TDECQ Calculation According to D3.1

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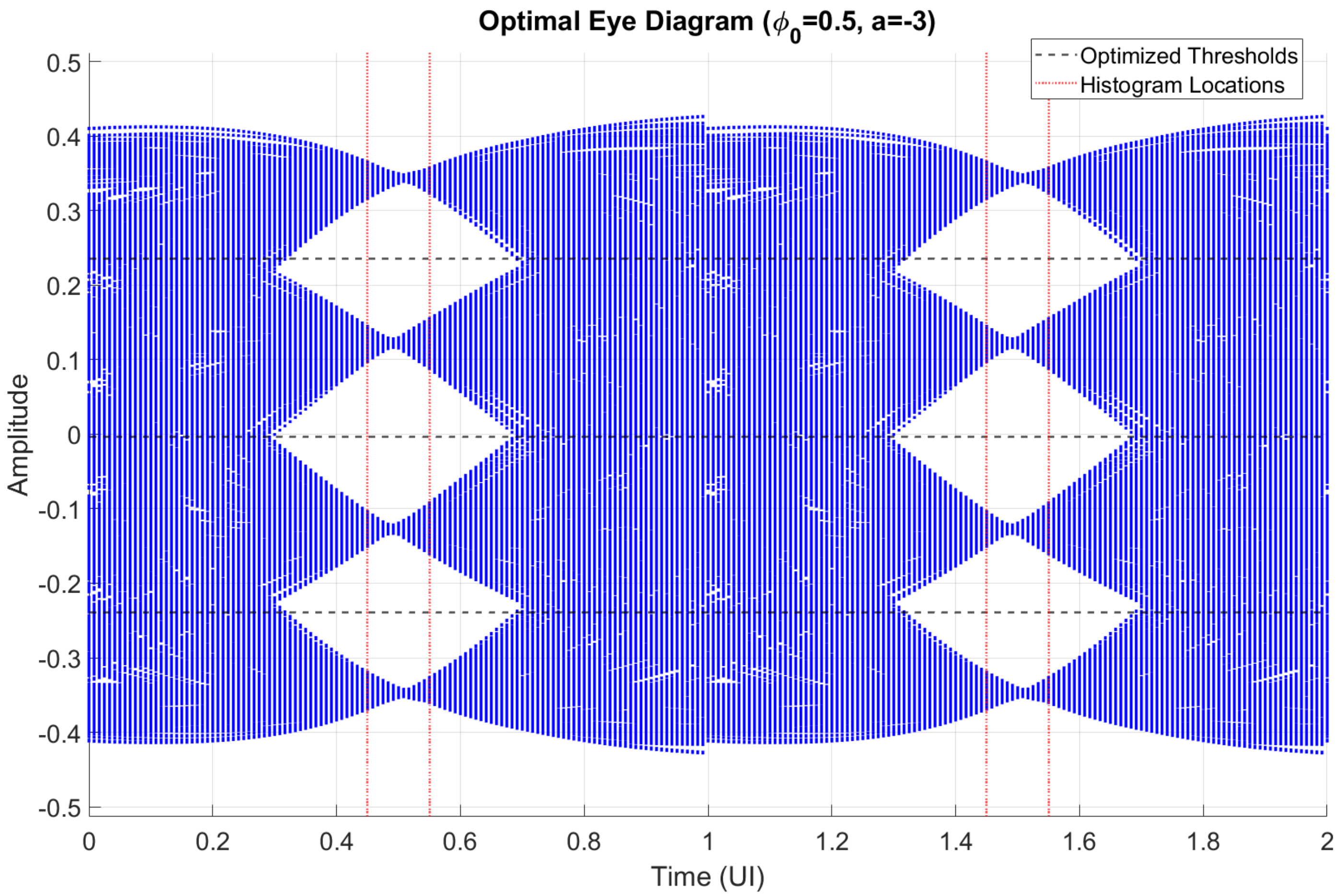
Comments addressed: 95, 96, 97, 103, 205, 227, 229, 230, 263, 264, 267, 276, 287

Eye Diagram Based on Scatter Plots

- R1-263: Don't interpolate
  - Based on phase steps of T/120
  - Density can be increased by choosing denser phase grid
- R1-97: Histogram locations need not be optimal, but must be centered around the discontinuity + T/2.
- As Comment R1-96 notes, an eye diagram is not required to construct the histograms.
- R1-96 suggests removing the description of how the eye diagram is created
  - The description helps the reader understand what is meant by an eye diagram when the reference equalizer is T-spaced and should be kept as long as the eye diagram is normative.
  - The text could be modified to say that this method is an example of how the eye diagram is created such that it does not prohibit other methods. (see next page).
  - If the eye diagram is illustrative, the explanation can be dropped



Comment(s) addressed: 96, 97, 263

Symbol Error Rate Estimation

- Refer to swenson_3dj_adhoc_01a_260624.pdf at https://www.ieee802.org/3/dj/public/adhoc/optics/0626_OPTX/swenson_3dj_adhoc_01a_260624.pdf
- Q-function-based method proposed in comments listed are straightforward, fast, easier to understand, and more accurate than the current histogram-based methodology
- R1-103 suggests a reorganization and rewrite that includes Q-function-based estimation of the symbol error rate with knowledge of the transmitted sequence
- R1-229 and R1-230 suggest using Q-function and knowledge of transmitted sequence, respectively

Comment(s) addressed: 103, 229, 230

Methodology

- Appendix 1 gives details on an MMSE algorithm that achieves repeatable results and has been implemented by several parties
- There is no evidence that blind search gives substantially better or different results
- The qualifier that better results can be used should be eliminated
  - Even if better results can be achieved, that should not be adopted unless a methodology is specified to enable repeatability
  - If the committee desires, algorithms to improve results could be investigated
  - For example, minimize mean squared error at multiple points:
    - ϕ_0 - .05T and ϕ_0 + .05 T $(e^2 = e_L^2 + e_R^2)$
    - or ϕ_0, ϕ_0 - .05T, and ϕ_0 + .05 T $(e^2 = e_0^2 + e_L^2 + e_R^2)$
    - Minor change from current algorithm

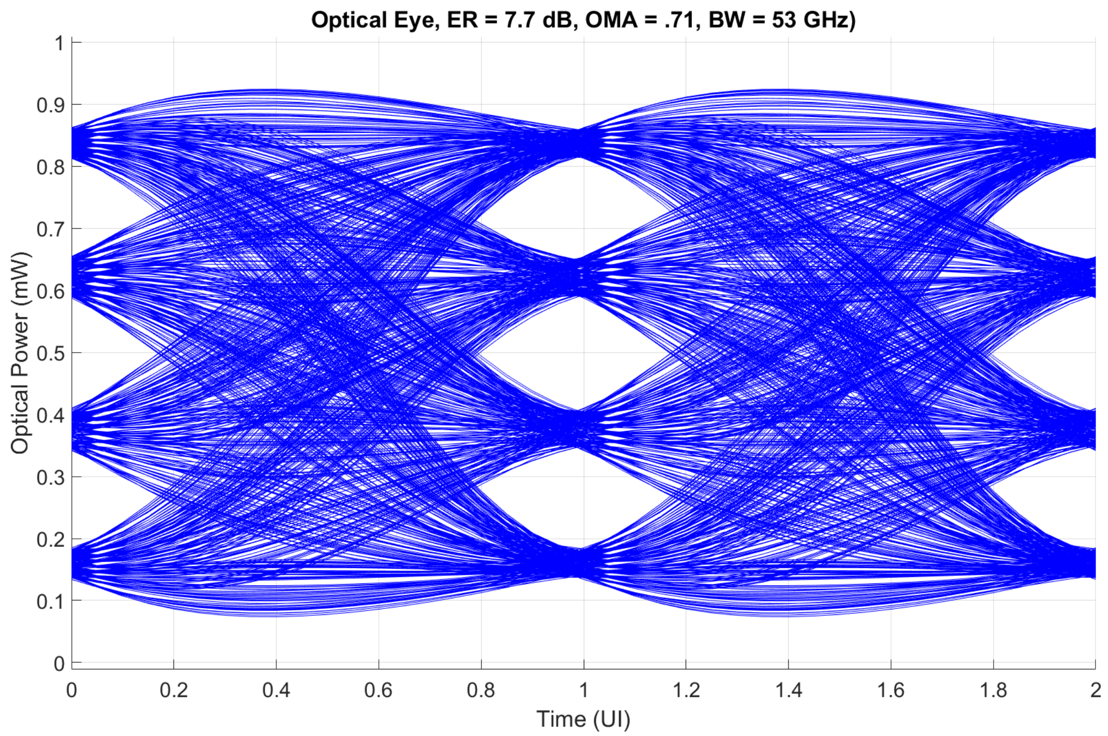
Comment(s) addressed: 95, 205, 287

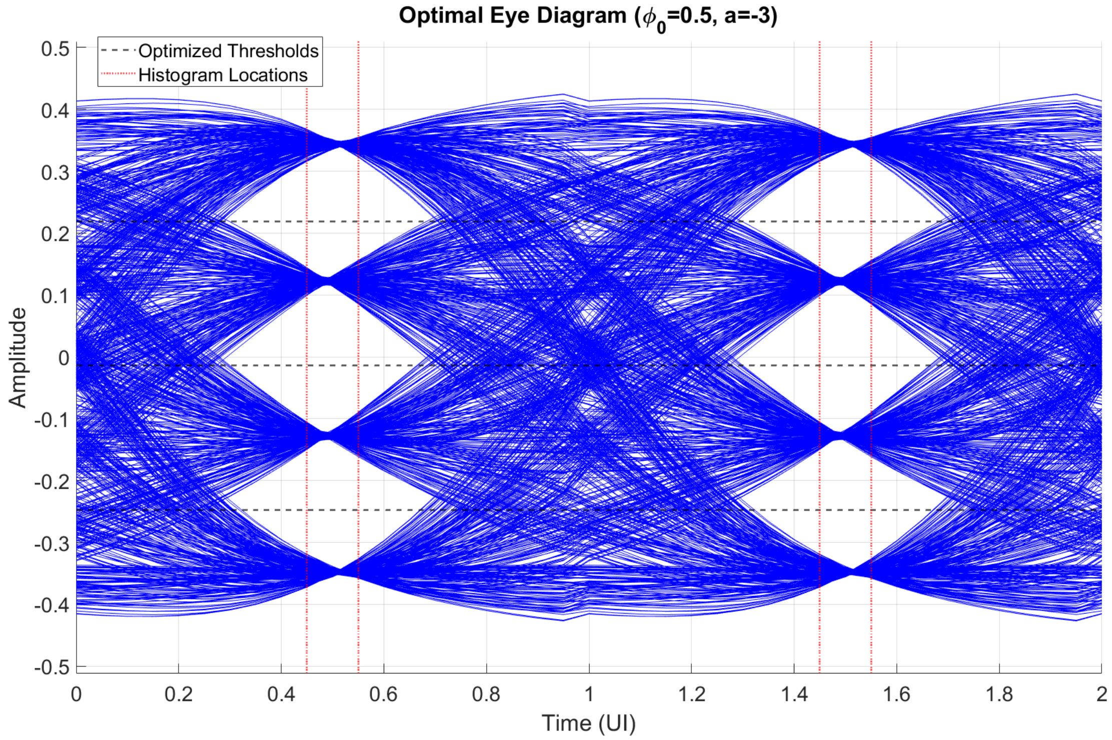
Tests with Simulated Waveforms

- Following includes testing with unconstrained equalizer tap optimization
  - Constrained optimization to be added
- Optical transmitter modeled
  - Square pulses into 3-pole Butterworth filter with specified 3-dB bandwidth
  - Electrical input into Mach-Zehnder push-pull modulator biased at quadrature
  - Various modulation depths and arm drive mismatch modeled

Results for Simulated Optical Waveform

Electrical BW = 53 GHz, MZM, OMA = .71 mW, ER = 7.7dB; TDECQ = .56 dB





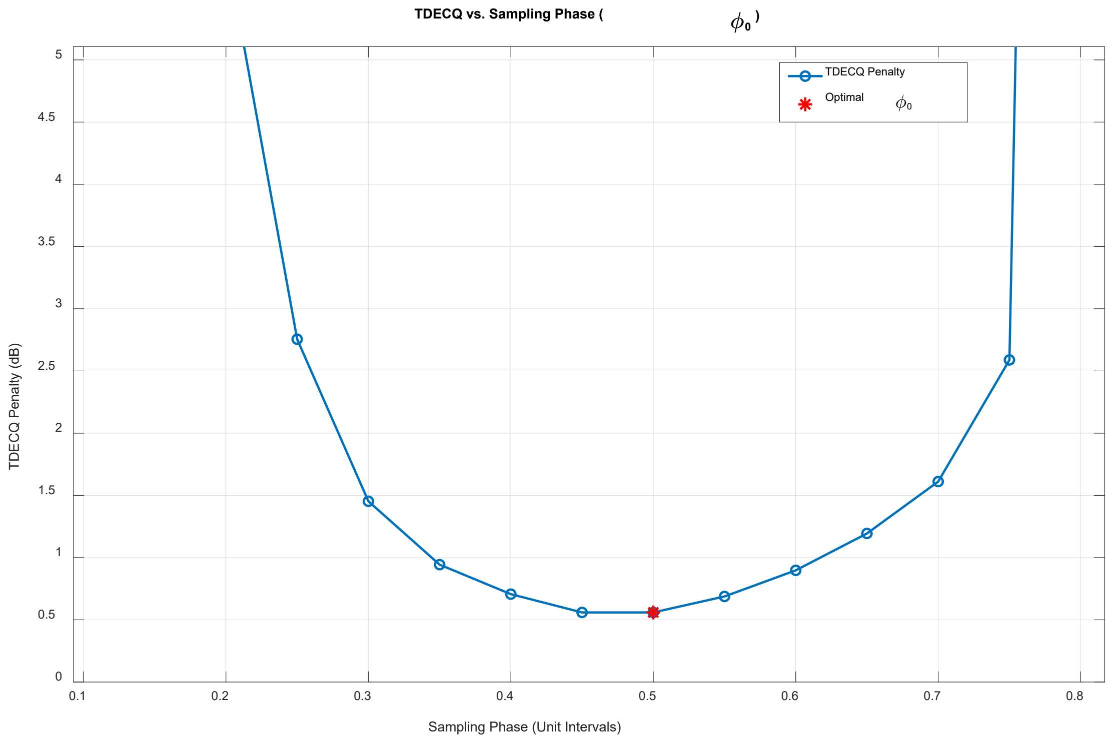
a_optimized = -3, but similar TDECQ results at a=-2, a=-1

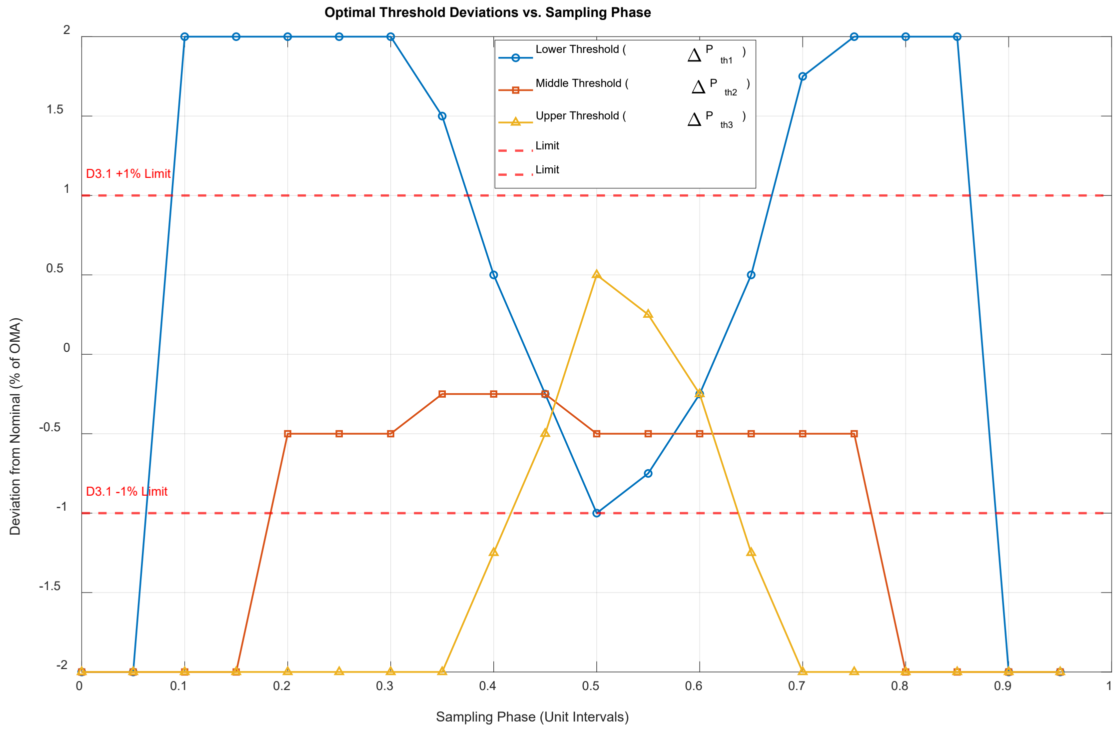
Feedfoward Taps: -0.0018595 0.0097551 -0.088159 1.012
0.066199 0.0016139 -0.0058612 0.0026031
0.00062424 -0.00016547 0.0011562 -0.0010536
0.00048185 0.0026784 -5.2641e-05

Feedback Tap: 0.0210

All taps compliant with Table 180-16

Sampling Phase Sensitivity and Threshold Deviation Limits





- Sampling phase sensitivity is well behaved (bowl shaped); No need to search over entire range
- Threshold deviations near optimal sampling phase are within ±1% limits
  - However, at optimal phase, deviation of lower threshold reaches limit. May want to expand limits

Comment(s) addressed: 276

Appendix 1

MMSE Calculation of Reference Equalizer Taps

for TDECQ

Acknowledgement to Hansel D'Silva for correcting numerical error in autocorrelation matrix

1 Introduction

This document outlines the Minimum Mean-Squared Error (MMSE) formulation for determining the reference equalizer coefficients for Clause 180.9.6 TDECQ calculations. By employing a direct Wiener-Hopf matrix solution, this approach eliminates the need for computationally intensive brute-force equalizer coefficient searches.

1.1 Alphabet Definition and Scope Limitation

- **Sequence Alphabet Definition:** The ideal transmitted SSPRQ target sequence $x(n)$ is mapped utilizing the standard symmetric, zero-mean PAM4 integer alphabet $\{-3, -1, 1, 3\}$.
- **Scope Limitation regarding Tap Bounds:** This mathematical derivation represents an unconstrained global optimization baseline. The strict tap coefficient boundary limits outlined in Table 180-16 are not enforced in this derivation. If the unconstrained solution yields coefficients exceeding the tap limits, the final tap values must instead be determined using a bounded, constrained optimization framework such as quadratic programming.

The reference receiver architecture uses a Decision-Feedback Equalizer (DFE) with 15 feedforward taps and 1 feedback tap. Assuming correct decisions are fed back under a known test pattern sequence, the feedback path uses the ideal transmitted symbols. The captured waveform samples natively incorporate deterministic inter-symbol interference (ISI) and signal-dependent transmitter noise, while background Additive White Gaussian Noise (AWGN) is shaped by a 4-pole Bessel-Thomson filter.

2 System Model and Notation

Following the notation of IEEE Draft P802.3dj/D3.1, the total continuous-time input signal to the sampler is expressed as:

$$z(t) = r(t) + \eta(t) \tag{1}$$

where $r(t)$ is the received optical signal captured by the sampling scope—which has already undergone filtering by the 4-pole Bessel-Thomson filter and naturally includes any transmitter noise—and $\eta(t)$ represents the filtered background Gaussian noise.

The discrete-time signal sampled at the symbol rate with a sampling phase ϕ_0 is given by:

$$z_n = z(nT + \phi_0) \tag{2}$$

The reference equalizer computes its output y_n using 15 feedforward taps $w(a+i)$ for $i \in \{0, 1, \ldots, 14\}$ and a single feedback tap $b(1)$:

$$y_n = \sum_{i=0}^{14} w(a+i) z_{n-a-i} - b(1)x(n-1) \tag{3}$$

where $x(n)$ denotes the ideal transmitted SSPRQ symbol sequence, and $a \in \{-3, -2, -1, 0\}$ denotes the number of pre-cursor taps.

We establish a compact matrix formulation by defining the compound input vector $\mathbf{z}_n$ and the consolidated coefficient vector $\mathbf{w}$:

$$\mathbf{z}_n = \begin{bmatrix} z_n & z_{n-1} & \ldots & z_{n-14} & x(n-1) \end{bmatrix}^T \tag{4}$$

$$\mathbf{w} = \begin{bmatrix} w(a) & w(a+1) & \ldots & w(a+14) & -b(1) \end{bmatrix}^T \tag{5}$$

This allows the equalizer output expression to be written as a vector inner product:

$$y_n = \mathbf{w}^T \mathbf{z}_n \tag{6}$$

3 MMSE Formulation and Wiener-Hopf Solution

The error signal e_n relative to the aligned target symbol $x(n)$ is defined as:

$$e_n = y_n - x(n) = \mathbf{w}^T \mathbf{z}_n - x(n) \tag{7}$$

$$J(\mathbf{w}) = E\left[e_n^2\right] = E\left[\left(\mathbf{w}^T \mathbf{z}_n - x(n)\right)^2\right] \tag{8}$$

Expanding the expectation yields:

$$J(\mathbf{w}) = \mathbf{w}^T E\left[\mathbf{z}_n \mathbf{z}_n^T\right] \mathbf{w} - 2\mathbf{w}^T E\left[\mathbf{z}_n x(n)\right] + E\left[x(n)^2\right] \tag{9}$$

Defining the input autocorrelation matrix $\mathbf{R}_{zz} = E\left[\mathbf{z}_n \mathbf{z}_n^T\right]$ and the cross-correlation vector $\mathbf{p}_{zx} = E\left[\mathbf{z}_n x(n)\right]$, the objective function simplifies to:

$$J(\mathbf{w}) = \mathbf{w}^T \mathbf{R}_{zz} \mathbf{w} - 2\mathbf{w}^T \mathbf{p}_{zx} + E\left[x(n)^2\right] \tag{10}$$

Taking the gradient with respect to the tap vector $\mathbf{w}$ and setting it to zero minimizes the cost function:

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = 2\mathbf{R}_{zz}\mathbf{w} - 2\mathbf{p}_{zx} = \mathbf{0} \tag{11}$$

This results in the classic Wiener-Hopf matrix equation:

$$\mathbf{R}_{zz}\mathbf{w}_{\text{opt}} = \mathbf{p}_{zx} \implies \mathbf{w}_{\text{opt}} = \mathbf{R}{zz}^{-1}\mathbf{p}_{zx} \tag{12}$$

4 Partitioned Matrix Structure and Sequence Constraints

The global (16×16) correlation matrix $\mathbf{R}_{zz}$ partitions into sub-blocks isolating the feedforward and feedback dimensions:

$$\mathbf{R}_{zz} = \left[\begin{array}{c|c} \mathbf{R}_{zz,\mathrm{f}} & \mathbf{R}_{zx,\mathrm{fb}} \\ \hline \mathbf{R}_{xz,\mathrm{fb}} & R_{xx} \end{array}\right] \tag{13}$$

4.1 Composition of the Feedforward Sub-block $\mathbf{R}_{zz,\mathrm{f}}$

The 15×15 sub-matrix $\mathbf{R}_{zz,\mathrm{f}}$ captures the combined statistics of the captured signal and background noise. Because $r(t)$ and $\eta(t)$ are mutually uncorrelated, the matrix splits into:

$$\mathbf{R}_{zz,\mathrm{f}} = \mathbf{R}_{rr} + \mathbf{R}_{\eta\eta} \tag{14}$$

4.1.1 Computation of $\mathbf{R}_{rr}$ from Captured Samples

Because $r(t)$ corresponds to the actual captured noisy waveform samples from the oscilloscope, it inherently includes any transmitter noise contributions alongside the deterministic ISI profile. For a periodic SSPRQ test pattern of length $N = 65,535$, the indices $j, k \in \{0, 1, \ldots, 14\}$ of $\mathbf{R}_{rr}$ are evaluated directly via time-averaging:

$$[\mathbf{R}_{rr}]_{j,k} = \frac{1}{N} \sum_{n=1}^{N} r(nT - jT + \phi_0) r(nT - kT + \phi_0) \tag{15}$$

4.1.2 Derivation and Numerical Baseline of Background Noise Matrix $\mathbf{R}_{\eta\eta}$

The background noise matrix $\mathbf{R}_{\eta\eta}$ is a symmetric Toeplitz matrix uniquely determined by its first row vector $\mathbf{r}_{\eta\eta} = [R_{\eta\eta}(0), R_{\eta\eta}(1), \ldots, R_{\eta\eta}(14)]$. The continuous-time transfer function of a fourth-order low-pass Bessel-Thomson filter is defined by:

$$H_{\mathrm{BT}}(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105} \tag{16}$$

To evaluate the normalization matching the specified 53.125 GHz -3 dB electrical power frequency cutoff at a symbol rate of 106.25 GBaud, the frequency scale factor maps via $s = j2\pi f \cdot \tau_0$ with $\tau_0 = \frac{2.113915}{2\pi \cdot 53.125\times 10^9}$.

The continuous power spectral density of the filtered Gaussian process is proportional to $|H_{\mathrm{BT}}(f)|^2$. Applying the Wiener-Khinchin theorem, the autocorrelation function at a continuous time lag τ is found by taking the inverse Fourier transform:

$$R_{\eta\eta}(\tau) = \sigma_G^2 \cdot \frac{\int_{\infty}^{\infty} |H_{\mathrm{BT}}(f)|^2 e^{j2\pi f\tau}\, df}{\int_{-\infty}^{\infty} |H_{\mathrm{BT}}(f)|^2 df} \tag{17}$$

Sampling this correlation profile at uniform discrete symbol intervals $\tau = mT$, where $T = \frac{1}{106.25\ \mathrm{GBaud}} \approx 9.41176$ ps, isolates the integer lag values $m = |j-k|$. Setting the total background noise power on-diagonal to $R_{\eta\eta}(0) = \sigma_G^2$, numerical integration of the normalized frequency domain structure yields the following unique row coefficients:

$$\mathbf{r}_{\eta\eta} = \sigma_G^2 \left[\ 1.0000,\quad 0.0206,\quad 0.0013,\quad 0.0151,\quad -0.0001,\quad \mathbf{0}_{1\times 10}\ \right] \tag{18}$$

where any entry for element index lag $m \geq 5$ decays completely below 10^{-4} and is truncated to zero.

4.2 Deterministic SSPRQ Cross-Correlation and Feedback Elements

Because SSPRQ is not perfectly independent and identically distributed (i.i.d.), the remaining variables must be computed directly from the pattern's structural indices:

- The true sequence variance is computed via:

$$R_{xx} = \frac{1}{N} \sum_{n=1}^{N} x^2(n-1) \tag{19}$$

- The j-th entry of the 15×1 cross-correlation block $\mathbf{R}_{zx,\mathrm{fb}}$ captures the non-zero correlation between the past transmitted symbol and the captured filtered waveform:

$$[\mathbf{R}_{zx,\mathrm{fb}}]_j = \frac{1}{N} \sum_{n=1}^{N} r(nT - jT + \phi_0)x(n-1) \tag{20}$$

- The j-th entry of the 16×1 global cross-correlation vector $\mathbf{p}_{zx}$ targeting the symbol $x(n)$ evaluates to:

$$[\mathbf{p}_{zx}]_j = \begin{cases} \frac{1}{N}\sum_{n=1}^{N} r(nT - jT + \phi_0)x(n) & \text{for } 0 \le j \le 14 \\ \frac{1}{N}\sum_{n=1}^{N} x(n-1)x(n) & \text{for } j = 15 \end{cases} \tag{21}$$

Appendix 2

Analytical Derivation: Noise Autocorrelation of a

4-Pole Bessel-Thomson Filter

1 The Normalized Transfer Function

The 4th-order Bessel-Thomson transfer function, normalized for a DC gain of 1 (where $\omega_{3\mathrm{dB,norm}} \approx 2.11392$ rad/s), is:

$$H(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105} \tag{1}$$

2 Poles and Residue Evaluation

The poles p_i are the roots of the denominator of $H(s)$. Using the partial fraction expansion $H(s) = \sum_{i=1}^{4} \frac{A_i}{s-p_i}$, the residues A_i are calculated as $A_i = \mathrm{Residue}[H(s), p_i] = \lim_{s\to p_i}(s-p_i)H(s)$.

i	p_i			A_i		
1	−2.103 79	$+j$	2.657 42	−1.663 39	$+j$	2.244 08
2	−2.103 79	$-j$	2.657 42	−1.663 39	$-j$	2.244 08
3	−2.896 21	$+j$	0.867 234	1.663 39	$-j$	8.3963
4	−2.896 21	$-j$	0.867 234	1.663 39	$+j$	8.3963

Table 1: Poles p_i and residues A_i.

3 Derivation of the Autocorrelation Function

The impulse response is defined as $h(t) = u(t)\sum_{i=1}^{4} A_i e^{p_i t}$, where $u(t)$ is the unit step function. The noise autocorrelation $R(\tau)$ is the convolution:

$$R(\tau) = \frac{N_0}{2}\int_{-\infty}^{\infty} h(t)h(t+\tau)dt = \frac{N_0}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\frac{A_iA_j}{-(p_i+p_j)}e^{p_j|\tau|} \tag{2}$$

4 Bandwidth Scaling and Sampling

For a filter with $f_{3\text{dB}} = 0.5R_s$ ($\omega_{3\text{dB}} = \pi R_s$), we must scale frequency by the scaling factor $\omega_{3\text{dB}}/\omega_{3\text{dB,norm}} \approx \pi R_s/2.11392$. This maps the sampling time $t_k = k/R_s$ to the normalized time $\tau_k = \pi k/2.11392 \approx 1.48615k$. Substituting this into (2):

$$r_k = \frac{N_0}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} \frac{A_i A_j}{-(p_i + p_j)} e^{p_j(1.48615k)} \tag{3}$$

5 Normalized Autocorrelation Sequence

Normalizing by r_0 (where $\rho_k = r_k/r_0$), we obtain the sequence:

Lag (k)	Normalized Autocorrelation (ρ_k)
0	1.0000
1	0.0206
2	0.0013
3	-0.0001
4	1.7288e-6