Partial-Response MMSE for MLSD: Targeted Updates to Annex 178A for 400 Gbps

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IEEE P802.3dv Task Force Meeting November 2026 Bangkok, Thailand

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Abstract

This contribution examines targeted changes to Annex 178A to accommodate partial-response equalization for MLSD with precursor and postcursor components.

The existing Annex 178A formulation has much of the required framework.

The intent is a basis for discussion and consensus building in IEEE P802.3dv and seeking feedback on technical sufficiency and suitability for inclusion in the baseline

*MMSE Minimum Mean Squared Error
*MLSD Maximum Likelihood Sequence Detector

Extend 178A.1.10 MLSD to evaluate the generalized PR response

- Explore partial-response equalization $\alpha_{-1}D^{-1} + 1 + \alpha_1 D^1$
- Replace DFE syntax with partial response (PR)
- Don't force MMSE to eliminate useful precursor energy.
- Allow MMSE to optimize a precursor + cursor + postcursor PR target.
- This can improve the equalized signal before MLSD is considered.
- MLSD enables the PR solution but is not the source of the MMSE benefit.
- Apply generalized partial-response cursor locations to MMSE in **Annex 178A.1.8.1** *Receiver discrete-time equalizer coefficients.*
  - Only minor changes in syntax required.
- Use Partial Response Equalization Maximum-Likelihood Sequence Detection based on MMSE and **178A.1.10** *Maximum likelihood sequence detection.*
  - Estimated approach with small changes to 178A.1.10

Overview of proposed Annex 178A edits

- Replace DFE syntax with a generalized partial response (PR) target (precursor + cursor + postcursor) in 178A.1.8.1
- Update the reference receiver block diagram: PR filter, COM_{DFE} becomes COM_{PR}
- Generalize equations and indexes in 178A.1.10 (MLSD) and 178A.1.10.2 (sequence noise distribution); add index k for expanded cursors
- Basic MMSE matrix equation unchanged; only minor syntax changes required

Replace generalize feedback filter in partial response in receiver block diagram

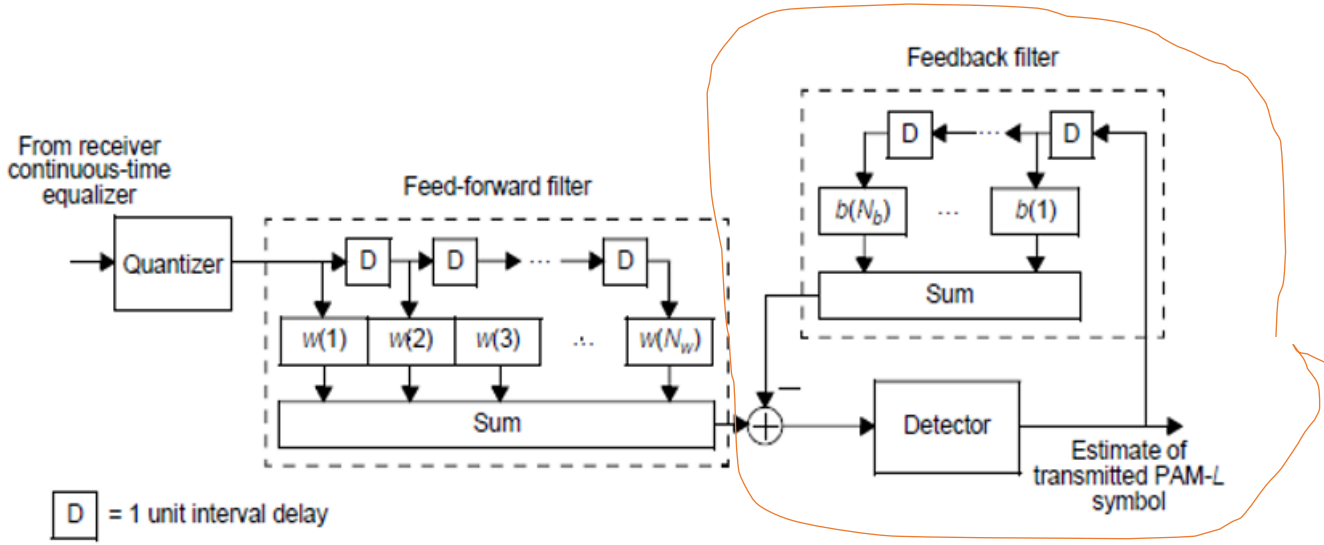
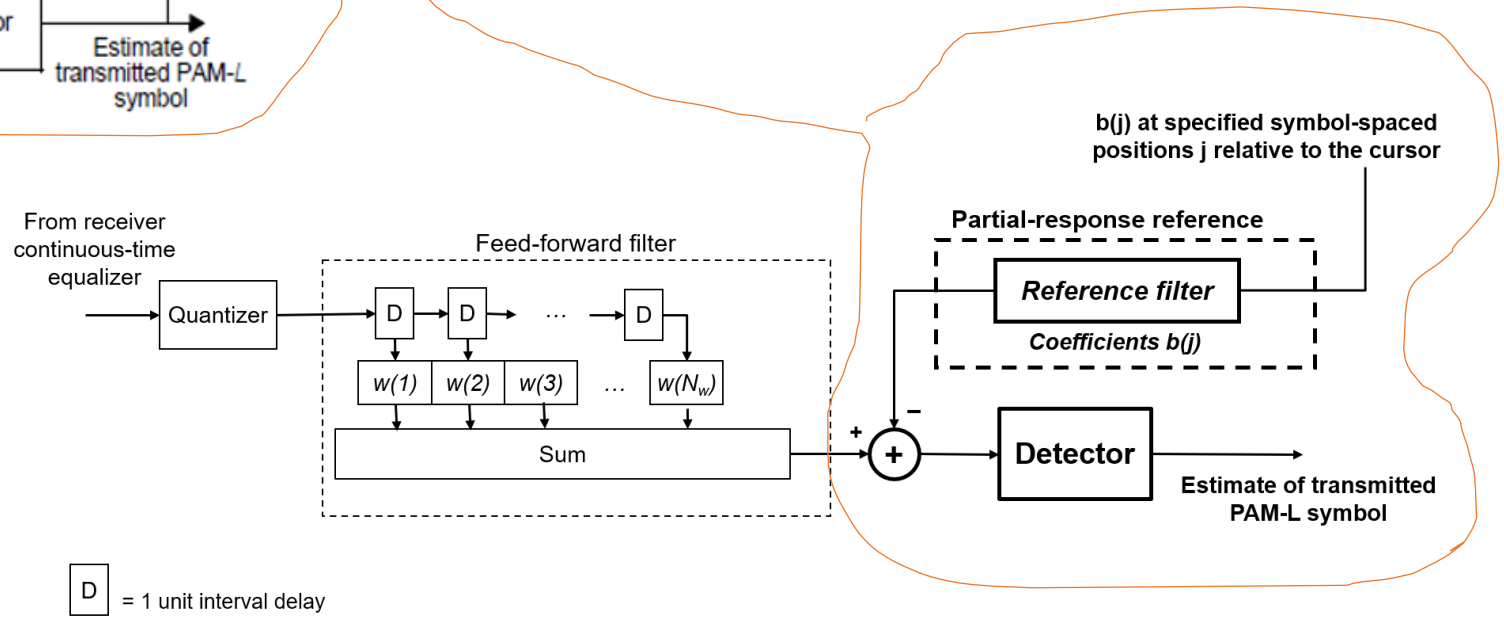


Figure 178A–10—Receiver discrete-time equalizer



Number pre-cursor coefficients, N_{bn}, added table

Table 178A–10—Summary of receiver discrete-time equalizer parameters

Parameter	Symbol	Units
Number of pre-cursor taps	d_w	—
Number of fixed-position taps	N_{fix}	—
Number of floating tap groups	N_{wg}	—
Number of taps per floating tap group	N_{wf}	—
Highest allowed tap index	N_{max}	—
Normalized upper limit on feed-forward coefficient $w(j)$	$w_{max}(j)$	—
Normalized lower limit on feed-forward coefficient $w(j)$	$w_{min}(j)$	—
Number of feedback taps	N_b	—
Normalized upper limit on feedback coefficient $b(j)$	$b_{max}(j)$	—
Normalized lower limit on feedback coefficient $b(j)$	$b_{min}(j)$	—

Table 178A_10 -- Summary of receiver discrete-time equalizer parameters

Parameter	Symbol	Units
Number of pre-cursor taps	d_w	
Number of fixed-position taps	N_{fix}	
Number of floating tap groups	N_{wg}	
Number of taps per floating tap group	N_{wf}	
Highest allowed tap index	N_{max}	
Normalized upper limit on feed-forward coefficient $w(j)$, where j is the feed-forward tap index	$w_{max}(j)$	
Normalized lower limit on feed-forward coefficient $w(j)$ where j is the feed-forward tap index	$w_{min}(j)$	
Number of positive partial-response coefficients	N_b	
Number of negative partial-response coefficients	N_{bn}	
Normalized upper limit on partial-response coefficient $b(j)$, where j is the partial-response coefficient index is relative to the cursor	$b_{max}(j)$	
Normalized lower limit on partial-response coefficient $b(j)$ where j is the partial-response coefficient index is relative to the cursor	$b_{min}(j)$	

Negative indexes are included for PR coefficients

No change to the basic matrix equation

$$\begin{bmatrix} w \\ b \\ \lambda \end{bmatrix} = \begin{bmatrix} R & -H_b^T & -h_0^T \\ -H_b & I_b & z_b^T \\ h_0 & z_b & 0 \end{bmatrix}^{-1} \begin{bmatrix} h_0^T \\ z_b^T \\ 1 \end{bmatrix} \quad (178A–33)$$

where

R	is the $N_w \times N_w$ matrix $H^T H + R_{nn}/\sigma_X^2$ and H^T is the matrix transpose of H
h_0	is row $d+1$ from H
H_b	is the $N_b \times N_w$ matrix formed from rows $d+2$ to $d+N_b+1$ from H
I_b	is a $N_b \times N_b$ identity matrix
z_b	is a $1 \times N_b$ vector of zeros

- No change to b_{lim}, w_{lim} , $\boldsymbol{\sigma_e^2}$, and COM computation processing

Refinement of H_b, I_b, and z_b

- $N_b + N_{bn}$ replaces N_b in H_b, I_b, and z_b.

$$\begin{bmatrix} w \\ b \\ \lambda \end{bmatrix} = \begin{bmatrix} R & -H_b^T & -h_0^T \\ -H_b & I_b & z_b^T \\ h_0 & z_b & 0 \end{bmatrix}^{-1} \begin{bmatrix} h_0^T \\ z_b^T \\ 1 \end{bmatrix} \quad (178A\text{-}33)$$

Where:

Symbol	Definition
R	is the $N_w \times N_w$ matrix $H^T H + R_{nn}$, where H^T is the transpose of H.
h_0	is row $d+1$ of H.
H_b	is the $(N_b + N_{bn}) \times N_w$ matrix formed from the rows of H at the partial-response coefficient indices k, where row $d+1+j$ corresponds to partial-response coefficient $b(j)$.
I_b	is the $(N_b + N_{bn}) \times (N_b + N_{bn})$ identity matrix.
z_b	is the $1 \times (N_b + N_{bn})$ vector of zeros.

Change COM_{DFE} to COM_{PR}

Syntax change accommodates partial response (PR)

$$COM = COM_{\mathrm{DFE}} + \underbrace{20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{\mathrm{MLSD}})}{A_s}\right]}_{\boldsymbol{\Delta COM}}$$

$$COM = COM_{\mathrm{PR}} + 20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{\mathrm{MLSD}})}{A_s}\right]$$

"*178A.1.10 Maximum likelihood sequence detection*" adjusted for partial response

The receiver discrete-time equalizer coefficients are determined using the procedure defined in 178A.1.8.1 using the parameters defined in Table 178A–10 but with the value of N_b set to 1. COM is then computed as defined in 178A.1.9 and the resulting value is labeled COM_{DFE}. The value of COM_{DFE} and the feedback filter coefficient $b_{lim}(1)$, along with the corresponding noise and residual intersymbol interference computed at the output of the feed-forward filter, are used to calculate a modification to COM_{DFE} that represents the advantage the MLSD-based receiver has over the DFE-based receiver. This modification is defined by Equation (178A–49).

$$COM = COM_{DFE} + 20\log_{10}(-P_{an}^{-1}(DER_{MLSD})/A_s) \quad (178A–49)$$

where $P_{an}(y)$ is defined in 178A.1.10.1 and DER_{MLSD} is defined by Equation (178A–50).

$$DER_{MLSD} = \sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j-1} P_j\left(-A_s\frac{(u_j^T u_j)^{3/2}}{(u_j^T V_j u_j)^{1/2}}\right) \quad (178A–50)$$

where

$P_j(y)$	is defined in 178A.1.10.2
u_j	is a $(j+1)\times 1$ vector whose elements are defined by Equation (178A–51)
V_j	is defined in 178A.1.10.3

$$u_j(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1-b_{lim}(1)) & 2 \le i \le j \\ (-1)^i b_{lim}(1) & i = j+1 \end{cases} \quad (178A–51)$$

If the value of COM calculated by Equation (178A–49) is less than COM_{DFE}, then the value of COM is set to be equal to COM_{DFE}.

The receiver discrete-time equalizer coefficients are determined using the procedure defined in 178A.1.8.1 and the parameters defined in Table 178A-10. Since b is partial response $1 \times (N_b + N_{bn})$ row vector coefficients, $k \in 1 \text{ to } (N_b + N_{bn})$;

COM is then computed as defined in 178 A.1.9 and the resulting value is labeled COM_{PR}. The value of COM_{PR} and the partial response filter coefficient $b_{lim}(k)$, along with the corresponding noise and residual inter symbol interference computed at the output of the feed-forward filter, are used to calculate a modification to COM_{PR} that represents the advantage the MLSD-based receiver has over the PR-based receiver. This modification is defined by Equation (178A-49).

$$COM = COM_{PR} + 20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{MLSD})}{A_s}\right] \quad (178A-49)$$

where P_{an} is defined in 178A.1.10.1 and DER_{MLSD} is defined by Equation (178A-50).

$$DER_{MLSD} = \sum_{k=1}^{N_{bn}+N_b}\sum_{j}^{\infty}\left(\frac{L-1}{L}\right)^{j-1} P_{j,k}\left(-A_s\frac{(u_{j,k}^T u_{j,k})^{3/2}}{(u_{j,k}^T V_j u_{j,k})^{1/2}}\right) \quad (178A-50)$$

Where:

$P_{j,k}$ is defined in 178A.1.10.2

$u_{j,k}$ is a (j + 1) × k vector whose elements are defined by Equation (178A-51)

V_j is defined in 178A.1.10.3

$$u_{j,k}(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1-b_{lim}(k)) & 2 \le i \le j \\ (-1)^j b_{lim}(k) & i = j+1 \end{cases} \quad (178A-51)$$

DER_{MLSD} is formed by summing the contributions of all partial-response coefficients k = 1 … N_b + N_{bn} and all sequence lengths j = 1 … j_{max} prior to the conversion to COM in Equation (178A-49). The index j is evaluated up to $j_{max} = \lfloor N_{ui}/2 \rfloor$, and the summation for each k may be terminated earlier when the contribution for the current j is less than 0.0001 times the accumulated sum.

If the value of COM calculated by Equation (178A-49) is less than COM_{PR}, then the value of COM is set to be equal to COM_{PR}.

Closer look: Index *k* added for expanded partial response cursors

where P_{an} is defined in 178A.1.10.1 and DER_{MLSD} is defined by Equation (178A-50).

$$DER_{\mathrm{MLSD}} = \sum_{k=1}^{N_{bn}+N_b} \sum_{j}^{\infty} \left(\frac{L-1}{L}\right)^{j-1} P_{j,k} \left(-A_s \frac{\left(u_{j,k}^T u_{j,k}\right)^{3/2}}{\left(u_{j,k}^T V_j u_{j,k}\right)^{1/2}} \right) \qquad (178A\text{-}50)$$

Where:

$P_{j,k}$ is defined in 178A.1.10.2

$u_{j,k}$ is a (j + 1) × k vector whose elements are defined by Equation (178A-51)

V_j is defined in 178A.1.10.3

$$u_{j,k}(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1 - b_{\mathrm{lim}}(k)) & 2 \le i \le j \\ (-1)^{j} b_{lim}(k) & i = j + 1 \end{cases} \qquad (178A\text{-}51)$$

DER_{MLSD} is formed by summing the contributions of all partial-response coefficients k = 1 … N_b + N_{bn} and all sequence lengths j = 1 … j_{max}, prior to the conversion to COM in Equation (178A-49). The index j is evaluated up to $j_{max} = \lfloor N_{ui}/2 \rfloor$, and the summation for each k may be terminated earlier when the contribution for the current j is less than 0.0001 times the accumulated sum.

If the value of COM calculated by Equation (178A-49) is less than COM_{PR}, then the value of COM is set to be equal to COM_{PR}.

- Index k added for expanded partial response cursors
- $k = N_b + N_{bn}$ and is positive because of the way N_b in H_b, I_b, and z_b *are determined*
- An extra sum is added to account for summing DERs as an estimate for the added cursors
- The MLSD error rate, DER_{MLSD} , is obtained by summing all contributions for all k and all j prior to conversion to COM

"*178A.1.10.2 Sequence noise distribution function* augmented for partial responses

178A.1.10.2 Sequence noise distribution function

The probability distribution function of the noise corresponding to a sequence of length j is denoted $p_j(y)$. It is initialized by the assignment defined in Equation (178A–54).

$$p_j(y) = \text{conv}\left[p_{an}(y), \frac{1}{|b_{lim}(1)|} p_{an}\left(\frac{y}{b_{lim}(1)}\right)\right] \quad (178A–54)$$

where $p_{an}(y)$ defined in 178A.1.10.1 and conv[u, v] denotes the convolution of u and v.

It is iteratively updated by repeating the assignment defined by Equation (178A–55) $j - 1$ times.

$$p_j(y) = \text{conv}\left[p_j(y), \frac{1}{|1 - b_{lim}(1)|} p_{an}\left(\frac{y}{1 - b_{lim}(1)}\right)\right]$$

The cumulative distribution function of the noise corresponding to a sequence of leng computed using Equation (178A–40) substituting $p_j(y)$ for $p(y)$.

Subscript k added

$$b_{lim}(1) \rightarrow b_{lim}(k)$$

178A.1.10.2 Sequence noise distribution function

The probability distribution function of the noise corresponding to a sequence of length j is denoted $p_{j,k}(y)$. It is initialized by the assignment defined in Equation (178A-54).

$$p_{j,k}(y) = \text{conv}\left[p_{an}, (y) \frac{1}{|b_{\lim}(k)|} p_{an}\left(\frac{y}{b_{\lim}(k)}\right)\right] \quad (178A-54)$$

where $p_{an}(y)$ defined in 178A.1.10.1 and conv[u, v] denotes the convolution of u and v.

It is iteratively updated by repeating the assignment defined by Equation (178A-55) (j-1) times for each k.

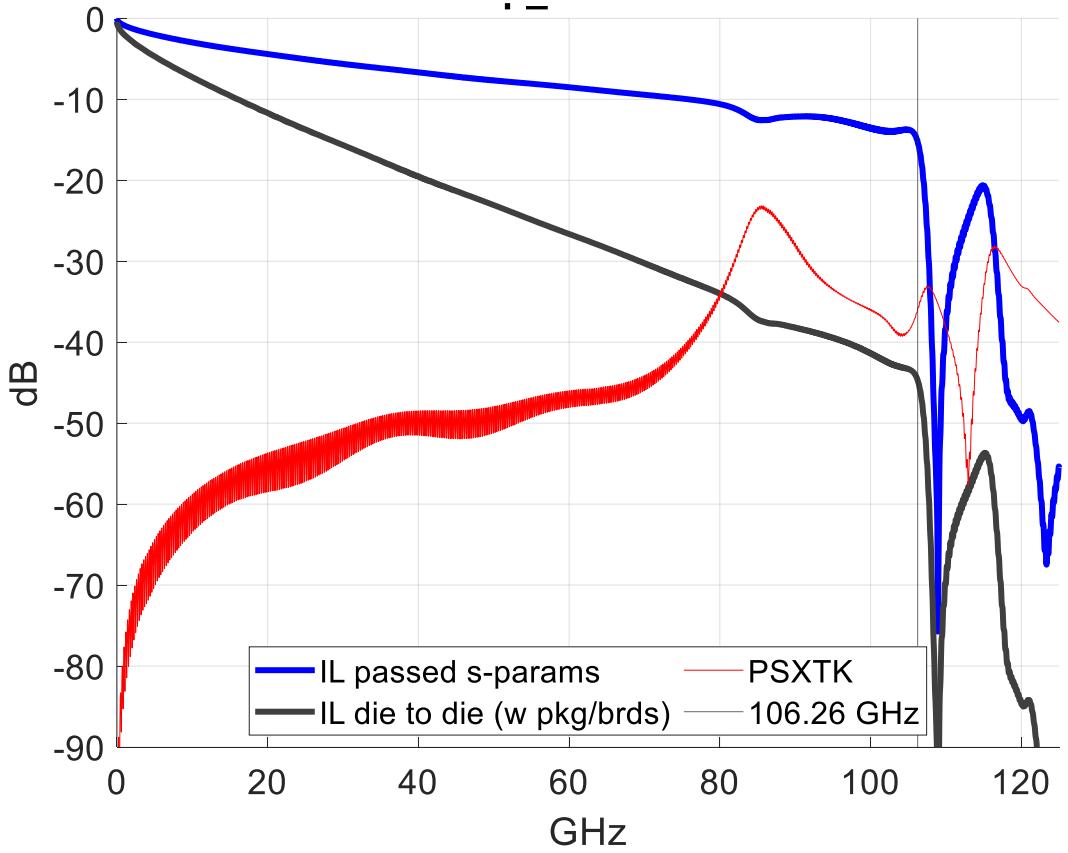
$$p_{j,k}(y) = \text{conv}\left[p_{j,k}, \frac{1}{|1-b_{\lim}(k)|} p_{an}\left(\frac{y}{1-b_{\lim}(k)}\right)\right] \quad (178A_55)$$

The cumulative distribution function of the noise corresponding to a sequence of length j, $p_{j,k}(y)$, is then computed using Equation (178A-40) substituting $p_{j,k}(y)$ for $p(y)$.

PR{1,1} improved COM from −0.8 dB to 5.5 dB on this 44 dB loss channel.

2 test cases

1. COM = −0.8 dB
   DFE1/FFE{8,6} float 2x4 case
   Equivalent syntax PR{0,1}/FFE{8,6}
2. COM = 5.3 dB
   PR{1,1}/FFE{8,6} float 2x4 case
   ΔCOM was 0.6 dB



This CPC to CPC channel has an IL of 44 dB at 106.25GHz with Tx/Rx 45 mm Packages (B).

gore_e4ai_02a_250529\C2C_CPC_EBW_l00mm_thru.s4p

COM test code is in branch MLSD_HO

COM configuration sheet and sweep table

Values and settings in the spread sheet may not be align with a standard. Please check before using

Table 93A-1 parameters			
Parameter	**Setting**	**Units**	**Information**
data rate	425	Gbit	
L	4	levels	
f_b	212.50	#VALUE!	#NUM!
f_min	0.05	GHz	
Delta_f	0.01	GHz	
R_0	46.25	s	
PKG_NAME	PKG_HiR_CLASSB PKG_HiR_CLASSB		TX RX
z_p select	3		
M	32		
C_d	[0.4e-4 0.9e-4 1.1e-4 ;0.4e-4 0.9e-4 1.1e-4]/1.4	nF	[TX RX]
L_s	[0.13 0.15 0.14; 0.13 0.15 0.14]/1.4	nH	[TX RX]
C_b	[0.3e-4 0.3e-4]/1.4	nF	[TX RX]
filter and Eq			
f_r	0.55	*fb	116.88GHz
c(0)	0.55		[min:step:max]
c(-1)	[-0.34:0.02:0]	[-0.34:0.02:0]	see if 0 is OK
c(-2)	[0:0.02:0.14]	[0:0.02:0.14]	see if 0 is OK
c(1)	[-0.2:0.02:0]	[-0.2:0.02:0]	see if 0 is OK
N_b	1	UI	
N_bn	0	1	2
bmax	[0.85]	[.25 0.85]	[.8 .9 .85 .5]
bmin	[0]	[0 0]	[0 0 0 0]
g_DC	[-20:1-0]	[-20:1-0]	
f_z	85.00	GHz	
f_p1	85.00	GHz	
f_p2	212.50	GHz	
g_DC_HP	[-6:1:-0]		see if 0 is OK
f_HP_PZ	1.328125	GHz	

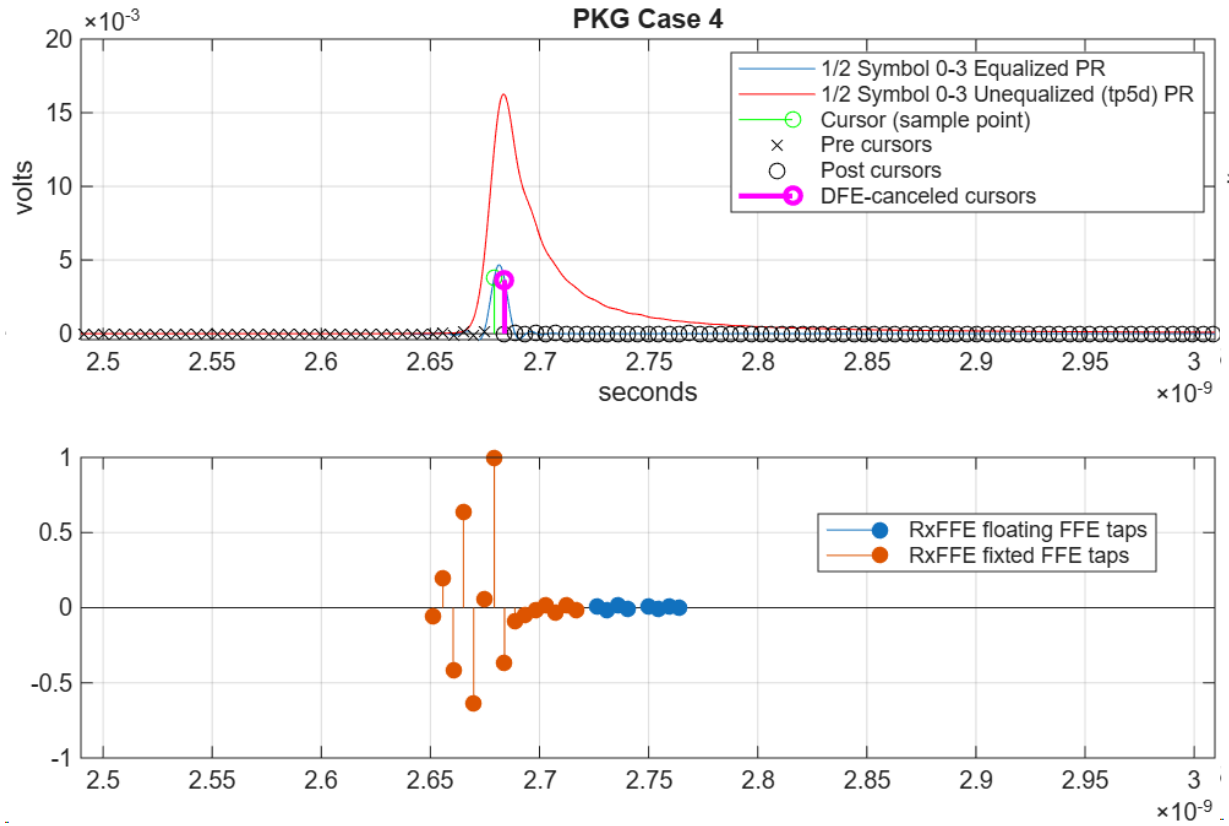
I/O control		
DIAGNOSTICS	1	logical
DISPLAY_WINDOW	1	logical
CSV_REPORT	0	logical
RESULT_DIR	.\results\KRCR_448G_{date}\	
SAVE_FIGURES	0	logical
Port Order	[1 3 2 4]	
RUNTAG	Test_448C2M_pam4_200G_stop_	
COM_CONTRIBUTION	0	logical
BREAD_CRUMBS	1	logical
no_com	0	logical
TDR and ERL options		
TDR	0	logical
ERL	0	logical
ERL_ONLY	0	logical
TR_TDR	0.01	ns
N	1600	logical
TDR_Butterworth	1	
beta_x	0	
rho_x	0.618	
TDR_W_TXPKG	0	UI
N_bx	0	
fixture delay time	[0 0]	
Tukey_Window	1	
Z_t	46.25	ohm
Noise, jitter		UI
sigma_RJ	0.01	UI
A_DD	0.02	V^2/GHz
eta_0	7.00E-09	dB
SNR_TX	33.5	
R_LM	0.95	
Txpskew	0	
Txnskew	0	
Rxpskew	0	
Rxnskew	0	

Floating Tap Control			
N_bg	2	0 1 2 or 3 groups	
N_bf	4	taps per group	
N_f	80	UI span for floating taps	
bmaxg	0.05	max DFE value for floating taps	
B_float_RSS_MAX	0.1	rss tail tap limit (not used)	
N_tail_start	9	(UI) start of tail taps limit	
Filter: Rx FFE			
ffe_pre_tap_len	6	UI	
ffe_post_tap_len	8	UI	
ffe_main_cursor_min	1		
ffe_pre_tap1_max	0.7	rss tail tap limit (not used)	
ffe_post_tap1_max	0.7		
ffe_tapn_max	0.7		
N_qb	0	like enob	
P_qc	1E-07		
Operational			
ERL Pass threshold	7.3	dB	
COM Pass threshold	3	db	
DER_0	2.00E-04		
T_r	0.00200	ns	
FORCE_TR	1	logical	
PMD_type	C2C		
EW	0		
MLSD	1	logical	
ts_anchor	1		
sample_adjustment	[-24 24]		
num_ui_RXFF_noise	2000	need enough UI to capture pulse delay dela	
zero_pad	1	recommended	only for
flim	200.0E+9	Hz	only for
zero_pad_tukey_window_in_fb	0.01	fb	only for
Impulse response truncation threshold	0.001	may speed up computation at the cost of r	
Local Search	2	use to get a handle on where the ts and ctl	
TS_SRCH_MODE	middle	not used if local search = 0	

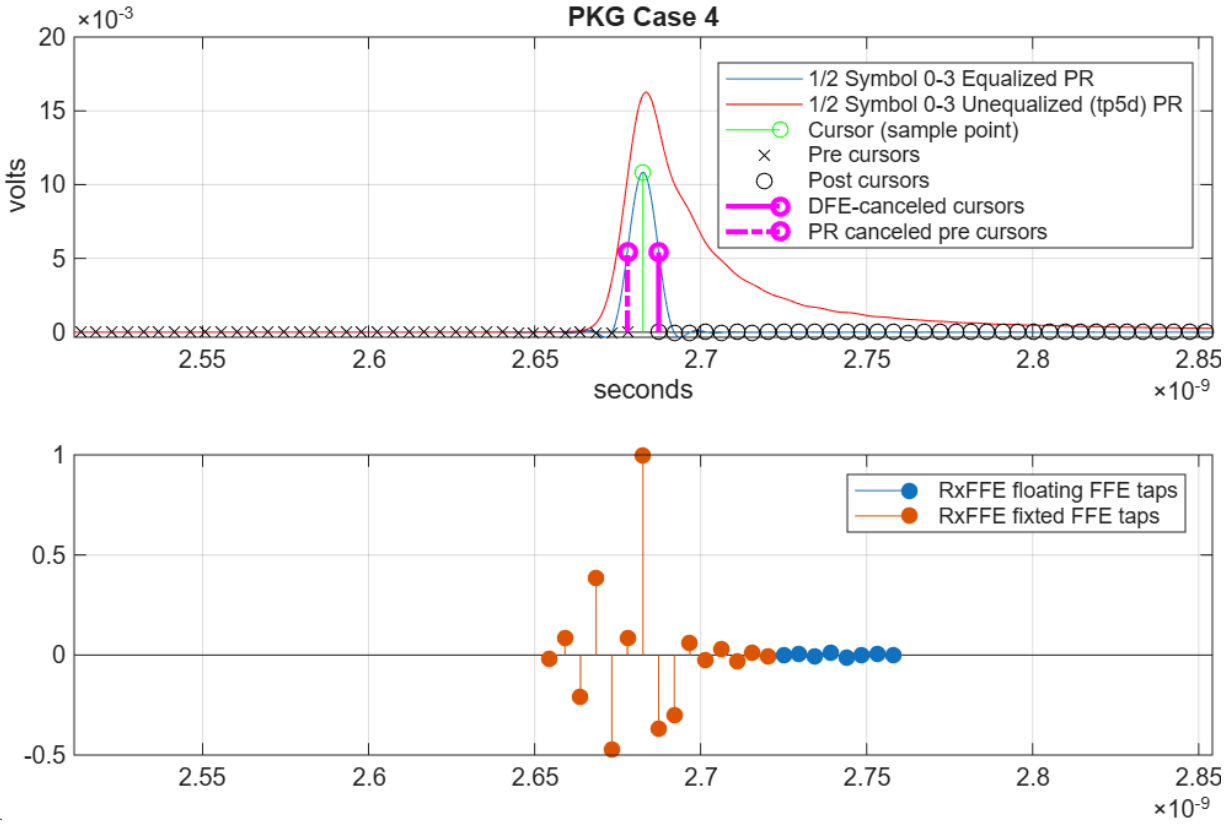
CASE	N_b	N_bn	bmax	bmin	N_qb
DFE	1	0	[0.85]	[0]	7
PR{1,1}	1	1	[0.5 0.5]	[0 0]	7

Partial response PR{1,1} equalized pulse response is larger than the DFE case

DF1/FFE{8,6} w float 2x4
b_{max} = 0.85



PR{1,1}/FFE{8,6} w float 2x4
b_{max}= [0.5 0.5]



Summary

- Results show improved COM for partial-response equalization $\alpha_{-1}D^{-1} + 1 + \alpha_1 D^1$
- Path from Annex 178A demonstrated
- Next steps
  - review
  - refinement
  - consensus

✓ COM test code is in branch MLSD_HO

Thank You!