TDECQ Considerations

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Review of Cl 121.8.5.3

Let x be the transmitted symbol, $x \in \{0,1,2,3\}$

Let y be the transmitted signal point measured

Three thresholds, Pth(1) < Pth(2) < Pth(3)

Four zones, or decision regions

Z[0] = {y|y<Pth(1)}

Z[1] = {y|Pth(1)<y<Pth(2)}

Z[2] = {y|Pth(2)<y<Pth(3)}

Z[3] = {y|y>Pth(3)}

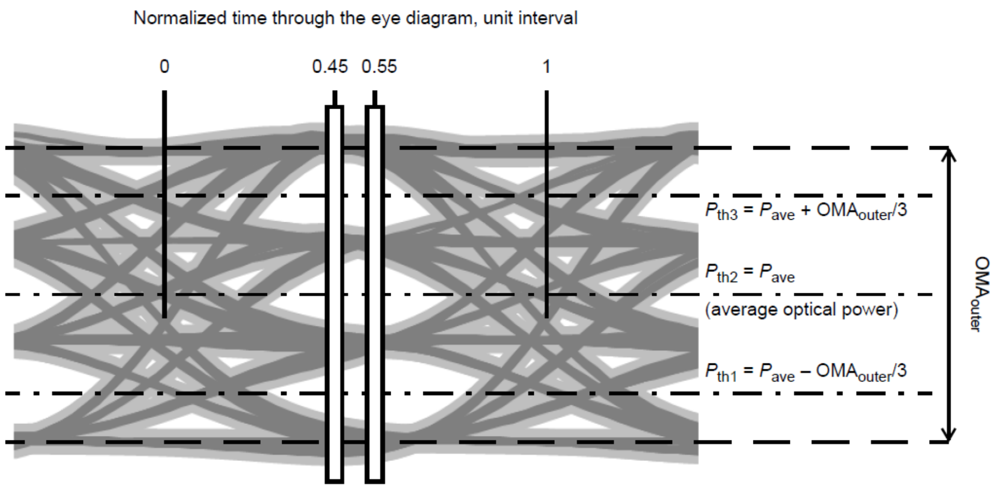


Figure 121–5—Illustration of the TDECQ measurement

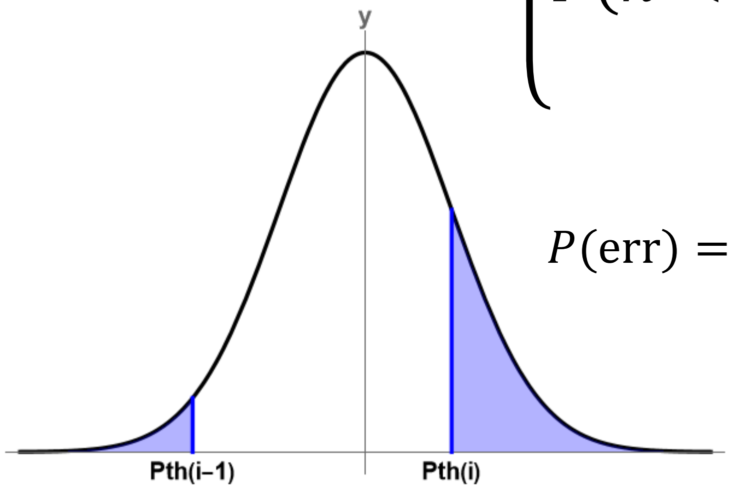
Let n be the noise added to y to give z, i.e., z=y+n

The decision rule is that the decided symbol based on the noisy z is $\hat{x}(z) = i$ where $z \in Z[i]$

Review of Cl 121.8.5.3 (cont'd)

- y is ***assumed*** to be in the correct zone to begin with ($\hat{x}(y) = x$)
  - This assumes that the eye is open
- An error is made if the noise takes z out of the zone y is in initially

$$P(\text{err}|y) = \begin{cases} P(n > \text{Pth}(1) - y) & y \in Z[0] \\ P(n < \text{Pth}(1) - y) + P(n > \text{Pth}(2) - y) & y \in Z[1] \\ P(n < \text{Pth}(2) - y) + P(n > \text{Pth}(3) - y) & y \in Z[2] \\ P(n < y - \text{Pth}(3)) & y \in Z[3] \end{cases}$$



$$P(\text{err}) = \sum_{y} P(y)P(\text{err}|y) = \sum_{i=0}^{3} \sum_{y_k \in Z[i]} P(y_k) \left[Q\left(\frac{y_k - \text{Pth}(i)}{\sigma}\right) + Q\left(\frac{\text{Pth}(i+1) - y_k}{\sigma}\right) \right]$$

$$\text{Pth}(-1) \equiv -\infty \quad \text{Pth}(4) \equiv \infty$$

Observations and Recommendation

- Clause 121.8.5.3 does not compute symbol error rate this way
- Instead, clause 121.8.5.3 attempts to condition the probability of error on the value of the noise and then sum over the different noise values.
- It does this for each of three thresholds, then sums those three probabilities for the final probability of symbol error
- The two methods are not mathematically equivalent, because the tail of the gaussian distribution is being ignored at the largest magnitude y_k values
- Ahmad El-Chayeb has indicated in a (semi) private communication that the two methods give the same result in practice at the SNRs considered (so far)
    - Depends on the SNR and how close y is to the nearest threshold
    - Depends also on the maximum magnitude y_k value considered
- This should be fixed for future readability and to ensure repeatability with different implementations

Consider a grid of y_i values equally spaced. Limit attention to the left histogram.

TDECQ considers three thresholds, $\{T_1, T_2, T_3\}$, that are the boundaries of four *zones*, or decision regions, for the PAM4 signal.

Consider first T_1 and $y_i > T_1$.

$CF_1(y_i) \approx \text{Prob}(T_1 \leq y \leq y_i)$ (approximately, because we are using discrete histograms)

Suppose the noise n is given by $n = -(y_i - T_1)$ (Note: n is negative).

If y is above T_1 and below y_i, then $y + n$ will be below T_1, which results in a symbol error *if* y was in the correct zone to begin with.

TDECQ computes the probability of a making a symbol error by crossing from above T_1 to below T_1 as

$$\sum_{\{y_i|y_i>T_1\}} \text{Prob}(n = T_1 - y_i)\text{Prob}(T_1 \leq y \leq y_i) = \sum_{\{y_i|y_i>T_1\}} \text{Prob}(n = T_1 - y_i) CF_1(y_i) \quad (1)$$

where n is Gaussian noise of the proper variance. The problem with this approach is when $y_i = y_{\text{max}} \equiv \max\{y_i\}$

In that case, $\text{Prob}(T_1 \leq y \leq y_{\text{max}})$ should not be weighted by $\text{Prob}(n = T_1 - y_{\text{max}})$

Instead it should be weighted by $\text{Prob}(n \leq T_1 - y_{\text{max}})$; otherwise we are ignoring the tail of the Gaussian distribution.

So the correct probability of crossing from above T1 to below T1 is given by

$$\sum_{\{y_i | T_1 < y_i < y_{\mathsf{max}}\}} \mathsf{Prob}(n = T_1 - y_i) CF_1(y_i) + \mathsf{Prob}(n \leq T_1 - y_{\mathsf{max}}) CF_1(y_{\mathsf{max}}) \quad (2)$$

Similarly, the correct probability of crossing from below T1 to above T1 is given by

$$\sum_{\{y_i | y_{\mathsf{min}} < y_i < T_1\}} \mathsf{Prob}(n = T_1 - y_i) CF_1(y_i) + \mathsf{Prob}(n \geq T_1 - y_{\mathsf{min}}) CF_1(y_{\mathsf{min}}) \quad (3)$$

(In this region, $CF_1(y_i) = \mathsf{Prob}(y_i \leq y \leq T_1)$)

Putting these together, assuming y was originally in the correct zone, the probability of making an error by crossing T_1 is given by

$$\sum_{\{y_i | y_{\mathsf{min}} < y_i < y_{\mathsf{max}}\}} \mathsf{Prob}(n = T_1 - y_i) CF_1(y_i) + \mathsf{Prob}(n \geq T_1 - y_{\mathsf{min}}) CF_1(y_{\mathsf{min}}) + \mathsf{Prob}(n \leq T_1 - y_{\mathsf{max}}) CF_1(y_{\mathsf{max}}) \quad (4)$$

The second problem with the computation of symbol error probability in TDECQ is that it computes the probability of making an error by crossing $T1$, $T2$, and $T3$ independently, then adds those probabilities together to get a total probability. But this is double or triple counting in some cases, since a given y value has a certain probability of crossing one (or sometimes two, if an internal region) thresholds, and it does not make another symbol error if it crosses the next threshold.

All of these problems could be avoided by using the more conventional error probability conditioned on the value of y and then using the Q function to determine the probability of crossing a threshold out of the original zone.