Explanation of IEEE 802.3, Clause 68 TWDP

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This paper describes the TWDP test methodology used by the test procedure and algorithm specified in IEEE 802.3 Clause 68. The penalty is defined as the difference (in dB) between a reference signal to noise ratio (SNR) and the equivalent SNR at the slicer input of a reference decision feedback equalizer (DFE) receiver for the measured waveform after propagation through a simulated fiber channel.

Reference SNR

The reference SNR is the SNR of an ideal channel with a matched filter receiver. For the ideal channel, rectangular on-off keyed pulses are transmitted, and the transmitter and fiber are assumed perfect. The received pulse is a rectangular pulse with an amplitude measured in OMA of $\mathrm{OMA}_{\mathrm{RCV}}$ and time duration of one unit interval *T*. The receiver has a perfect matched filter front end matched to the rectangular receive pulse. The output of the matched filter is sampled once per unit interval (without timing error) and presented to the decision element (binary slicer).

The signal to noise at the slicer input is given by

$$\mathrm{SNR} = \mathrm{OMA}_{\mathrm{RCV}} \sqrt{T/\left(2N_0\right)} \tag{1}$$

where N_0 is the one-sided power spectral density of the additive white Gaussian noise assumed for the receiver. The bit error rate is given by

$$\mathrm{BER} = Q(\mathrm{SNR}) \tag{2}$$

where *Q*( ) is the Gaussian error probability function

$$Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{3}$$

6.5 dB has been assigned as budget to allow for optical power penalties related to dispersion. Therefore, the reference SNR, $\mathrm{SNR}_{\mathrm{REF}}$, is set 6.5 dB optical (13 dB electrical) above the level required to give a BER of 10^{-12} for the ideal channel. Hence

$$\mathrm{SNR}_{\mathrm{REF}} = 8.47 + 6.5 = 14.97\,\mathrm{dBo} \tag{4}$$

For a given $\mathrm{OMA}_{\mathrm{RCV}}$, $\mathrm{SNR}_{\mathrm{REF}}$ determines a particular N_0 through equation (1). Without loss of generality, $\mathrm{OMA}_{\mathrm{RCV}}$ is normalized to 1, and N_0 is set accordingly.

Waveform measurement and processing

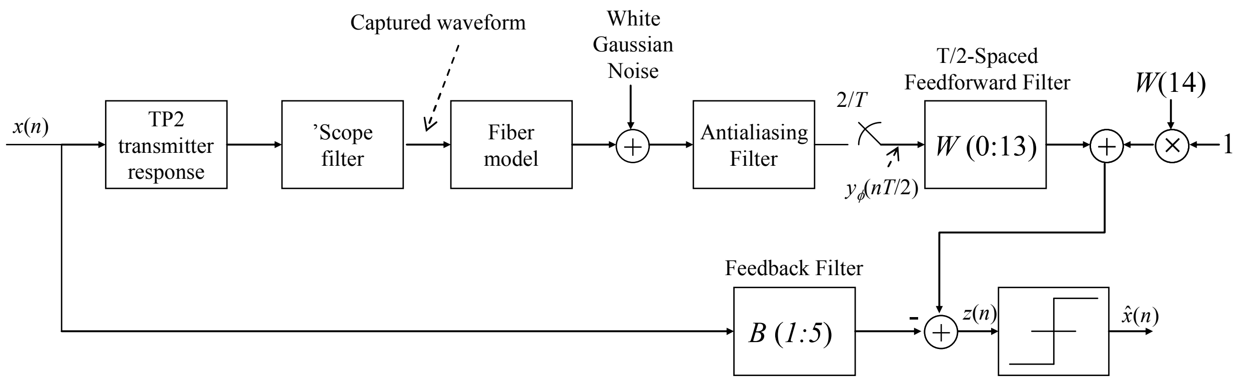

Figure 1 - Model for TWDP calculation

The TWDP is calculated using a model as shown in Figure 1. The transmitted waveform from the system under test (SUT) is captured with a sampling oscilloscope. The data sequence driving the SUT is a periodic PRBS9 or similar data pattern. The scope is set to capture the signal with at least seven samples per unit interval. The scope has a fourth-order Bessel-Thomson response with 3 dB electrical bandwidth of 7.5 GHz to filter the waveform. The scope is set to average out noise in the waveform.

The inputs to the algorithm are the following:

- One complete cycle of the captured waveform (re-sampled, if necessary) - e.g. one complete cycle of the waveform for a periodic PRBS9 input sequence. The re-sampled waveform has 16 samples per unit interval.

- One complete cycle of the data sequence used to generate the transmitted waveform. The data sequence and the captured waveform must be aligned (i.e., a rectangular pulse train based on the data sequence is aligned with the captured waveform within one unit interval). The transmitted sequence is denoted $\{x(n)\}$ and is periodic with period N (e.g., $N = 511$ for PRBS9).

The captured waveform is processed as follows:

1. The OMA and baseline (zero-level) of the captured waveform are estimated. The zero-level is subtracted from the waveform and the waveform is scaled such that the resulting OMA is 1. N_0 is set such that $\mathrm{SNR}_{\mathrm{REF}}$ is 14.97 dBo, as described in the previous section.

2. Three fiber channels are simulated, each corresponding to a defined stressor. The waveform is passed though each of these simulated channels to compute a "Trial TWDP" for each channel. The TWDP reported is the maximum of the three Trial TWDP values. The remainder of this description describes the processing done for each simulated fiber.

3. In the model of Figure 1, white Gaussian noise is shown added to the channel output and the sum is passed through an antialiasing filter. A fourth-order Butterworth filter of bandwidth 7.5 GHz is used as the antialiasing filter. The model in Figure 1 is useful for describing the TWDP calculation, but it does not depict the actual signal processing performed by the code. In the TWDP code, the signal (without added noise) is passed through the antialiasing filter to produce a deterministic output. The autocorrelation of the filtered noise is separately computed, and that autocorrelation is used to solve for the optimal equalizer coefficients and the equivalent SNR as described more fully below.

4. The antialiasing filter output is sampled at rate $2/T$ with phase ϕ. In the model, the input to the sampler consists of distorted signal plus noise. Denote the model output of the sampler at time $nT/2$ as $y_\phi(nT/2)$, which has a deterministic component and a random component given by

$$y_\phi(nT/2) = r(n) + \eta(n) \tag{5}$$

The deterministic sequence $\{r(n)\}$ is computed by the TWDP code. $\{r(n)\}$ is periodic and is the sampled version of the filtered output of the reference fiber. $\{x(n)\}$ repeats after N bits, therefore $\{r(n)\}$ repeats after $2N$ samples (two samples per bit). The sequence $\{\eta(n)\}$ is a discrete-time noise sequence which would result from passing the white Gaussian noise through the anti-aliasing filter and sampling.

5. The sampled signal is processed by a fractionally-spaced MMSE-DFE receiver with 14 feed-forward taps (at $T/2$ spacing) and 5 feedback taps. The feedforward filter is augmented with a 15th tap that adjusts to optimize a constant offset. The feedforward and feedback tap coefficients are calculated using a least-squares approach that minimizes the mean-squared error at the slicer input for the given captured waveform, assuming the white Gaussian noise has a noise power spectral density of N_0 as determined in the previous section.

   Figure 1 shows the channel and equalizer model used for the least-squares calculation. The reference DFE consists of a feedforward filter $\{W(0),\ldots,W(13)\}$, a constant offset coefficient $W(14)$, and a feedback filter $\{B(1),\ldots,B(5)\}$. A conventional DFE would feedback decisions $\{\hat{x}(n)\}$. In this case, we assume that decisions are correct and replace the decided bits with the transmitted bits $\{x(n)\}$.

   The feedback filter is symbol spaced and strictly causal, feeding back the five bits prior to the current bit $x(n)$. The input sequence on which the slicer makes decisions is denoted $\{z(n)\}$, where

$$\begin{aligned} z(n) &= \sum_{k=0}^{13} W(k) y_\phi(nT + DT - kT/2) + W(14) - \sum_{k=1}^{5} B(k)x(n-k) \\ &= \sum_{k=0}^{13} W(k)\big(r(2n+2D-k) + \eta(2n+2D-k)\big) + W(14) - \sum_{k=1}^{5} B(k)x(n-k) \end{aligned} \tag{6}$$

   In (6), D is an integer such that the number of anticausal $T/2$-spaced taps in the feedforward filter is $2D$. DT is referred to as the equalizer delay. The equalizer delay can also be expressed as the number of feedforward anticausal taps, $2D$.

The least-squares solution for the feedforward and feedback filters minimizes the quantity

$$\mathrm{MSE} = \mathrm{E}\left[\sum_{n=0}^{N-1}\left(z(n)-x(n)\right)^2\right] \tag{7}$$

where the expectation operator, E, refers to the random sequence generated by the additive white Gaussian noise, filtered through the antialiasing filter and feedforward filter. (Here MSE is actually N times the mean-squared error when averaged over the input sequence $\{x(n)\}$.) The optimal filter coefficients are computed using the deterministic sequence $\{r(n)\}$ and the autocorrelation of the random sequence $\{\eta(n)\}$. The algorithm finds the optimal W vector, B vector, and equalizer delay DT such that the mean-squared error is minimized in an efficient manner (see [1]). The optimal sampling phase ϕ for the $T/2$ sampler is found by a brute-force search over the 16 sampling phases in the unit interval.

6. Once the MSE is minimized by finding the optimal sampling phase, equalizer delay, and tap coefficients, the bit-error rate is calculated by the semi-analytic method, as follows:

   a. The Gaussian noise variance at the input to the slicer is calculated.

   b. For each bit in the data sequence, the equalized signal at the slicer input is calculated and the probability of error is determined based on the amplitude of the equalized signal, the variance of the noise at the slicer input, and the value of the slicer threshold.

   c. The probabilities of error are averaged over all slicer inputs (bits) in the sequence to compute a total probability of error $\mathrm{BER}_{\mathrm{SUT}}$.

7. The equivalent SNR in optical dB is deduced from the $\mathrm{BER}_{\mathrm{SUT}}$ as follows:

$$\mathrm{SNR}_{\mathrm{EQUIV}} = 10\log_{10}\left(Q^{-1}(\mathrm{BER}_{\mathrm{SUT}})\right) \tag{8}$$

8. The Trial TWDP for each simulated fiber is equal to the difference (in optical dB) between the reference SNR and the equivalent SNR, i.e.

$$\text{Trial TWDP} = \mathrm{SNR}_{\mathrm{REF}} - \mathrm{SNR}_{\mathrm{EQUIV}} \text{ (dBo)} \tag{9}$$

The TWDP reported is the maximum of the three Trial TWDP values.

References

[1] P. A. Voois, I. Lee, and J. M. Cioffi, "The effect of decision delay in finite-length decision feedback equalization," *IEEE Transactions on Information Theory,* vol. 53, pp. 618-621, Mar. 1996.