TDECQ Considerations Revisited

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10 March 2026

Supporters

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Introduction

- swenson_3dj_01a_2509.pdf identified deficiencies in Cl 121.8.5.3 for computing TDECQ
- Discussion at that meeting indicated that TDECQ would be completely rewritten in Cl 180.9.6.4
  - The author understood that the Q function, as described in swenson_3dj_01a_2509.pdf would be used to compute probability of error versus the method used in Cl 121.8.5.3
  - The new Cl 180.9.6.4 uses the same methodology as 121.8.5.3 leading to the same inaccuracies and repeatability issues
- Recommend changing the calculation to improve repeatability and readability
- Remainder of this presentation is based on swenson_3dj_01a_2509.pdf
  - Please see that presentation for additional mathematical details

Review of Cl 180.9.6.4

Let x be the transmitted symbol, $x \in \{0,1,2,3\}$

Let y be the transmitted signal point measured

Three thresholds, Pth1< Pth2 < Pth3

Four zones, or decision regions

$Z[0] = \{y|y<P_{th1}\}$

$Z[1] = \{y|P_{th1}<y<P_{th2}$

$Z[2] = \{y|P_{th2}<y<P_{th3}\}$

$Z[3] = \{y|y>P_{th3}\}$

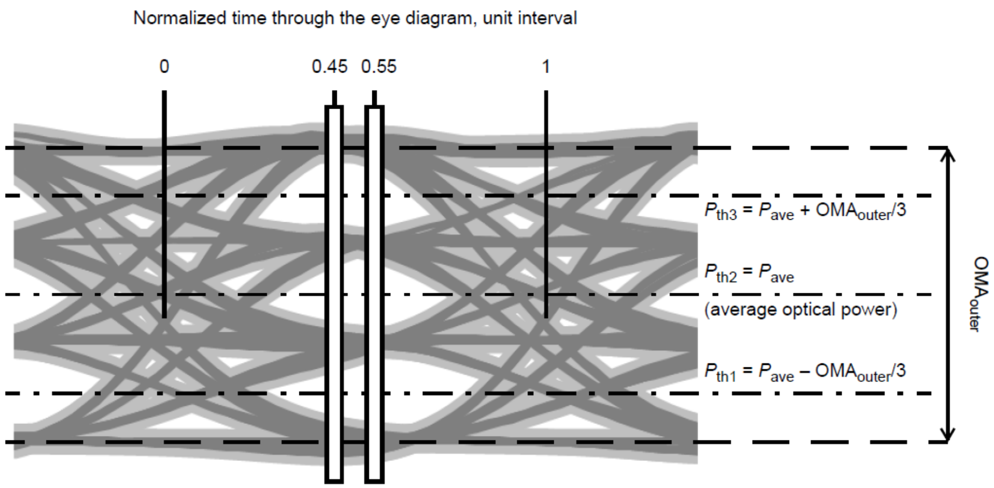


Figure 121–5—Illustration of the TDECQ measurement

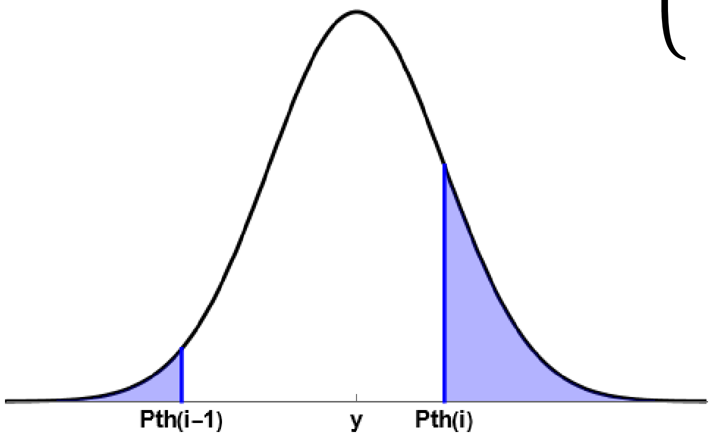
Let n be the noise added to y to give z, i.e., z=y+n

The decision rule is that the decided symbol based on the noisy z is $\hat{x}(z) = i$ where $z \in Z[i]$

Review of Cl 180.9.6.4 (cont'd)

- y is ***assumed*** to be in the correct zone to begin with ($\hat{x}(y) = x$)
  - This assumes that the eye is open
  - (Note: This assumption may be violated for TDECQ = 3.4dB)
- An error is made if the noise takes z out of the zone y is in initially

$$\mathrm{Prob}(\mathrm{err}|y) = \begin{cases} \mathrm{Prob}(n > \mathrm{P_{th1}} - y) & y \in Z[0] \\ \mathrm{Prob}(n < \mathrm{P_{th1}} - y) + \mathrm{Prob}(n > \mathrm{P_{th2}} - y) & y \in Z[1] \\ \mathrm{Prob}(n < \mathrm{P_{th2}} - y) + \mathrm{Prob}(n > \mathrm{P_{th3}} - y) & y \in Z[2] \\ \mathrm{Prob}(n < \mathrm{P_{th3}} - y & y \in Z[3] \end{cases}$$



$$\mathrm{Prob}(\mathrm{err}) = \sum_{y} \mathrm{Prob}(y)\mathrm{Prob}(\mathrm{err}|y)$$

$$= \sum_{i=0}^{3} \sum_{y_k \in Z[i]} \mathrm{Prob}(y_k) \left[Q\left(\frac{y_k - \mathrm{P_{th(i)}}}{\sigma}\right) + Q\left(\frac{\mathrm{P_{th(i+1)}} - y_k}{\sigma}\right) \right]$$

$\mathrm{P_{th0}} \equiv -\infty$

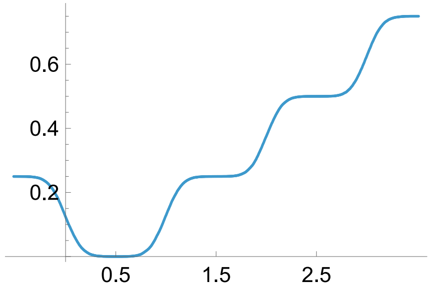
$\mathrm{P_{th4}} \equiv \infty$

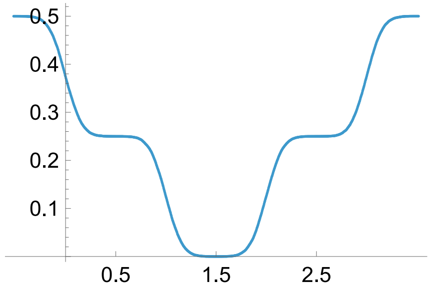
Review of Cl 180.9.6.4 (cont'd)

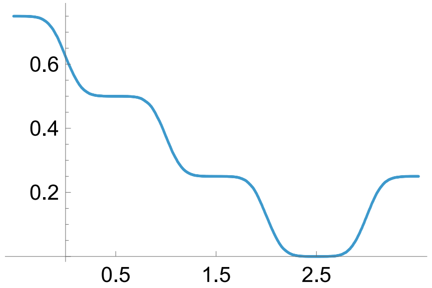
- Clause 180.9.6.4 does not compute symbol error rate as described on the previous slide
  - Instead, for each threshold, it calculates a "Cumulative Distribution Function"

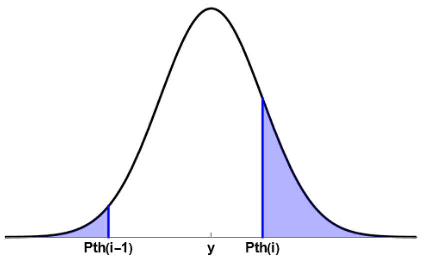
$$\mathrm{CDF}(y_0, P_{th(i)}) = \begin{cases} \mathrm{Prob}(P_{th(i)} \leq y \leq y_0), & y_0 > P_{th(i)} \\ \mathrm{Prob}(y_0 \leq y < P_{th(i)}), & y_0 < P_{th(i)} \end{cases}$$

  - It then computes a weighted sum of the $\mathrm{CDF}(y_0, P_{th(i)})$ values over all y_0, where each value is weighted by the probability that the Gaussian noise is $P_{th(i)} - y_0$
- It does this for each of three thresholds, then sums those three probabilities for the final probability of symbol error









Observations and Recommendation

- The two methods are not mathematically equivalent, because the tail of the gaussian distribution is being ignored at the largest magnitude y_k values (may be small effect)
- Also, the method computes the probability of error for each threshold as though that were the only threshold (a binary decision). It double counts or triple counts errors, depending on the region that the sample is in (a larger effect)
- Recommendation: Modify calculation of symbol error rate to that shown on slide 4 to improve readability and to ensure repeatability with different implementations
  - Study to quantify effect is ongoing