Equalizer Optimization for TDECQ

Norman L. Swenson
Affiliations: Nokia and Point2
Adee Ran
Affiliation: Cisco
Guangcan Mi
Affiliation: Huawei

Roberto Rodes
Affiliation: Coherent
Mike Dudek
Affiliation: Marvell

Version 1.1g
IEEE 802.3dj May Meeting
14 May 2026

Comments Addressed: 92, 395, 396, 478

Supporters

- Chris Cole, Coherent
- Richard Mellitz, Samtec
- Ali Ghiasi, Ghiasi Quantum
- Karl Muth, Broadcom

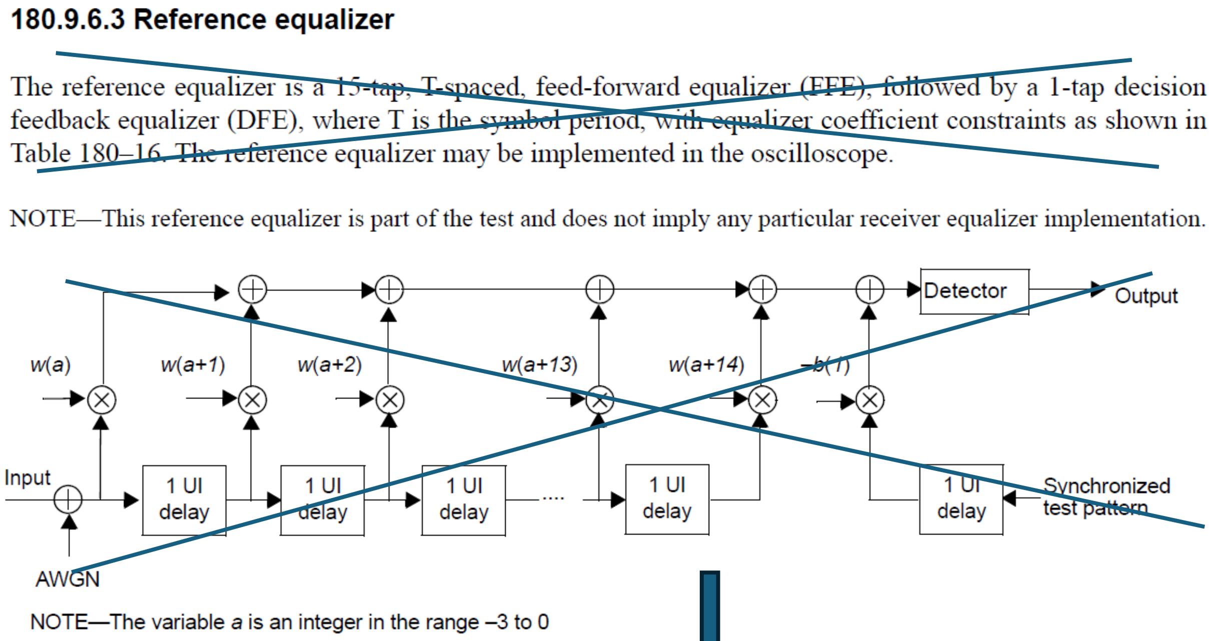
Preview

- This presentation suggests changes to the draft specification that enable:
  - Well-defined equalizer convergence
  - Implementation with rapid execution times
- The goal was to remove ambiguity and facilitate repeatability while leaving room for alternative implementations
- This method optimizes mean squared error at a given sampling phase and finds the trial TDECQ at that sampling phase, then optimizes over the sampling phase to achieve the final TDECQ

Proposed Changes to p.479

The reference equalizer is a T-spaced discrete-time equalizer with 15 feedforward taps and a single decision feedback tap, where T is the symbol period. Equalizer coefficient constraints are given in Table 180–16. The reference equalizer may be implemented in the oscilloscope. The decisions fed back in the equalizer are modeled as being correct, so the equalizer is modeled as shown in Figure 180-10, where $x(n)$ is the n^{th} symbol in the test pattern sequence, $r(t)$ is the output of the reference receiver defined in 180.9.2, $\eta(t)$ is colored Gaussian noise, $z(t) = r(t) + \eta(t)$, $z_n = z(nT + \phi)$, where $0 \le \phi < T$, and y_n is the equalizer output corresponding to $x(n)$. The Bessel-Thomson (BT) filter shown is identical to that in 180.9.2. The received signal $r(t)$ is aligned with the test pattern such that $r(nT)$ corresponds to $x(n)$.

This slide was presented in swenson_3dj_01_2605.pdf

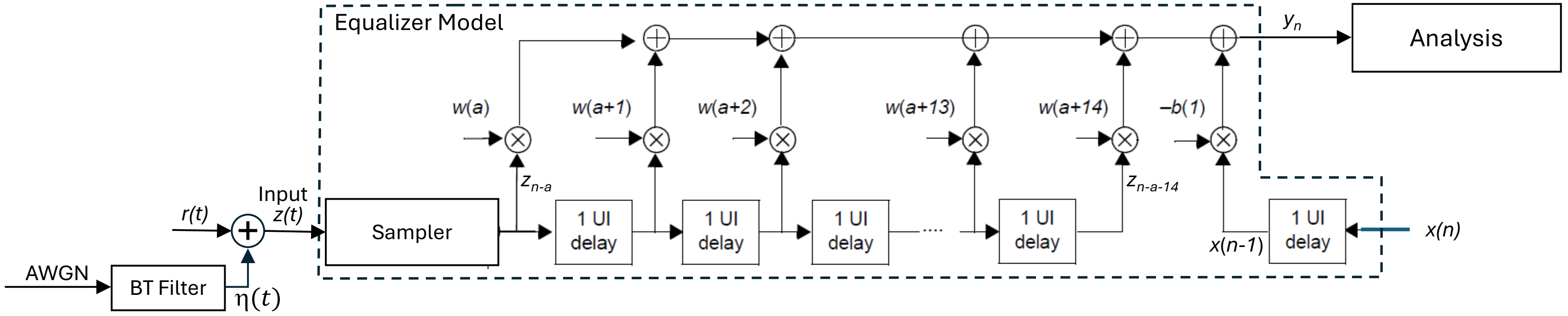


180.9.6.3 Reference equalizer

~~The reference equalizer is a 15-tap, T-spaced, feed-forward equalizer (FFE), followed by a 1-tap decision feedback equalizer (DFE), where T is the symbol period, with equalizer coefficient constraints as shown in Table 180–16. The reference equalizer may be implemented in the oscilloscope.~~

NOTE—This reference equalizer is part of the test and does not imply any particular receiver equalizer implementation.

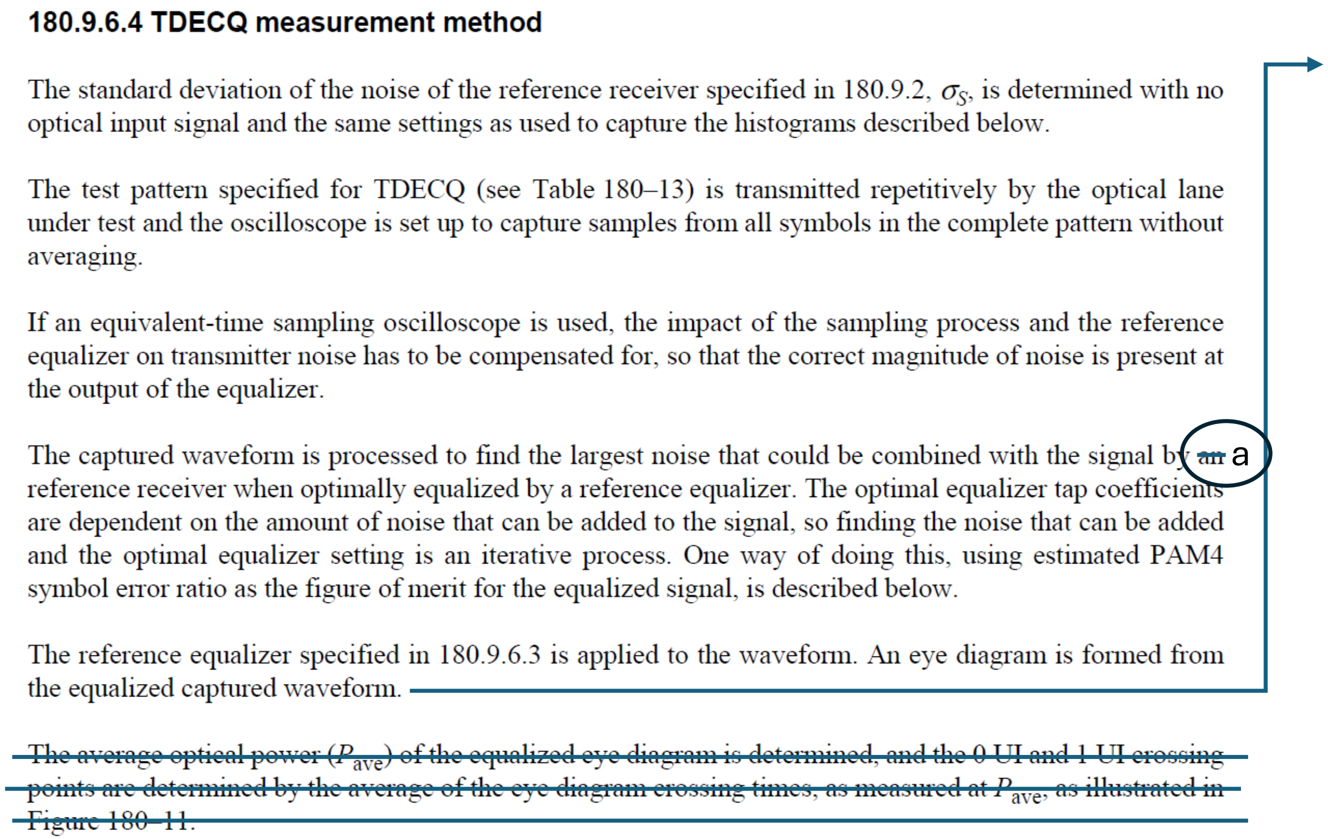
NOTE—The variable *a* is an integer in the range –3 to 0



NOTE—The variable *a* is an integer in the range –3 to 0

Figure 180-10

Proposed changes to p.480



180.9.6.4 TDECQ measurement method

The standard deviation of the noise of the reference receiver specified in 180.9.2, σ_S, is determined with no optical input signal and the same settings as used to capture the histograms described below.

The test pattern specified for TDECQ (see Table 180–13) is transmitted repetitively by the optical lane under test and the oscilloscope is set up to capture samples from all symbols in the complete pattern without averaging.

If an equivalent-time sampling oscilloscope is used, the impact of the sampling process and the reference equalizer on transmitter noise has to be compensated for, so that the correct magnitude of noise is present at the output of the equalizer.

The captured waveform is processed to find the largest noise that could be combined with the signal by ~~an~~ a reference receiver when optimally equalized by a reference equalizer. The optimal equalizer tap coefficients are dependent on the amount of noise that can be added to the signal, so finding the noise that can be added and the optimal equalizer setting is an iterative process. One way of doing this, using estimated PAM4 symbol error ratio as the figure of merit for the equalized signal, is described below.

The reference equalizer specified in 180.9.6.3 is applied to the waveform. An eye diagram is formed from the equalized captured waveform.

~~The average optical power (P_{ave}) of the equalized eye diagram is determined, and the 0 UI and 1 UI crossing points are determined by the average of the eye diagram crossing times, as measured at P_{ave}, as illustrated in Figure 180–11.~~

as follows. For a given sampling phase ϕ_0 and assumed variance σ_G^2 for $\eta(t)$, calculate the equalizer coefficients $\boldsymbol{w} = [w(a)\ w(a+1)\ \ldots\ w(a+14)]$ and b that result in the minimum mean squared error (MMSE) between y_n and x(n), subject to the requirements of Table 180–16. With the coefficients fixed, sweep the sampling phase from $\phi_0 - \frac{T}{2}$ to $\phi_0 + \frac{T}{2}$, and for each phase plot the scatter plot of the y_n values resulting from the test pattern sequence. An example eye diagram is shown in Figure 180-11. The feedback symbol *x(n)* changes at phase $\phi = \phi_0 - \frac{T}{2}$. Note: the Gaussian noise η(t) added to r(t) in Figure 180-10 is not reflected in the eye diagram; it is accounted for separately when estimating the SER in the analysis block.

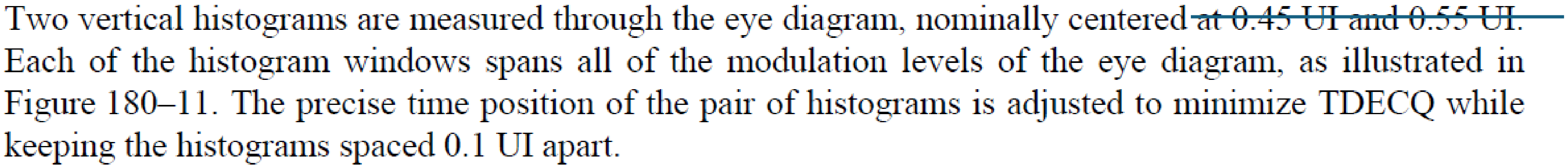
Comment(s) Addressed: 92, 395, 396

This slide is a modified version of slide 6 in swenson_3dj_01_2605.pdf. New text is highlighted.

Proposed changes to p. 481

This slide proposes changes to p. 481 in addition to those proposed in slide 7 of swenson_3dj_01_2605.pdf.

.05 UI before and after sampling phase ϕ_0



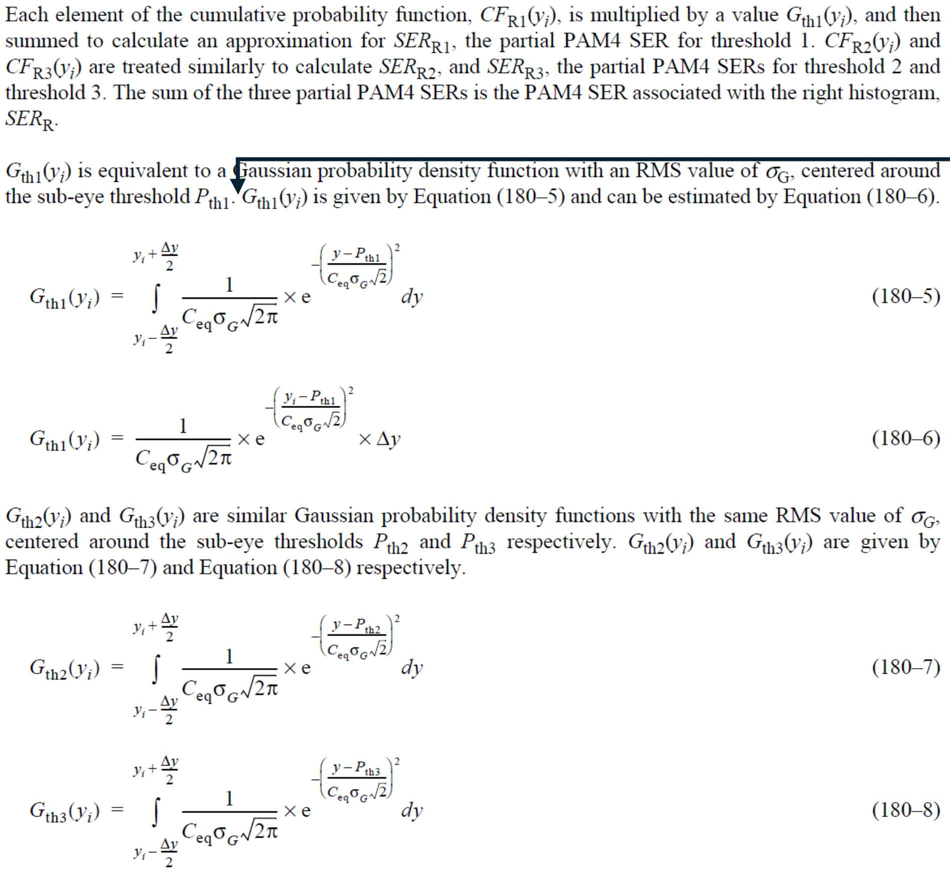
Two vertical histograms are measured through the eye diagram, nominally centered ~~at 0.45 UI and 0.55 UI.~~ Each of the histogram windows spans all of the modulation levels of the eye diagram, as illustrated in Figure 180–11. The precise time position of the pair of histograms is adjusted to minimize TDECQ while keeping the histograms spaced 0.1 UI apart.

After the paragraph above, insert the following paragraph:

> As described further below, σ_G quantifies the amount of noise that can be added at the receiver while still achieving the target symbol error rate. For a given value of σ_G , the symbol error rate for the left histogram, SER_L, and that for the right histogram, SER_R, are estimated as described below using the tap coefficients optimized to minimize mean squared error at ϕ_0 for that given value of σ_G . The nominal thresholds used in the estimation of the SER are the average power level P_{ave} and $P_{ave} \pm OMA_{TDECQ}/3$, as shown in Figure 180-11. When the SER is estimated, the thresholds are adjusted within 1% of their nominal values to minimize max(SER_L, SER_R). If max(SER_L, SER_R) is not within 1% of the target PAM4 SER of 4.56×10^{-4}, the process is iterated, changing the value of σ_G and reoptimizing the equalizer taps until max(SER_L, SER_R) is within 1% of the target. Having found the value of σ_G that causes the SER to fall within the target range, TDECQ(ϕ_0) is calculated as described below. TDECQ(ϕ_0) is then minimized over ϕ_0 , which finds the optimal location of the histograms. TDECQ is the minimum value thus found.

Comment(s) Addressed: 92, 395, 396, 478

Proposed changes to p. 482



Each element of the cumulative probability function, $CF_{R1}(y_i)$, is multiplied by a value $G_{th1}(y_i)$, and then summed to calculate an approximation for SER_{R1}, the partial PAM4 SER for threshold 1. $CF_{R2}(y_i)$ and $CF_{R3}(y_i)$ are treated similarly to calculate SER_{R2}, and SER_{R3}, the partial PAM4 SERs for threshold 2 and threshold 3. The sum of the three partial PAM4 SERs is the PAM4 SER associated with the right histogram, SER_R.

$G_{th1}(y_i)$ is equivalent to a Gaussian probability density function with an RMS value of σ_G, centered around the sub-eye threshold P_{th1}. $G_{th1}(y_i)$ is given by Equation (180–5) and can be estimated by Equation (180–6).

$$G_{th1}(y_i) = \int_{y_i-\frac{\Delta y}{2}}^{y_i+\frac{\Delta y}{2}} \frac{1}{C_{eq}\sigma_G\sqrt{2\pi}} \times e^{-\left(\frac{y-P_{th1}}{C_{eq}\sigma_G\sqrt{2}}\right)^2} dy \qquad (180–5)$$

$$G_{th1}(y_i) = \frac{1}{C_{eq}\sigma_G\sqrt{2\pi}} \times e^{-\left(\frac{y_i-P_{th1}}{C_{eq}\sigma_G\sqrt{2}}\right)^2} \times \Delta y \qquad (180–6)$$

$G_{th2}(y_i)$ and $G_{th3}(y_i)$ are similar Gaussian probability density functions with the same RMS value of σ_G, centered around the sub-eye thresholds P_{th2} and P_{th3} respectively. $G_{th2}(y_i)$ and $G_{th3}(y_i)$ are given by Equation (180–7) and Equation (180–8) respectively.

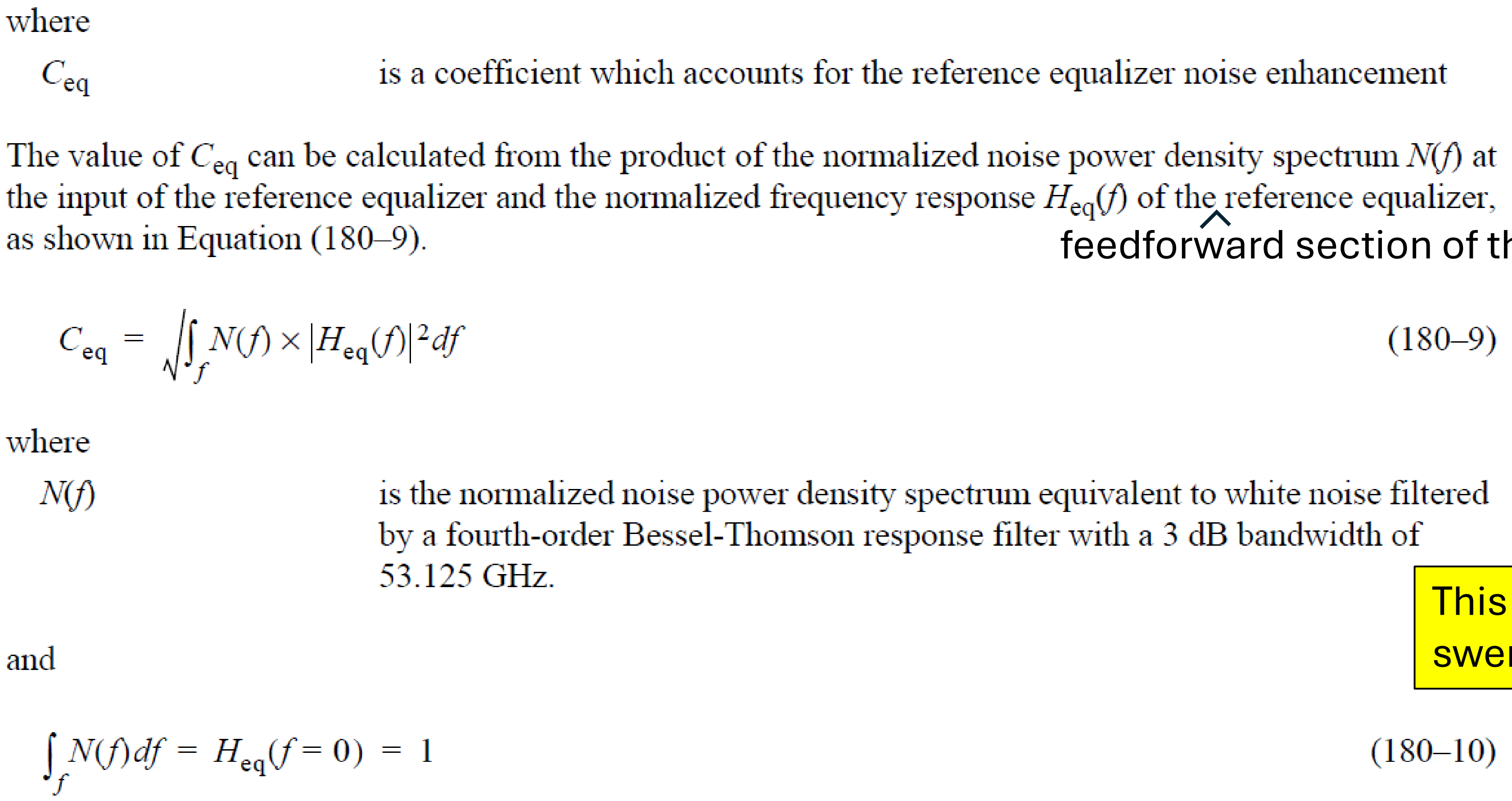
$$G_{th2}(y_i) = \int_{y_i-\frac{\Delta y}{2}}^{y_i+\frac{\Delta y}{2}} \frac{1}{C_{eq}\sigma_G\sqrt{2\pi}} \times e^{-\left(\frac{y-P_{th2}}{C_{eq}\sigma_G\sqrt{2}}\right)^2} dy \qquad (180–7)$$

$$G_{th3}(y_i) = \int_{y_i-\frac{\Delta y}{2}}^{y_i+\frac{\Delta y}{2}} \frac{1}{C_{eq}\sigma_G\sqrt{2\pi}} \times e^{-\left(\frac{y-P_{th3}}{C_{eq}\sigma_G\sqrt{2}}\right)^2} dy \qquad (180–8)$$

This accounts for the Gaussian noise $\eta(t)$ added at the input of the reference equalizer in Figure 180-10.

This slide was presented in swenson_3dj_01_2605.pdf

Proposed changes to p. 483



where

C_{eq}	is a coefficient which accounts for the reference equalizer noise enhancement

The value of C_{eq} can be calculated from the product of the normalized noise power density spectrum $N(f)$ at the input of the reference equalizer and the normalized frequency response $H_{eq}(f)$ of the reference equalizer, as shown in Equation (180–9).

feedforward section of the

$$C_{eq} = \sqrt{\int_f N(f) \times |H_{eq}(f)|^2 df} \tag{180–9}$$

where

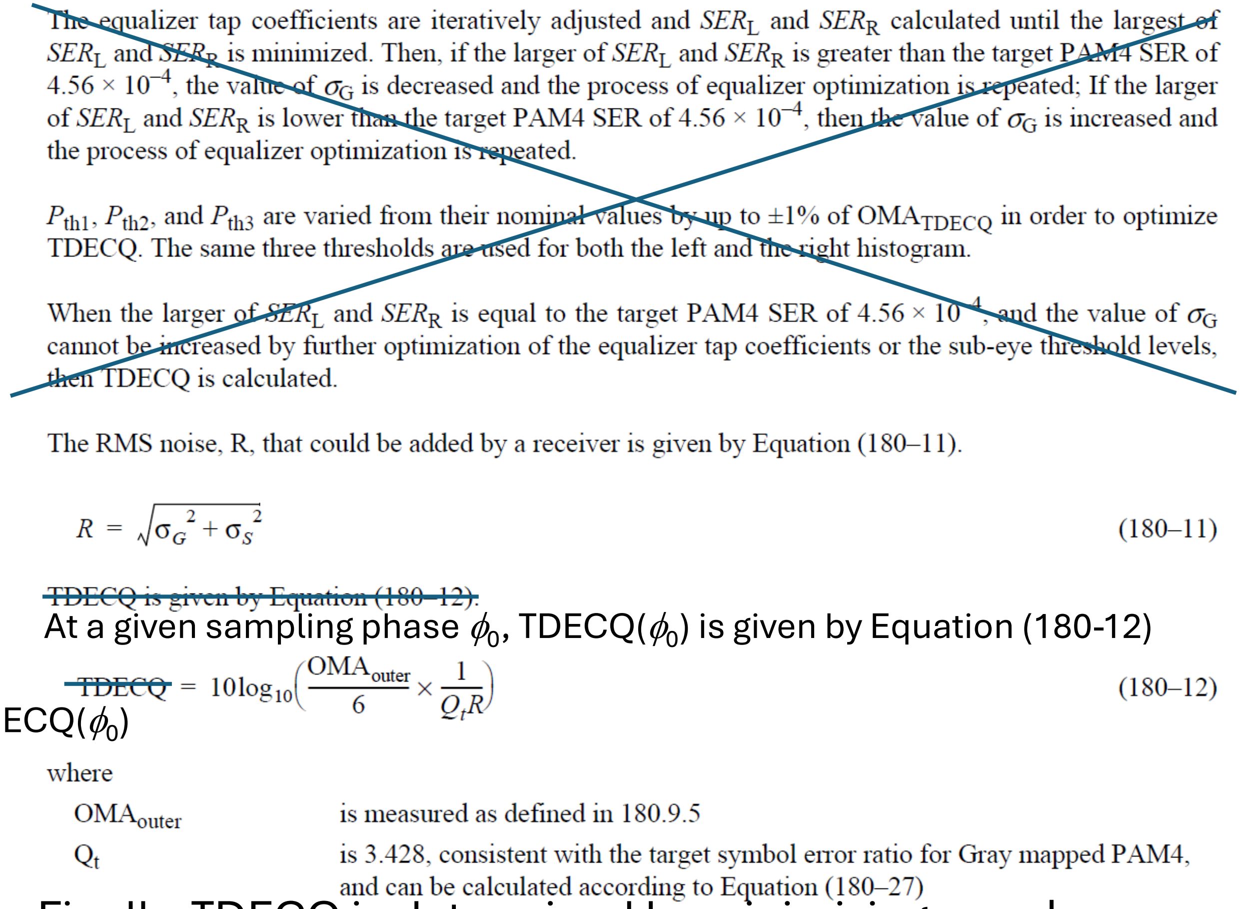
$N(f)$	is the normalized noise power density spectrum equivalent to white noise filtered by a fourth-order Bessel-Thomson response filter with a 3 dB bandwidth of 53.125 GHz.

and

$$\int_f N(f) df = H_{eq}(f = 0) = 1 \tag{180–10}$$

This slide was presented in swenson_3dj_01_2605.pdf

Proposed changes to p. 483 (cont'd)



~~The equalizer tap coefficients are iteratively adjusted and SER_L and SER_R calculated until the largest of SER_L and SER_R is minimized. Then, if the larger of SER_L and SER_R is greater than the target PAM4 SER of 4.56×10^{-4}, the value of σ_G is decreased and the process of equalizer optimization is repeated; If the larger of SER_L and SER_R is lower than the target PAM4 SER of 4.56×10^{-4}, then the value of σ_G is increased and the process of equalizer optimization is repeated.~~

~~P_{th1}, P_{th2}, and P_{th3} are varied from their nominal values by up to ±1% of OMA_{TDECQ} in order to optimize TDECQ. The same three thresholds are used for both the left and the right histogram.~~

~~When the larger of SER_L and SER_R is equal to the target PAM4 SER of 4.56×10^{-4}, and the value of σ_G cannot be increased by further optimization of the equalizer tap coefficients or the sub-eye threshold levels, then TDECQ is calculated.~~

The iteration of σ_G and the tap values to hit the target SER are covered in the text on slide 6.

The RMS noise, R, that could be added by a receiver is given by Equation (180–11).

$$R = \sqrt{\sigma_G^{\ 2} + \sigma_S^{\ 2}} \qquad (180–11)$$

~~TDECQ is given by Equation (180–12).~~

At a given sampling phase ϕ_0, TDECQ(ϕ_0) is given by Equation (180-12)

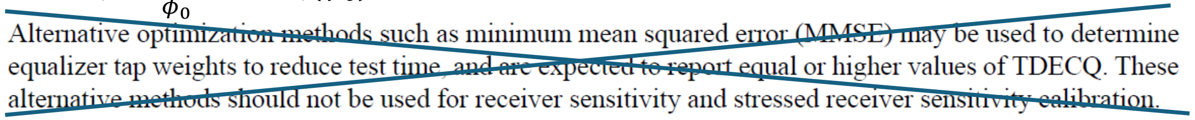
$$\cancel{\mathrm{TDECQ}}\ \mathrm{TDECQ}(\phi_0) = 10\log_{10}\left(\frac{\mathrm{OMA_{outer}}}{6} \times \frac{1}{Q_t R}\right) \qquad (180–12)$$

where

OMA_{outer}	is measured as defined in 180.9.5
Q_t	is 3.428, consistent with the target symbol error ratio for Gray mapped PAM4, and can be calculated according to Equation (180–27)

Finally, TDECQ is determined by minimizing over ϕ_0:

$$\mathrm{TDECQ} = \min_{\phi_0} \mathrm{TDECQ}(\phi_0)$$



~~Alternative optimization methods such as minimum mean squared error (MMSE) may be used to determine equalizer tap weights to reduce test time, and are expected to report equal or higher values of TDECQ. These alternative methods should not be used for receiver sensitivity and stressed receiver sensitivity calibration.~~

Comment(s) Addressed: 92, 395, 396, 478

Replace with: "Alternative optimization methods that achieve similar TDECQ values are valid."

Thank You

Backup (Presented Earlier)

Background of Reference Equalizer for TP2 Compliance Testing

- The first reference equalizer for 802.3 compliance testing was TWDP (Transmitter Waveform and Dispersion Penalty) in Clause 68.6.6 for 10GBASE LRM
  - December 2004 contribution by Swenson, Voois, Lindsay, and Zeng
    - https://grouper.ieee.org/groups/802/3/aq/public/tools/
    - N. Swenson, et al., “Explanation of IEEE 802.3, Clause 68 TWDP,” 2006, available as Clause_68_TWDP.pdf at https://standards.ieee.org/wp-content/uploads/2022/07/802.3-2022_downloads.zip
    - N. L. Swenson, et al., "Standards Compliance Testing of Optical Transmitters Using a Software-Based Equalizing Reference Receiver," in *OFC and NFOEC*, 2007, paper NWC3. https://opg.optica.org/abstract.cfm?URI=NFOEC-2007-NWC3
  - The final version of the TWDP reference equalizer was a T/2-spaced DFE with 14 feedforward taps and 5 feedback taps
- TDEC was subsequently introduced for binary NRZ waveforms using a T-spaced FFE reference equalizer
- TDECQ followed for PAM-4, also based on a feed forward equalizer

The Search

Comments I-395, I-396 (Swenson), Comment I-92 (Adee)

Suggested Resolution: Specify that an MMSE solution is compliant whenever TDECQ or TECQ is evaluated.

Test Definition Must Be Well-Defined and Implementable for Repeatability

- It is impractical to do a brute force search to find a global optimum with 16+ degrees of freedom*
- MMSE optimization is well established for DFE optimization – why aren't we using it?
  - Algorithm is well-defined, implementable, and readily available

---

* There are (arguably) 23 degrees of freedom to search over:16 (taps) +1 (number of precursors) + 1 (phase) + 1 (noise level) + 3 (threshold levels) + 1 (histogram selection)

Brute Force Search vs MMSE for Decision Feedback Equalizer (DFE) optimization with 15 feedforward taps and 1 feedback tap (16 taps total)

Feature	**Brute Force Search**	**MMSE (Analytical Solution)**
Mathematical Goal	Exhaustive search for the lowest Error/BER.	Direct solution of the Wiener-Hopf equations.
Search Space	$16-$dimensional hyper-grid.	Single point (intersection of 16 hyper-planes).
Complexity Class	$O(S^N)$ where S is steps and $N = 16$.	$O(N^3)$, where $N = 16$.
Estimated Operations	$\approx 1.2 \times 10^{24}$(assuming 32 values/tap).	$\approx$ 4,096 floating-point operations.
Compute Time	$\approx$ 38,000 years (at 1 THz test rate).	$\approx$ 100 microseconds (on a standard PC).
Channel Knowledge	None required (blind testing).	Requires Pulse Response or Training Data.
Global Optimality	Guaranteed (eventually).	Guaranteed (instantly) for Mean Square Error.
Noise Handling	Evaluates noise impact per iteration.	Accounts for noise via diagonal regularization.