Noise and Interpolation Considerations for TDECQ Calculation

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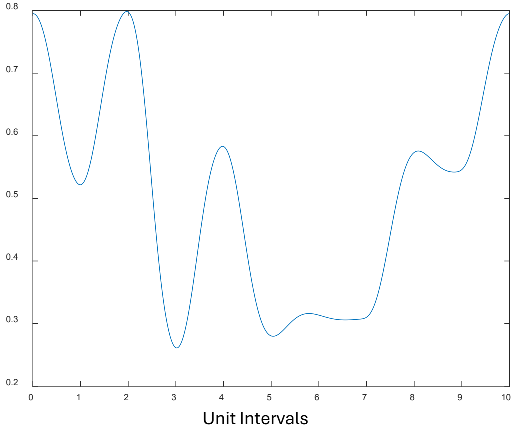
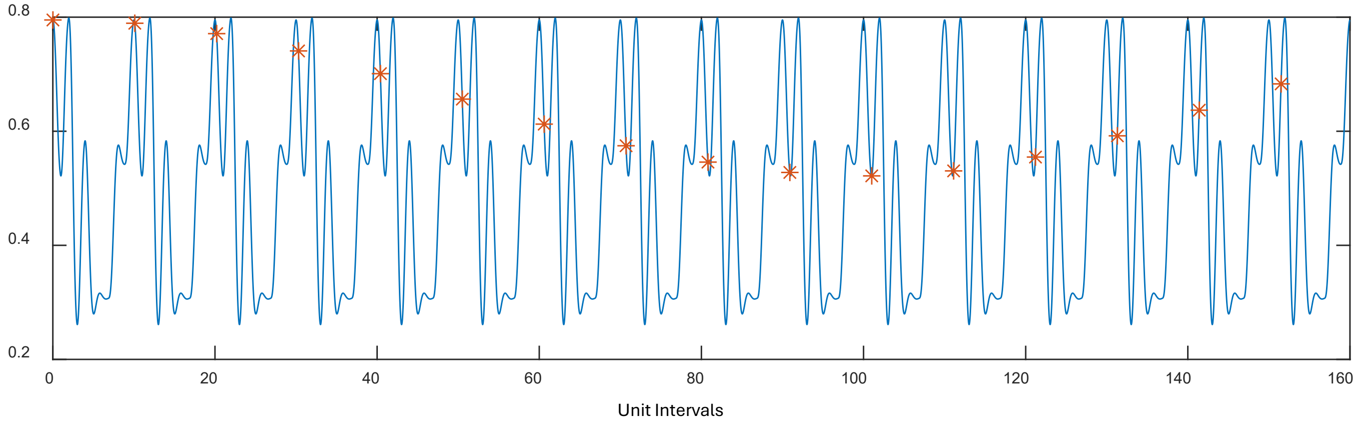
Comments addressed: 264, 267, 284

Sampling Scope Operation

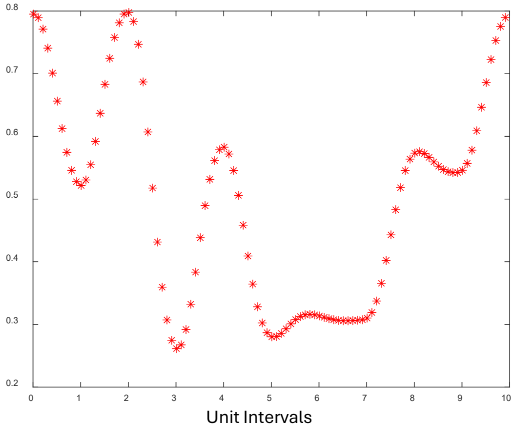
- Sampling scope works with repeating waveform
- Waveform is sampled at a relatively low rate compared to the baud rate of the waveform
- Collects samples at (for example) each repetition of the pattern at a different phase
- Reconstructs waveform by putting samples together
  - Effective sampling rate is determined by spacing of reconstructed samples, much greater than 2 x Nyquist
- Because sampling is not real time, the statistics of the noise change
  - Noise is uncorrelated sample to sample



Example: 10UI PAM4 Waveform



16 Repetitions of Waveform, 1 Sample per Repetition



Reconstructed from 160 Repetitions

Sampler in Scope ≠ Sampler in Reference Equalizer

- The sampler described on the preceding page is not the sampler in the reference equalizer in Figure 180-10
- The reference equalizer is a T-spaced discrete time equalizer that samples once per symbol period
  - This is a model that stands in for equalizers realized in practice
- The scope samples the waveform and the samples are used to calculate what the output of the reference equalizer would be

Addresses Comment R1-284

Interpolation

- Because the effective sampling rate is so high (compared to Nyquist), interpolation is not a problem to get a sample of the deterministic signal at an arbitrary phase
  - Can use sinc interpolation, spline interpolation, or simple linear interpolation
- However, the noise statistics will be affected by the interpolation
- BUT – the noise statistics have ALREADY been modified by the sampling process

Sampling Scope Effect on Noise

- The reference receiver includes a 4-pole Bessel Thomson filter with 3-dB bandwidth of 1/2T, where T is the symbol period
- The transmitter noise and the scope noise can be represented as white Gaussian noise filtered by the BT filter
- After a few symbol periods, the autocorrelation of the BT filter output goes to zero*
- Since the sampling period of the sampling scope is much greater than a few symbol periods, the noise on the waveform is uncorrelated (white) Gaussian noise
- To accurately model noise on the captured waveform, one must measure the variance of the apparent white noise, remove the white noise, then add back noise of the correct variance filtered by the BT filter

* See contribution: N. Swenson, *Analytical Derivation: Noise Autocorrelation of a 4-Pole Bessel-Thomson Filter*, 2 July 2026; Appendix 2 of companion presentation Swenson_01.

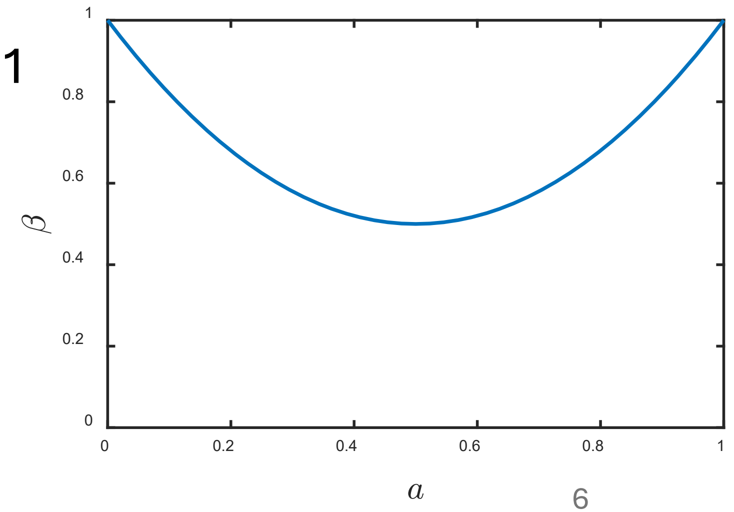
Interpolation Effect on Noise

- As described above, the noise on the waveform captured by the sampling scope appears white (uncorrelated from sample to sample)
- Interpolation is a linear filtering operation that will reduce the noise variance
- Example: linear interpolation
  - y_1+n_1 at time t_1 and y_2+n_2 at time t_2 , where n_1 and n_2 are zero-mean uncorrelated Gaussian noise samples, each of variance σ^2 .
  - Desire sample y_ϕ at time $t_\phi = t_1 + a(t_2 - t_1) = t_1 (1-a) + a\, t_2$, $0 \le a \le 1$

$$y_\phi = y_1(1-a) + y_2 a \qquad\qquad n_\phi = n_1(1-a) + n_2 a$$

$$E\left[n_\phi^2\right] = \sigma^2(1-a)^2 + \sigma^2 a^2 = \sigma^2\beta$$

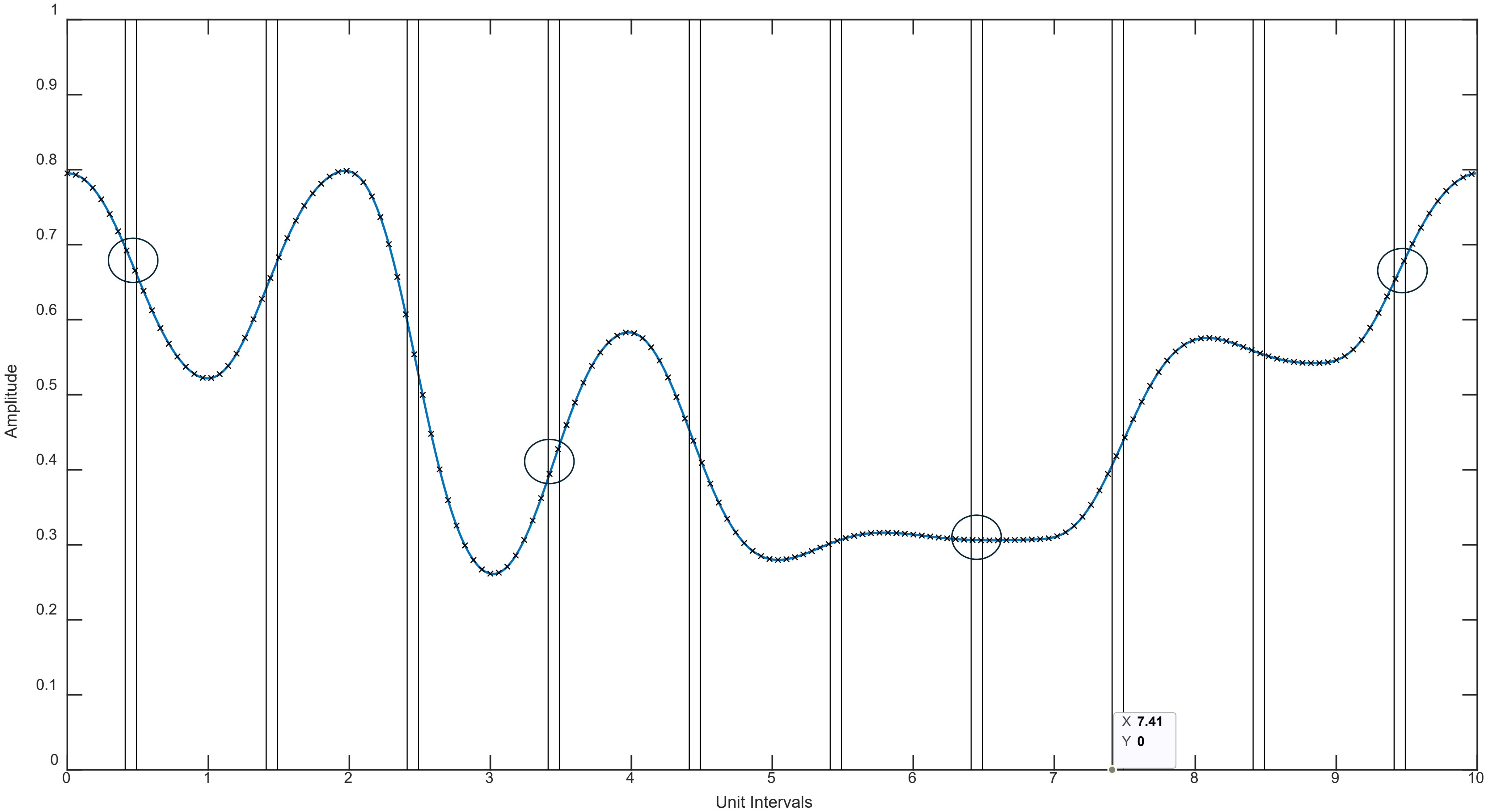
where $\beta = (1 - 2a + 2a^2)$ $\qquad$ $1/2 \le \beta \le 1$



Why We Want to Interpolate

- If we don’t interpolate and use finite width histogram bins, we risk missing samples for some symbols or having multiple samples for certain symbols
- Suppose the histogram slice width is T/M. (Currently, M=25.) If the effective sample rate is:
  - Greater than M/T, some slices will capture multiple samples for a given symbol
  - Less than M/T, some slices will have no samples for some symbols
  - Equal to M/T, then there is no need to have a finite width; you get one sample of each symbol in the slice at the same phase

Example: Effective rate > M/T



Noise for Equalizer Optimization and SER Estimation should be Treated Differently

- The y_n samples used for MMSE tap optimization should be as accurate as possible to find the best tap values
  - Recommend using scope averaging for this collection
  - Recommend interpolation to achieve value at ϕ_0
- The samples for the histogram should include transmitter noise
  - A finite width histogram window results in oversampling or undersampling certain symbols in the test pattern, skewing the histogram
  - Interpolation for a zero-width window will reduce the noise variance on some samples, but this can be compensated for
  - Noise whitening due to sampling scope must be compensated for anyway

Summary and Recommendations

- Specify that the histograms are collected at $\phi_0 \pm .05\ T/2$
- Properly account for noise
  - Account for the effects of interpolation, if used
  - De-whiten the noise captured by the sampling scope
  - Details can be added, or we could simply load it into a statement as we do presently:

    "If an equivalent-time sampling oscilloscope is used, the impact of the sampling process and the reference equalizer on transmitter noise has to be compensated for, so that the correct magnitude of noise is present at the output of the equalizer."
  - We could add a sentence:

    "Similarly, if interpolation is used, the impact of the interpolation process on the statistics of the transmitter noise must also be compensated for so that the correct magnitude of noise is present at the output of the equalizer."
- If zero-width histogram window is not accepted, we should add a qualifier that each symbol in the test pattern is represented once for each collection of the test pattern sequence.

Thank you