Codeword Error Rate and (Hyper)-Spherical TDECQ

Addressing comments 1, 343, 345, 347 and 349 against P802.3dj D2.0

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AGENDA

1. TDECQ Challenges
2. TDECQ in the Current Draft (P802.3dj D2.0)
3. Codeword Error Rate TDECQ
4. A Geometric Perspective
5. Approximating CER TDECQ – Hyper-Spherical TDECQ
6. Experimental Data

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TDECQ Challenges

- Concerns are being raised around TDECQ's correlation to link performance has been raised amongst members of the IEEE 802.3dj task force.
- chayeb_3dj_01_2505 documented some of the TDECQ challenges and potential improvements.
- chayeb_3dj_01_2505 demonstrated experimental data showing lack of correlation between test methodology (TDECQ) and actual link performance (post FEC errors).
- chayeb_3dj_01_2505 and ghiasi_3dj_03_2501 proposed a Codeword Error Rate TDECQ calculated at a target CER (codeword error rate) instead of the current SER (symbol error rate) target.
- This presentation proposes a TDECQ implementation that captures correlation of errors due to effects that are present in the waveform in an economically feasible way.
- This presentation addresses comments ***1, 343, 345, 347*** and ***349*** against IEEE P802.3dj D2.0.

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TDECQ in the Current Draft (P802.3dj D2.0)

Goal of TDECQ: Find the maximum intrinsic receiver noise that still achieves the desired error performance (currently specified as SER).

The methodology specified in the standards:

- Combines samples from each sample location, into histograms which represent an estimated pdf of the signal levels at the sample location, $$f_{y_k}(y_k)$$
- Let $n \sim N(0, \sigma^2)$ be a normal gaussian, then the SER is a function of σ defined as:

$$SER(\sigma) = \sum_{l=0}^{l=3} \sum_{k} f_{y_k}(y_k) \left[Q\left(\frac{y_k - P_{th(l-1)}}{\sigma} \right) + Q\left(\frac{P_{th(l)} - y_k}{\sigma} \right) \right]$$

- The maximum σ is found such that $SER(\sigma) \leq SER_{target}$

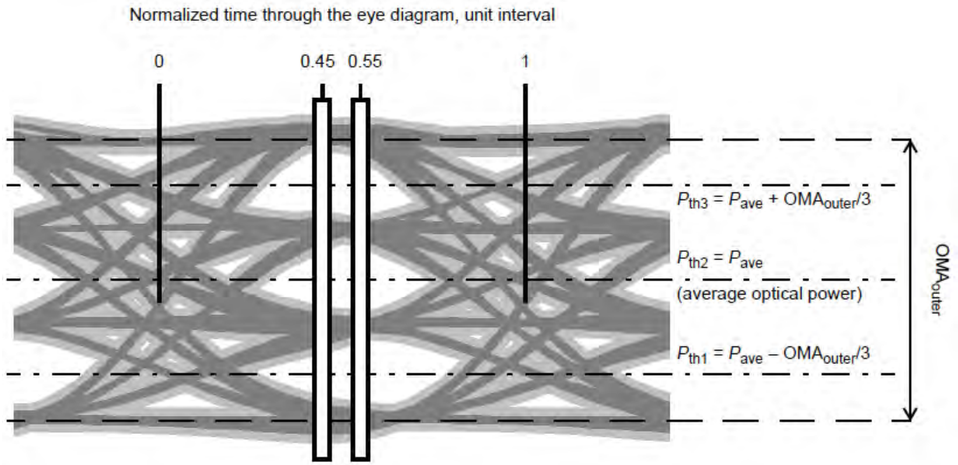

Figure 121–5—Illustration of the TDECQ measurement

IEEE 802.3bs – Clause 121.8.5.1

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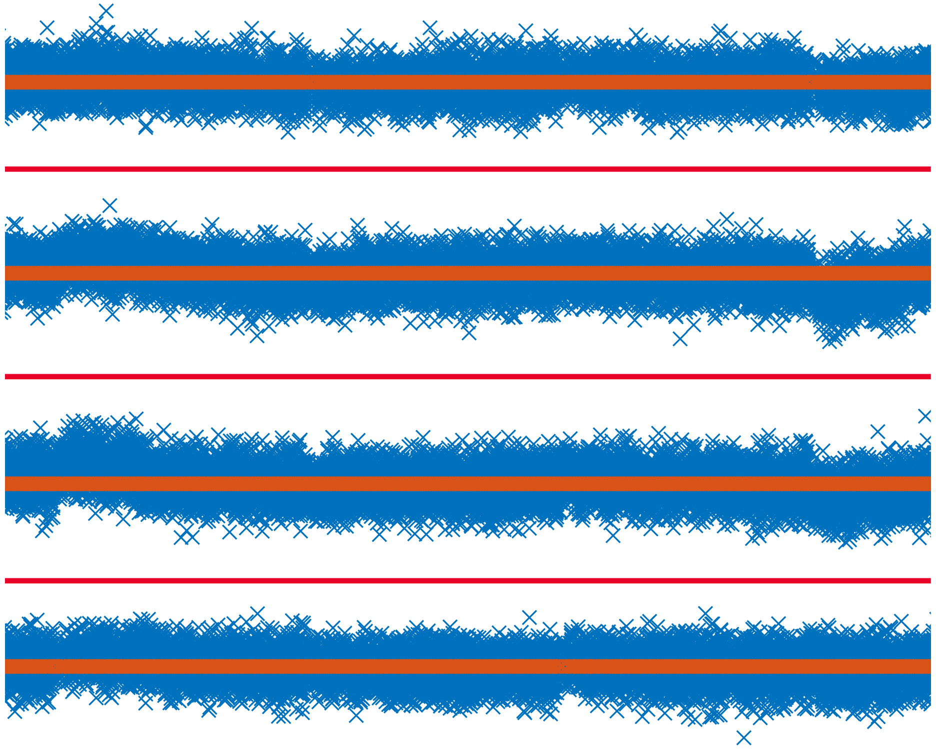
Codeword Error Rate TDECQ

Unwrapping the Histogram

- An alternative, but less efficient, methodology is to unwrap the level histograms as follows:

- Let:
  - i be the symbol index
  - x_i a sample from the target region of the symbol
  - l_i the symbol value (0,1,2,3)
  - N the number of symbols
  - $V(l_i)$ is the nominal level of the symbol
  - th_{l_i} is the optimal threshold below the l_i level

- The figure shows x_i in blue, $V(l_i)$ in orange, and th_{l_i} in red.



Codeword Errors

Unwrapping the Histogram

- We can define $SER(\sigma) = \frac{1}{N}\sum_{i=0}^{i=N-1}\left[Q\left(\frac{x_i - th_{l_i-1}}{\sigma}\right) + Q\left(\frac{th_{l_i} - x_i}{\sigma}\right)\right]$
- The above can be dissected to get a probability of error per symbol i assuming σ:
- $P_{err,i}(\sigma) = SER_i(\sigma) = \left[Q\left(\frac{x_i - th_{l_i-1}}{\sigma}\right) + Q\left(\frac{th_{l_i} - x_i}{\sigma}\right)\right]$
- $P_{err,i}(\sigma)$ is then the probability of error for the specific instance of x_i assuming receiver noise power σ
- Each x_i includes ISI, transmitter and scope noise.
- If symbols are organized into codewords of length d and we assume that the FEC can correct up to K errors a probability of codeword error for a given σ can be calculated using $P_{err,i}(\sigma)$

Codeword Errors

Probability of k Errors in a Codeword of length d, $C_{k,d}(\sigma)$

- For a set of d symbols, $P_{err,i}(\sigma)$ is the corresponding probability of error of the ith symbol
- The number of errors in the codeword follows a Poisson Binomial distribution
  - An extension of the Binomial distribution, where the probabilities of each trial are not identical
- This distribution, $C_{k,d}(\sigma)$, can be calculated by convolving the individual symbol PMF's together:
  - The ith symbol has the PMF: $p_i(k) = \begin{cases} 1 - P_{err,i}(\sigma) & k = 0 \\ P_{err,i}(\sigma) & k = 1 \end{cases}$, where 1 indicates an error
  - $C_{k,d}(\sigma) = p_0(k) * p_1(k) * \cdots * p_{d-1}(k)$
- Once the PMF is calculated, the probability of a correctable codeword is found by summing the first K entries of $C_{k,d}(\sigma)$
- And the probability that the jth codeword is in error is:

$$P_{fail,j}(\sigma) = 1 - \sum_{k=0}^{K-1} C_{k,d}(\sigma)$$

Codeword Errors (con't)

- A d symbol codeword can be made up of any d distinct symbols from the pattern
  - ➢ There will be $M = \left\lfloor \frac{N}{d} \right\rfloor$ available blocks
  - ➢ The Codeword Error Rate is then the mean of $P_{fail,j}(\sigma)$ over all blocks:

$$CER(\sigma) = \frac{1}{M} \sum_{j=0}^{M-1} P_{fail,j}(\sigma)$$

Codeword Error Rate Complexity

- The inner FEC codeword is 64 symbols long
  - An SSPRQ is $2^{16} - 1$ symbols
  - So, at least $M = 1023$ codewords need to be evaluated
  - Each codeword PMF requires 63 convolutions given a specific σ value
  - So, for each iteration using a σ value 64,449 convolutions are required

- The existing TDECQ method requires 1 convolution per σ value
  - There is additional computation to convert samples into histograms

- The complexity of the algorithm to exactly compute the Codeword Error Rate appears to be daunting

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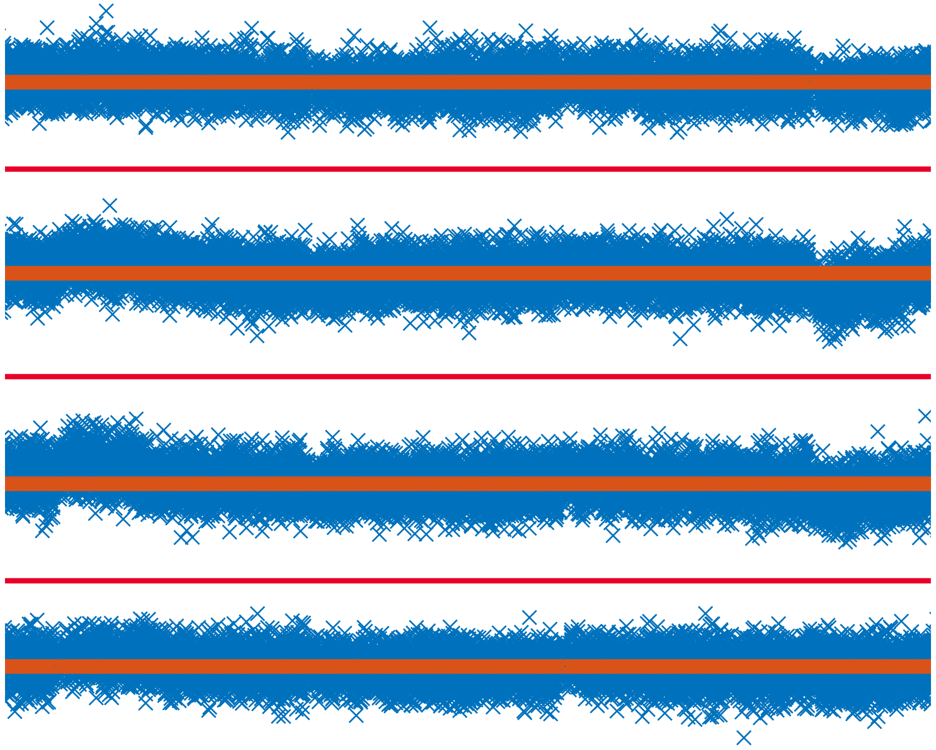
A Geometric Perspective

Treat codewords as vectors in a d-dimensional space

- Let:
  - i be the symbol index
  - x_i a sample from the target region of the symbol
  - l_i the symbol value (0,1,2,3)
  - N the number of symbols
  - $V(l_i)$ is the nominal level of the symbol
  - th_{l_i} is the optimal threshold below the l_i level

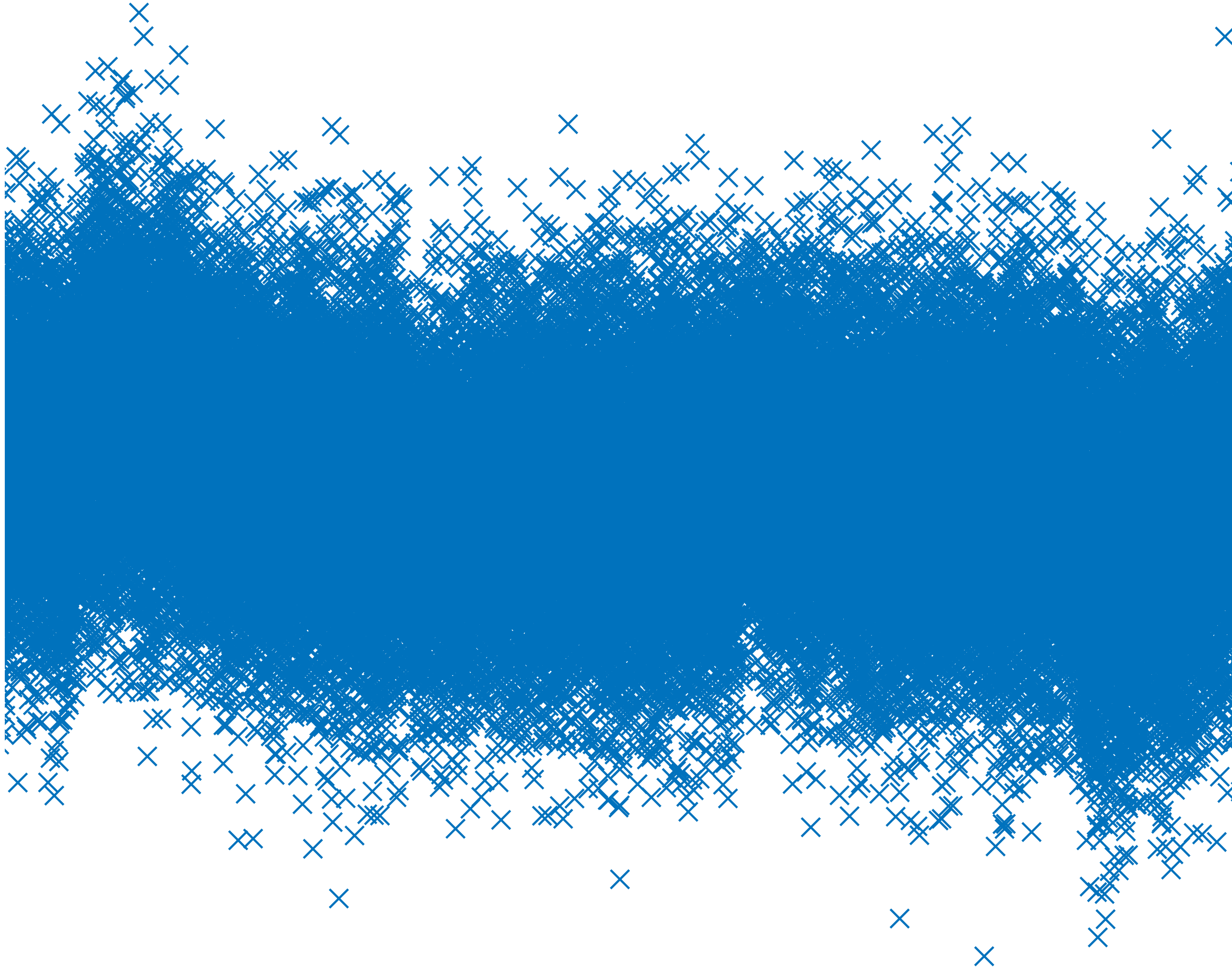
- The figure shows x_i in blue, $V(l_i)$ in orange, and th_{l_i} in red.



Defining the Residual Vector

Center analysis at the origin

- Define the residual, $\mu_i = x_i - V(l_i)$
- We can group the residuals to create a d-dimensional residual vector,
  - eg $\bar{\mu} = [\mu_0, \mu_8, \ldots \mu_{8(d-1)}]$
- The above example uses the interleaving of 8 symbol as specified in the standard for FECi.



A Geometric Perspective

The Noise Vector

- Let $\bar{n} \sim N(0, \Sigma)$ be a d-dimensional vector of the noise to be added, Σ is the covariance matrix of the added noise.
  - Assumed to be AWGN filtered by a 4th Order Bessel Thompson and any specified equalizers

- Combining the residual vector and noise vector results in the random variable $\bar{y} = \bar{\mu} + \bar{n}$
  - Where the residual is the mean and having a pdf, $f_{\bar{y}}(\bar{y}) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} e^{-\frac{1}{2}(\bar{y}-\bar{\mu})^T \Sigma^{-1} (\bar{y}-\bar{\mu})}$

- **Note:** If the data samples come from sufficiently separated UI and only linear equalizers are used then independence of noise samples can be assumed, i.e. $\Sigma \approx \sigma^2 \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix.

Probability of k Errors

- Can "easily" calculate the probability of 0 errors as:

$$P(0\ errors) = \int_R \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} e^{-\frac{1}{2}(\bar{y}-\bar{\mu})^T\Sigma^{-1}(\bar{y}-\bar{\mu})} d\bar{y}$$

  - where R is a d-dimensional region centered at 0, where each dimension is bound by the upper and lower threshold (or +/- infinity) for that symbol.
  - **NOTE: Each dimension has its own limit based on $\boldsymbol{l_i}$**
- For k>0, things are much more complicated
  - Need to take all regions R such that k dimensions are outside the thresholds and the other d-k dimensions are inside their respect thresholds.
  - If $\Sigma \approx \sigma^2\boldsymbol{I}$ this results in the same results as the previous section
- For simplicity of analysis, assume that all thresholds are $\pm\frac{OMA}{6}$ from the nominal symbol level
  - Now, all symbol errors can be though of as either too high or too low.

Probability of k Errors

Two-Dimensional Visualization

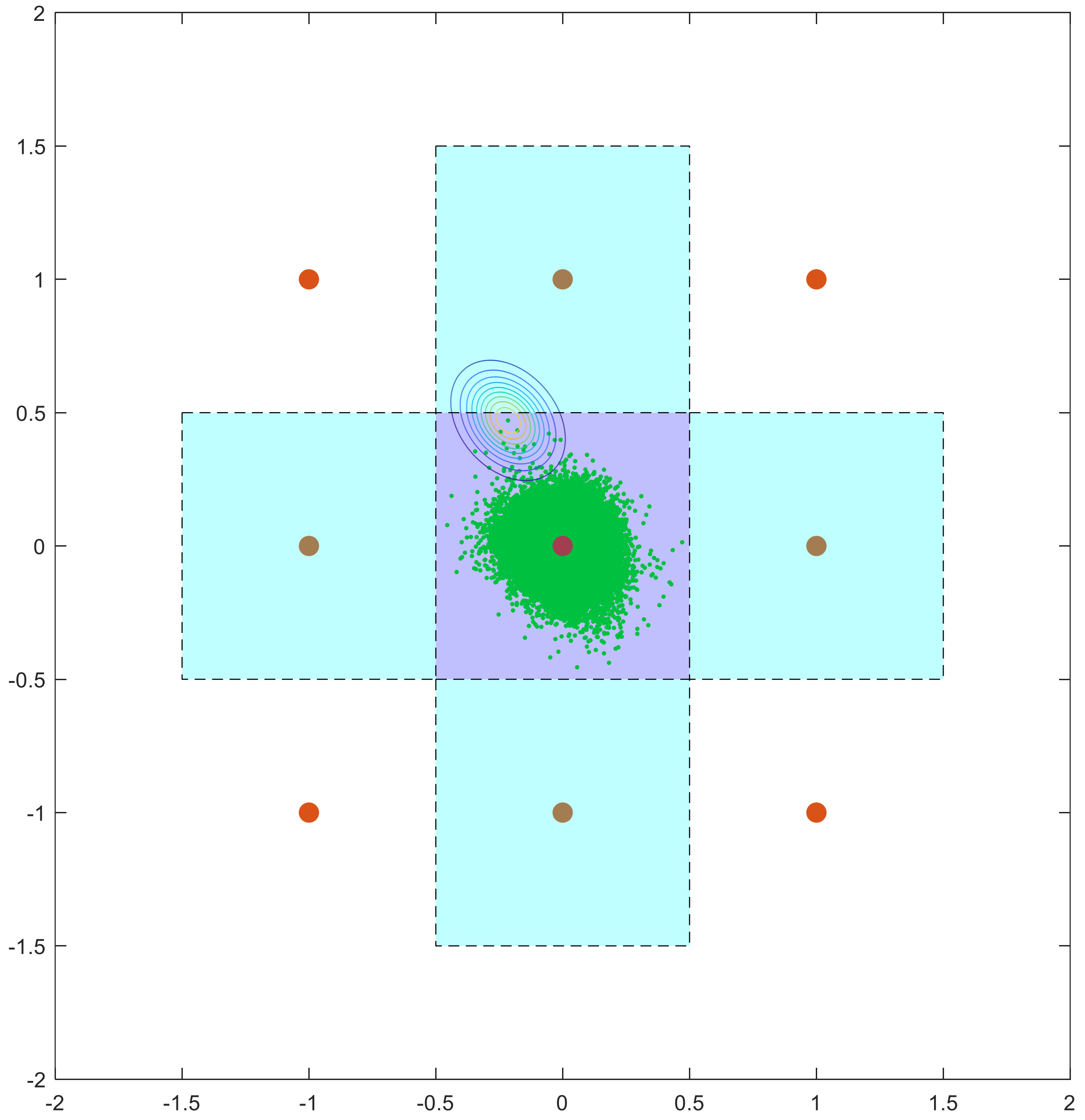
- The probability of exactly k errors is:

$$p_k = \sum_{c \in C_d(k)} \int_c \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} e^{-\frac{1}{2}(\bar{y}-\bar{\mu})^T \Sigma^{-1} (\bar{y}-\bar{\mu})} d\bar{y}$$

- Where $C_d(k)$ is a d-dimensional hypercube over the codeword space with exactly k symbol errors.

- Then the probability of k or more errors is

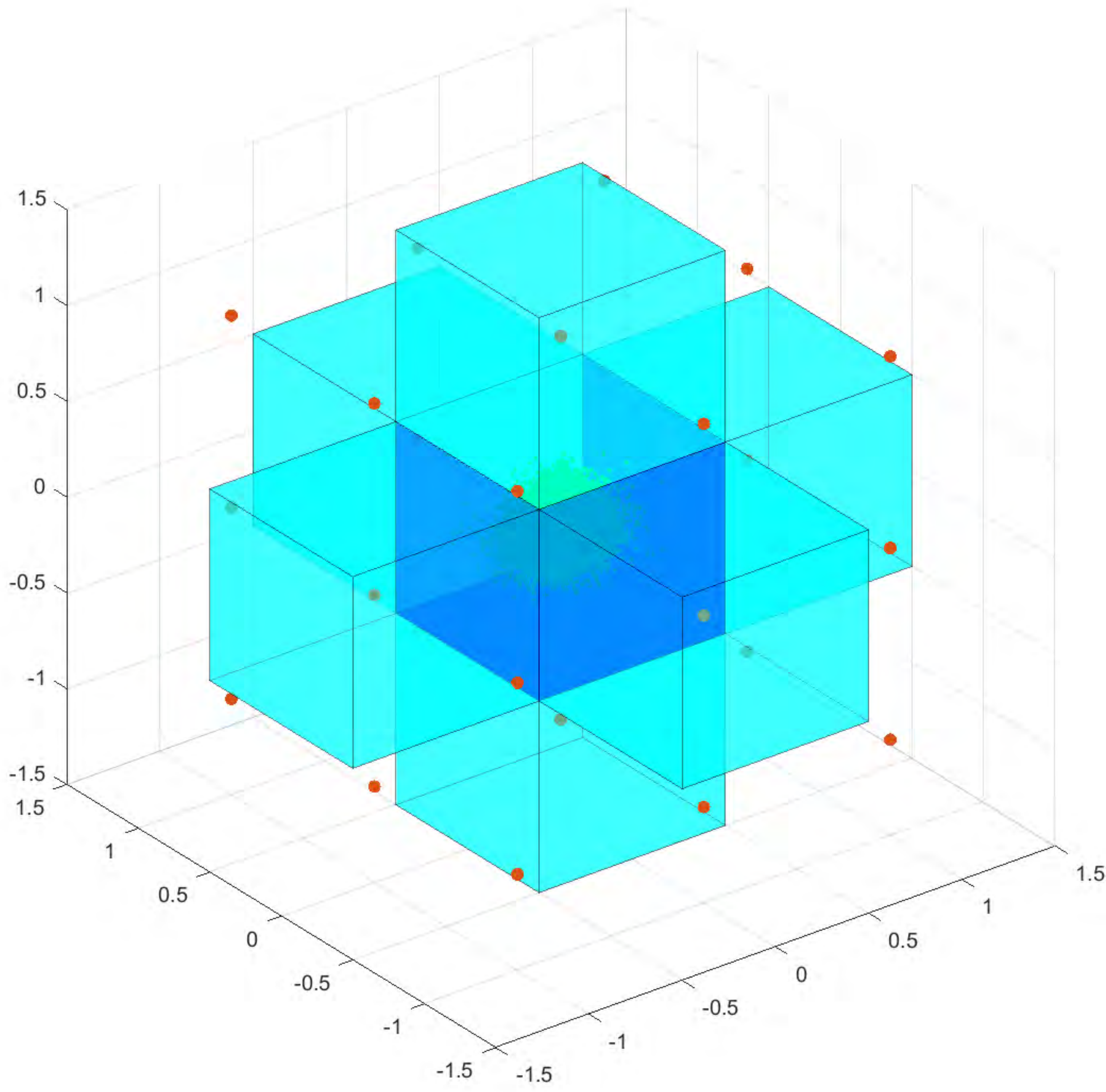
$$P_{fail} = 1 - \sum_{j=0}^{k-1} \mathrm{p_k}$$



Probability of k Errors

Three-Dimensional Visualization

- An example of our simplified model using just d=3 length codewords.
- The Red dots are the potential codewords
- Blue box indicates region of 0 errors per code word
- Cyan boxes indicate region with 1 error per code word
- Green dots are the residuals, $\bar{\mu}$, from data samples



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Regions with k Errors

Two-Dimensional Visualization

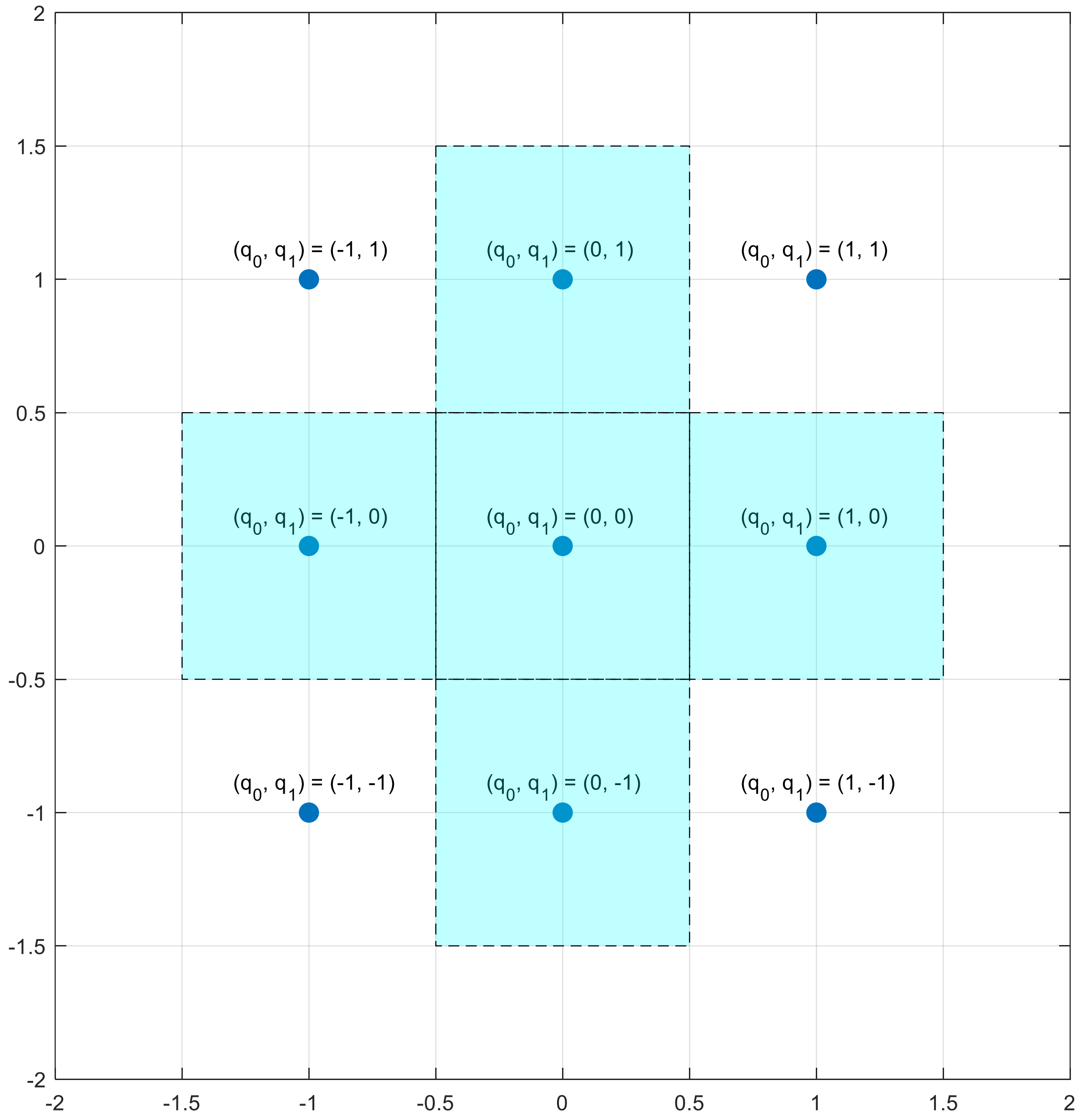
- Define a set of d-dimensional hypercubes, $C_d(k)$, all with sides of length $\frac{OMA}{3}$
- Each region is centered at the point

$$\frac{OMA}{3}[q_0, q_1, \ldots q_{d-1}], q_i \in \{-1,0,1\}$$

  where,

$$\sum_{i=0}^{d-1} |q_i| = k$$

- The number of such regions is $|C_d(k)| = 2^k \binom{d}{k}$
- 64 symbols and up to 3 errors results in over 333312 regions
- This appears to be an even more complex process



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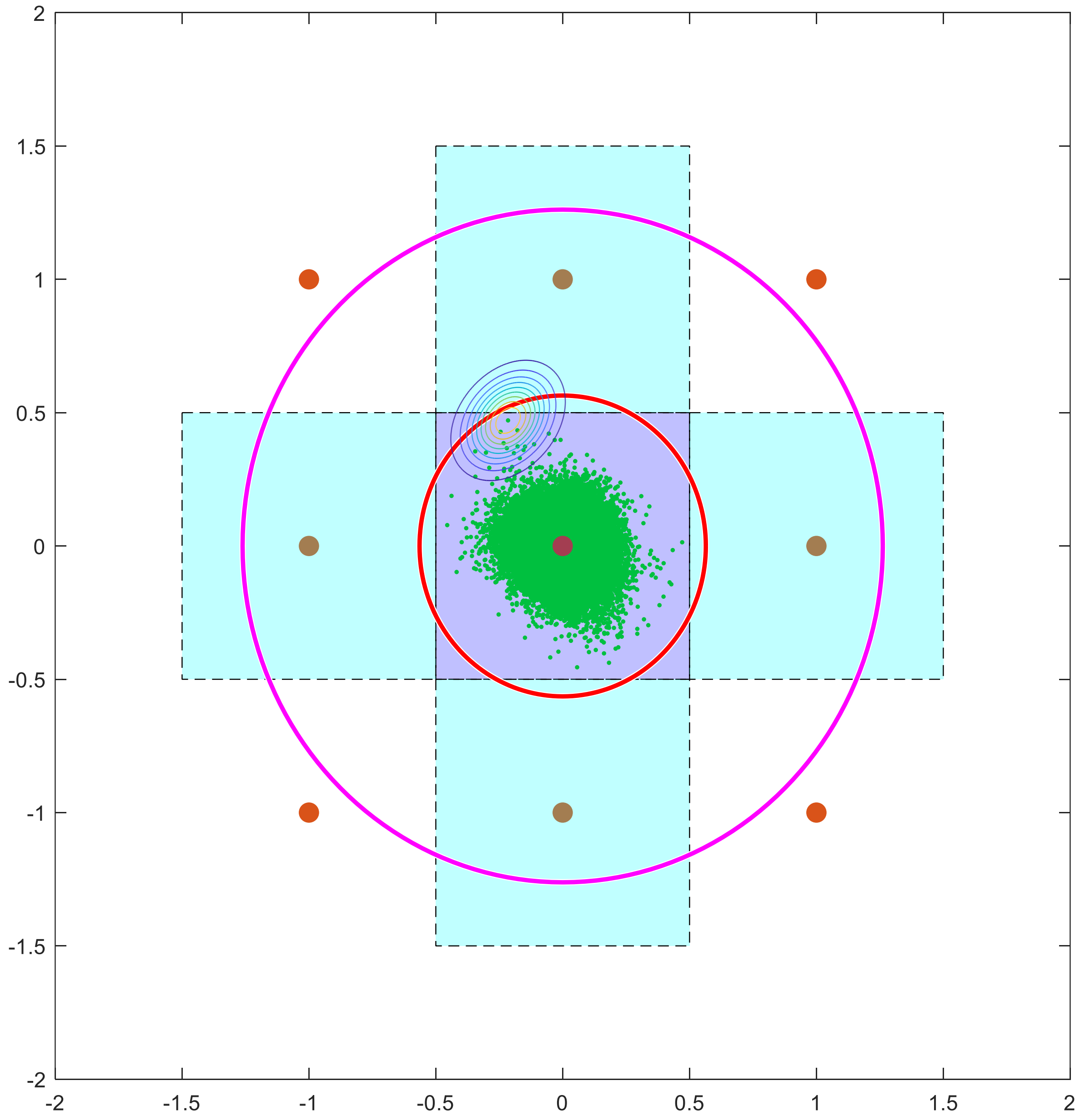
Can We Make it simpler?

Going Back to the Two-Dimensional Example

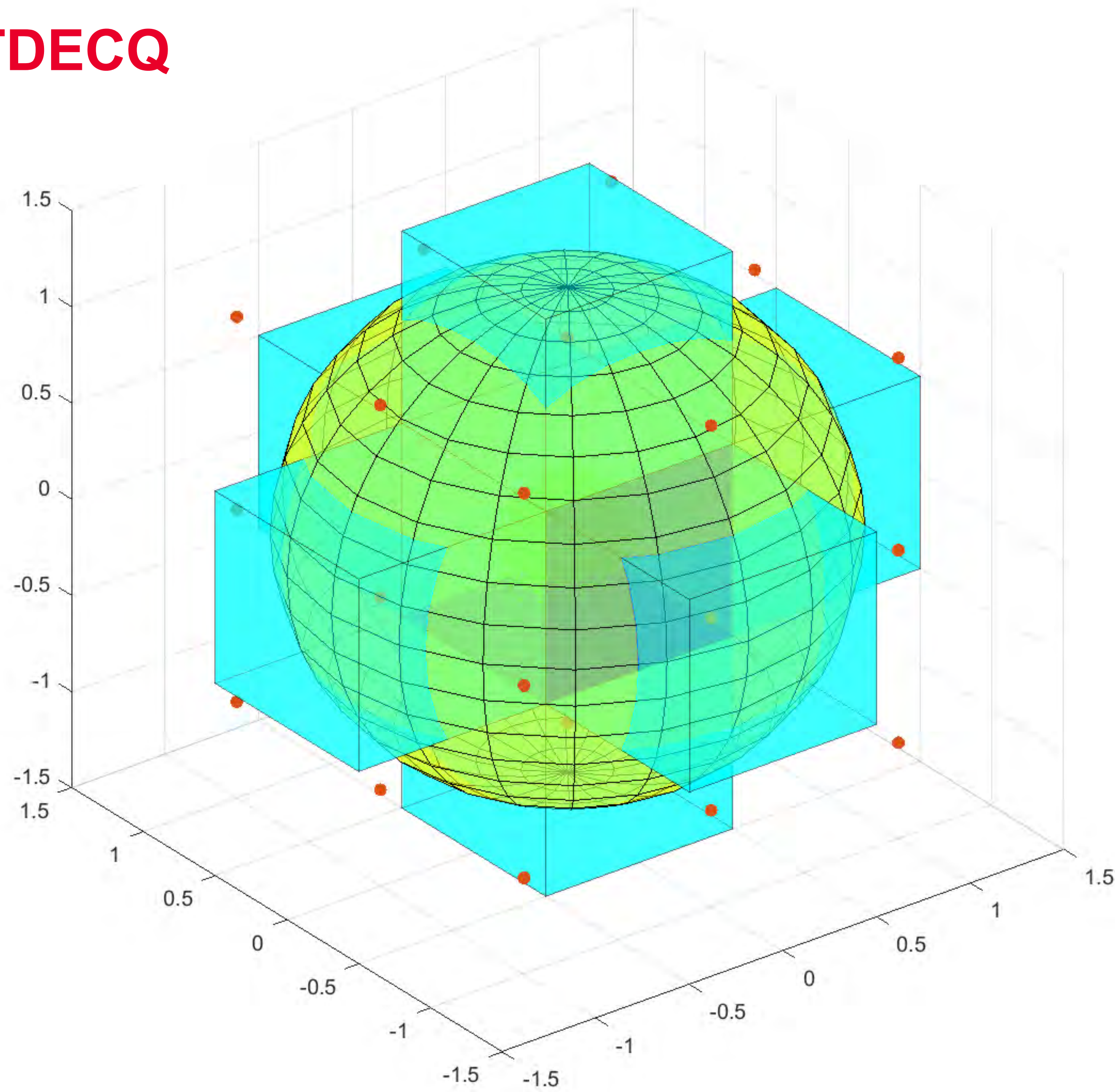
- Approximate the set of hypercubes as a single hypersphere
- Focus instead on the Euclidean distance from the ideal codeword location
- An estimate for the codeword error rate could be the probability that $\bar{y} = \bar{\mu} + \bar{n}$ is contained within a sphere of radius r_k centered at the ideal codeword location
- Essentially, CER $\approx 1 - \Pr\{\bar{y}^T \bar{y} \leq r_k^2\}$



Approximating Codeword Error Rate TDECQ

Reduction to a Singular Dimension

- Define a new variable: $r^2 = \frac{\bar{y}^T \bar{y}}{\sigma^2}$
- For the case where $\bar{y} \sim N(\bar{\mu}, \Sigma)$, r^2 follows a generalized χ^2-distribution
- If $\Sigma \approx \sigma^2 \boldsymbol{I}$, then r^2 follows the non-central χ^2-distribution with d degrees of freedom and non-centrality parameter $\lambda = \frac{\bar{\mu}^T \bar{\mu}}{\sigma^2}$
- With cdf $F_{r^2}(r^2; d, \lambda)$
- Then an approximation for the probability of at most k errors per codeword can be written as: $F_{r^2}\left(\frac{r_k^2}{\sigma^2}; d, \lambda\right)$. Where r_k is the radius of the circle representing at most k errors.



An Algorithm

(Hyper)-Spherical TDECQ

- Let r_k be the target radius for up to k errors per codeword
- Let $\bar{\mu}_i$ be the ith vector of sampled residuals out of N total vectors
- Define the probability of a correctable codeword for a given σ as:

$$G(\sigma) = \frac{1}{N} \sum_{i=0}^{N-1} F_{r^2}\left(\frac{r_k^2}{\sigma^2}; d, \frac{\bar{\mu}_i{}^T \bar{\mu}_i}{\sigma^2}\right)$$

  - ➢Find the maximum noise, σ_g, such that $\left(1 - \mathrm{G}(\sigma_g)\right) \le CER_{target}$
  - ➢Calculate $\sigma_s = \sqrt{C_{eq}^{-2}\sigma_g^2 + \sigma_c^2}$ to remove noise gain and include intrinsic channel noise

- $TDECQ = 10\log_{10}\left(\frac{\sigma_{ref}}{\sigma_s}\right)$, where σ_{ref} is the noise margin of an ideal transmitter
- Note: For d=1, this can be reduced to an equivalent calculation of traditional TDECQ

Additional Considerations

Choice of r_k

- Choose $\boldsymbol{r_k}$ such that $1 - CER_{target} = F_{r^2}\left(\frac{r_k^2}{\sigma_{ref}^2}; d, 0\right)$
  - ➢The probability of a successful codeword is equal to the probability of being within the hypersphere of radius $\frac{r_k^2}{\sigma_{ref}^2}$
  - ➢$r_k = \sigma_{ref}\sqrt{F_{\chi_d}^{-1}(1 - CER_{target})}$

- σ_{ref} is calculated similarly to the existing method, assuming an idealized system
  - ➢Assume symbol errors are iid with error probability P_e
  - ➢$\sigma_{ref} = \frac{OMA}{6\sqrt{2}\,\text{erfc}^{-1}\left(\frac{4}{3}P_e\right)}$
  - ➢P_e can be derived from CER_{target} as the solution to $1 - CER_{target} = \sum_{i=0}^{k}\binom{n}{i}P_e^i(1 - P_e)^{n-i}$

Additional Considerations

Efficient Calculation of Non-Central Chi-Square CDF

- The non-central Chi-Square CDF, $F_{r^2}(r^2; d, \lambda)$, does not have a closed form solution
- "Exact" Numerical solutions exist, but are computationally complex
- Tables could be generated for pairs of values, (r, λ), to speed up computations
- For calculations in this presentation the following estimate[1] is used:

$$F_{r^2}(r^2; d, \lambda) \approx \frac{1}{2}\operatorname{erfc}\left[\frac{1}{\sqrt{2}}\left(r - \sqrt{\lambda} - \frac{d-1}{2(r-\sqrt{\lambda})}\left(\log(r) - \log\left(\sqrt{\lambda}\right)\right)\right)\right]$$

1 - Fraser, D.A.S. & Wu, Jianrong & Wong, Augustine. (1998). An approximation for the noncentral chi-squared distribution. Communications in Statistics - Simulation and Computation. 27. 275-287. 10.1080/03610919808813480.

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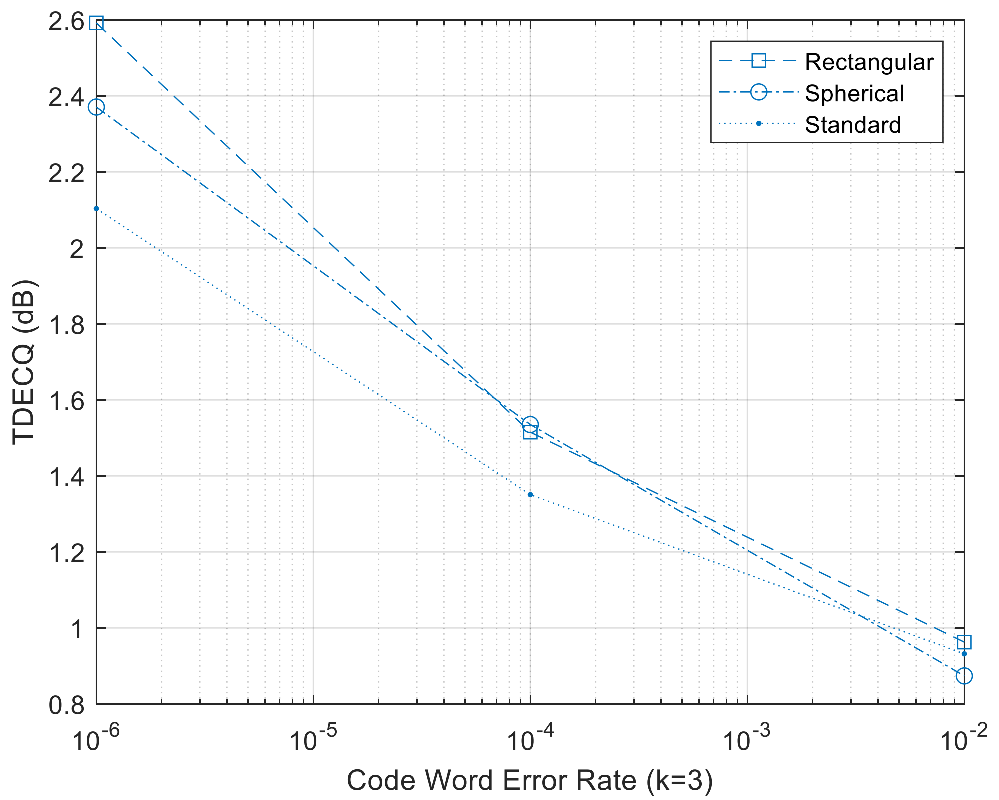
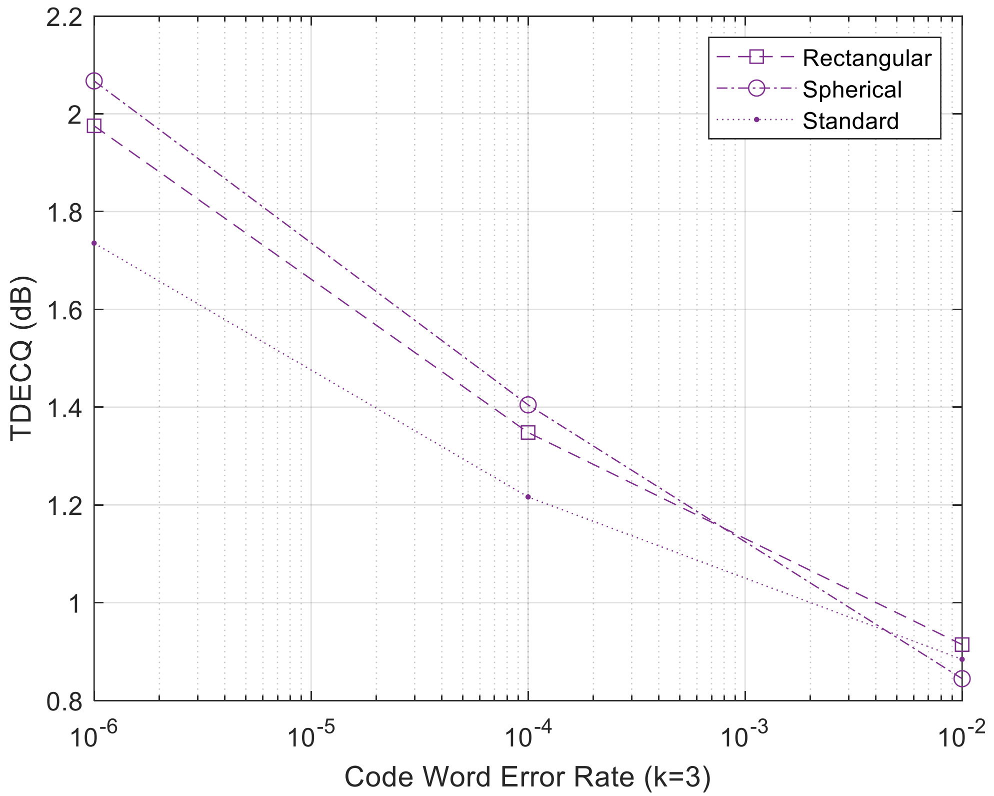
Comparison of Methods

Two Different Waveforms

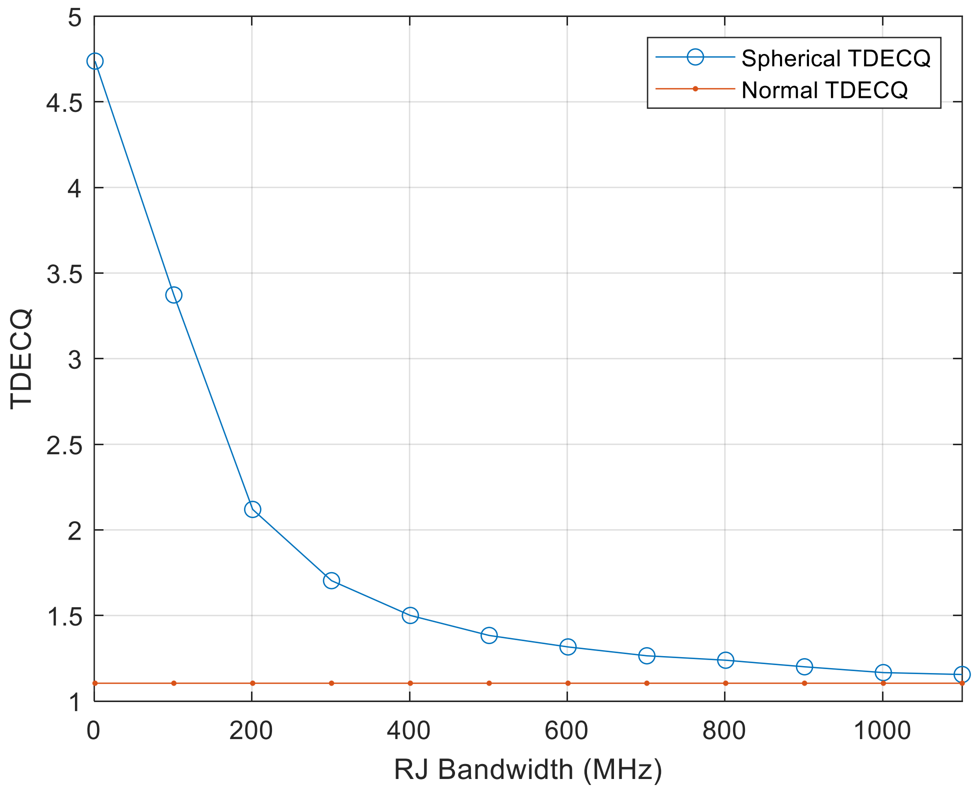




Bandlimited RJ

Simulated Test Case

- ran_3dj_02a_2407 and oif2024.449.02 demonstrated receivers with performance issue related to bandlimited random jitter
- Using a captured real signal and applied 400fs of bandlimited RJ in MATLAB
- Comparison of Normal TDECQ and Spherical TDECQ.
- Spherical TDECQ is sensitive to the lower frequency bandlimited RJ
- NOTE: On a Sampling Oscilloscope, the spectrum of the RJ will be aliased, and will appear uncorrelated.



Thank you