without PR equalization

Performance Optimization

- When we transmit ternary N-tuples of the type proposed, the minimum distance between the associated PR sequences is $\sqrt{2}$
  - The receiver may exploit this to achieve an effective SNR gain of up to 3dB



Maximum Likelihood Detection

- The receiver may use a maximum likelihood (ML) detector operating on N PR samples at a time to determine the most likely ternary N-tuple
  - The detector can take advantage of the fact that the last ternary value in each N-tuple is non-zero
  - The ML detector may be reinitialized every N cycles of the symbol clock so that errors do not propagate from the detection of one N-tuple to the next
  - As ML detection may be formulated as a shortest-path problem, the complexity increases only linearly with N
  - Any ML detector operates by computing a measure of likelihood. In principle this may be used to flag erasures, if FEC is used.
- An effective SNR gain of about 2.8dB has been observed when simulating the proposed 8b6T line code
  - In this simulation the ML detector was reinitialized at the start of each 6-tuple and a decision was made at the end of each 6-tuple
  - There is no error propagation, and the latency is low

Additional DFE Feedback Coefficients

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- So far, we have assumed that the DFE FFF shapes the system response to the exact target response, 1+D
  - In practice we expect to see additional post-cursor inter-symbol interference (ISI) from older symbols (symbols with delay greater than 1)
  - This may be dealt with by having a DFE FBF where the 1st coefficient is set to 0
  - Limiting the remaining DFE FBF coefficients is not expected to cause excessive noise enhancement
  - Therefore, error propagation from older symbols is expected to be a lesser problem
- Any ML detector keeps several candidate transmitted symbol sequences under consideration
  - Subtracting the estimated ISI from older symbols separately for each such sequence is a well-known technique to mitigate error propagation

Summary

- We have shown how to generate line codes which have the following properties
  - The signal on the line is PAM-3
  - Running disparity can be controlled at the transmitter
  - 1+D partial response (PR) equalization can be used at long lengths to eliminate error propagation associated with the 1st DFE feedback coefficient
  - Each PHY may decide whether to implement 1+D PR equalization in the receiver without requiring the cooperation of its link partner
  - Maximum likelihood (ML) detection can be used to achieve close to 3dB of effective SNR gain in conjunction with 1+D PR equalization
  - The performance gain may be achieved even if the ML detector is reinitialized, and final decisions are made every N cycles of the symbol clock